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NEW ZERO CROSS CORRELATION CODES BASED ON ZECHMETHOD'S FOR OCDMA SYSTEMS

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ABSTRACT

A new structure code for Spectral Amplitude Coding Optical Code Division Multiple Access (SAC–OCDMA) system with zero cross-correlation (ZCC) is presented in this paper. The principle idea of this method is based on the use of Zech's logarithms in the construction of the code sequences to ensure orthogonality and compare their performances with previously reported codes. The maximum cross correlation of this code is zero signifying that Multi Access Interference (MAI) and Phase Induced intensity noise (PIIN) effects are completely eliminated thus the system performances are improved. The BER simulation results against the total number of active users show a significant improvement in performance of the proposed code over preceding reported codes. The system can accommodate more than 90 simultaneous users compared to others codes at a bit error rate of 10^{-9}. *Furthermore, the construction method offers a good flexibility in the choice of the number of users, the weight and the code length*

1. INTRODUCTION

Optical code-division multiple accesses (OCDMA) is one of the most important techniques to solve the problem of traffic growth and multiple user access on the internet. In this multiplexing technique, each user in the communication channel is allocated a distinguishable and solely optical code at the transmitter [1,2]. The optical bandwidth is shared by several users simultaneously in the same time slot and the same frequency so as to increase the transmission capacity of an optical fiber.

This technique has many advantages besides OTDMA and WDM as higher security, flexibility and simplicity of the network control [1]. However, the performance of the technique OCDMA is mainly dependent on a specifically codes used for encoding. A proper choice of code family and code length is of great importance dictated by different design considerations at the transmitter as well as at the receiver. An effective OCDMA system is based on the choice of efficient codes with good or at most zero correlation properties for encoding the source [3].

The performance of the OCDMA system is limited by shot noise, beat noise, thermal noise, a dark current, and a phase-induced intensity noise (PIIN) [4]. The most important noise is the so-called Multi-Access Interference (MAI) originating from the cross correlation existing among multiple active users on a common channel. As the number of active users increases, the BER performance degrades due to increase in MAI. Therefore, the MAI and PIIN effects can be reduced effectively using low cross correlation codes; which increases the SAC-OCDMA system performance. Thus, the design of the code sequence with respect to cross-correlation is the important property for reducing the MAI contribution to the total optical received power. High auto-correlation and zero cross-correlation properties are required in the design of ZCC code to improve the system performance [5].

In the literature, various code schemes have been proposed to overcome the MAI problem among multiple users for the SAC-OCDMA systems, such as M-sequence codes, optical orthogonal codes, prime codes, double weight codes, and random diagonal (RD) codes [1,8,9,10,11]. The construction of the code is limited by the code length parameters, as is the case for the modified quadratic congruence and Dynamic Cyclic shift codes. For the case of prime codes and random codes, cross-correlation increases with the weight number while the code length is large for KS and EDW codes. As a result, code design cannot be used for any number of simultaneously users or high data rates in the SAC-OCDMA systems.

In this paper, we proposed a new construction method of Zero Cross-Correlation ZCC codes for OCDMA systems. The new proposed method allows the number of users, weight and code length to be chosen independently. The code cardinality is increased and the cross-correlation equal to zero. The rest of the paper is organized as follows: we will first describe mathematically the construction method for the proposed ZCC code. Then, we will introduce the code comparison with others codes from the references and show the efficiency of our proposed code compared to the others codes such as MDW, Hadamard and ZCC [17, 18]. Next, we validate the performance of the proposed ZCC code by using simulations under Optisystem software. Finally, a conclusion is given to summarize our work.

CONSTRUCTION METHOD OF ZCC CODE

In this technique, A ZCC code sequence $\boldsymbol{S_i}$ can be constructed as follows: Let $\boldsymbol{\alpha}$ be a root of a $\boldsymbol{m^{th}}$ degree primitive polynomial $\boldsymbol{f(x) \in GF(p)[x]}$. All nonzero elements of $\boldsymbol{GF[p^m]}$ are successive powers of $\boldsymbol{\alpha}$ and the multiplicative order of $\boldsymbol{\alpha}$ is $\boldsymbol{ord[\alpha] = p^m - 1}$, such that : $\boldsymbol{\alpha \in GF[p^m] = \{1, \alpha, \alpha^2, \dots, \alpha^{p^m-2}\}}$, where $\boldsymbol{q = p^m}$ with $\boldsymbol{p}$ a prime and $\boldsymbol{m}$ an integer.

The Zech logarithm also referred to as Jacobi's logarithms is used in the finite field $\boldsymbol{GF[q]}$ [12]. The nonzero element $\boldsymbol{\beta}$ is defined as: $\boldsymbol{\beta = \gamma^i}$ where $\boldsymbol{0 \le i \le q-2}$

For $i \le j$, $\begin{cases} \gamma^i + \gamma^j = \gamma^i \gamma^{j-i} = \gamma^k \\ \text{where } k \equiv i + z_{j-i}\ (modq - 1) \end{cases}$ (1)

For each element z_j know as the Zech's logarithm, we calculate:

$$\gamma^{z_j} = 1 + \gamma^j \qquad (2)$$

We suppose that γ^i and γ^j and also their sum are both non zero. Then, we compute the Zech logarithm tothe base $\gamma = \alpha$. For the singular point when the result value is undetermined, we suppose that z_j is null ($z_j = 0$).

The polynomial representation for a finite field $GF[p^m]$ as coefficients in the finite field $GF(p)$. Clearly $GF[p^m]$ can thus be interpreted as a vector space over $GF(p)$. The set $\{1, \alpha, \dots, \alpha^{m-1}\}$ can be used as a basis for the vector space. based on these assumptions, we can determine the number of user's as:

$$K \leq \left\lfloor \frac{q-1}{w} \right\rfloor, \qquad (3)$$

Where $(q-1)$ denotes the ZCC code length and w its weight. When the elements z_j are determined, the set of $(q-1)$ elements are partitioned onto$K$ sub-sets. Each constructed sub-set's has w elements corresponding to a specific user. The ZCC sequence is constructed as:

$$S_i = \begin{cases} \left(z_0, z_1, \dots, z_{(i*w+k)}, \dots, z_{(i+1)*w-1}\right) \\ 0 \leq i \leq K-1 \; and \; 0 \leq k \leq w-1 \end{cases} \qquad (4)$$

Finally, the i^{th} binary code of the ZCC code with length $(q-1)$ is deduced by replacing each number element from the ZCC sequence by ''1''. The resulting $(K \times (q-1))$ matrix of the ZCC code is given as follows:

$$C_{ZCC} = \begin{bmatrix} c_{0,0} & \cdots & c_{0,(q-1)} \\ \vdots & \ddots & \vdots \\ c_{(K-1),0} & \cdots & C_{(K-1),q-1} \end{bmatrix}_{(K \times (q-1))} \qquad (5)$$

An example of ZCC code matrix with: $q = 8$, $w = 2$ and$K = \left\lfloor \frac{8-1}{2} \right\rfloor = 3$ is derived below:

Let $\alpha \in GF(2^3) \equiv GF(2)$ be a root of a primitive polynomial: $f(x) = x^3 + x + 1$. Since α is a root of $f(x)$, then $\alpha^3 + \alpha + 1 = 0$ which is equivalent to $\alpha^3 = \alpha + 1$ in the $GF(2)$ finite field. So, we can obtain all the polynomial representations of γ^i with $(\gamma = \alpha)$ as follows: 1, $\alpha, \alpha^2, \alpha + 1, \alpha + \alpha^2, \alpha^2 + \alpha + 1$ and $\alpha^2 + 1$.Table 1 show all the elements of the Galois field $GF(2^3)$generated by the polynomial$f(x) = x^3 + x + 1$. Examples of the product and sum of two elements in this field are calculated as follows:

$$\alpha^3 = \alpha + 1,$$

$$\alpha^4 = \alpha^3.\alpha = \alpha(\alpha + 1) = \alpha^2 + \alpha,$$

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$\alpha^5 = \alpha^4.\alpha = \alpha\,(\alpha^2+\alpha) = \alpha^3+\alpha^2 = \alpha^2+\alpha+1, \alpha^6 = \alpha^5.\alpha = \alpha\,(\alpha^2+\alpha+1) = \alpha^3 + \alpha^2+\alpha = \alpha+1+\alpha^2+\alpha = \alpha^2+1.$

Table 1: The Galois field $\mathbf{GF(2^3)}$ generated by $\mathbf{f(x) = x^3 + x + 1}$

i	0	1	2	3	4	5	6
γ^i	1	α	α^2	$\alpha+1$	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$	α^2+1

Then, the Zech logarithms to the base $\gamma = \alpha$ are the $z_j's$ elements: 0, 3, 6, 1, 5, 4 and 2. The vector of $z_j's$ elements is partitioned with respect to the number of users and weight leading to the ZCC sequences shown in Table 2. The undetermined value for j=0 is set equal to zero ($z_j = 0$).

Table: 2 Zech logarithms to the base $\boldsymbol{\gamma = \alpha}$

j	0	1	2	3	4	5	6
z_j	0	3	6	1	5	4	2

Finally, the pairs (0, 3), (6, 1), (5, 4) corresponding to the weight code of each user obtained from equation (3) are replaced by the ''1' in their positions as is shown in Table 3. As we can see, the number of users and weight are too related with the code length. If the number of users increases, the weight code is decreased. For the code example in Table 1, $K > w$.

Table 3: ZCC code obtained with $\mathbf{q = 8, K = 3, 2}$ and $\mathbf{w = 2}$, respectively.

C_{ZCC}	K	Code length L	w	Code matrix
Code ZCC	3	7	2	$\begin{bmatrix} 1\,0\,0\,1\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1\,1\,0 \end{bmatrix}$

For the first code in Table 1, $K > w$. It is possible to increase to further increase the number of users using the matrix mapper below:

$$ZCC(i=2) = \begin{bmatrix} Zeros(K \times L) & ZCC(1) \\ ZCC(1) & Zeros(K \times L) \end{bmatrix} \quad (6)$$

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$$ZCC(i=3) = \begin{bmatrix} Zeros(2K \times 2L) & ZCC(2) \\ ZCC(2) & Zeros(2K \times 2L) \end{bmatrix} \quad (7)$$

Where Kdenotes the number of users, $L = (q-1)$ is the code length and Zeros (.) denotes a matrix of zeros and $ZCC(1)$ is the basic matrix with the dimension$(K \times (q-1)) = K \times L$. From this mapping, the code length is increasing with the number of users. The number of rows in the new generated matrix determines the number of users as follows: $K_{Total} = K.2^{i-1}$ and the length code that represents the number of colons in the new generated matrix is given by: $L_{Total} = L.2^{i-1}$

3. CODE COMPARISON

In this section, a comparison is given to contrast the performance of the proposed ZCC code method and some other existing optical codes. All the mathematical relationships between parameter codes are shown in Table 4.

Table 4: Expressions parameters of the proposed code and some others optical codes

Codes	No. of users	Code length	λ
MDW [1]	n	$3K + 8/3\,[sin(K.\pi/3)]^2$	$\lambda = 1$
ZCC [13,14,18]	$(w+1)$	$w.(w+1)$	$\lambda = 0$
Hadamard [15]	$(2^i - 1)$	2^i	$\lambda = 1$
Proposed ZCC	$K.2^{i-1}$	$L.2^{i-1}$	$\lambda = 0$

From Table 4, we can conclude that our proposed code gives the minimum cross-correlation which is equal to zero with the ZCC code of reference [13, 14, 16].The others methods MDW and Walsh-Hadamard have a unity cross-correlation. However, using the same number of users as is shown in Table 5, we can conclude that the code length and weight of the ZCC code of references are much higher than the proposed one. Walsh-Hadamard gives the minimum code length and a high weight but a unity cross-correlation. Consequently, the proposed method gives the best cross-correlation value with more flexibility in the choice of increased number of users and the weight. The code length increase with the same factor of proportionality of the number of users but is still too short compared to others optical methods. The method of construction of the proposed ZCC code is less complicated and can be generated using pre stored tables. The code length of the proposed method is calculated using equation (3) for the basic code length of $L = q - 1 \geq K \times W$. Then, we find the minimum code length given within the set of all possibilities of the prime number to get the total number of users, in our case $K_{Total} = 30$. The result is the set of $i \in \{1, \ldots, 6\}$, which can be used to find K_{Total} and the minimum code length L_{Total} (in our case $L = 125$).

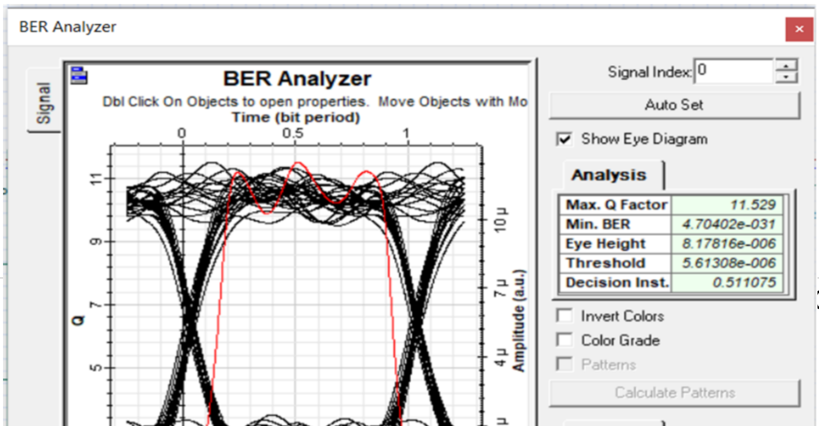
Table 5: The proposed code in comparison with some others optical codes for $\mathbf{K_{Tot} = 30}$

Codes	No. of users	Code length	Weight
MDW	30	90	$w = 4$
ZCC [13-14-16]	30	870	$w = 29$
Hadamard	30	32	$w = 16$
ZCC [17-18]	30	120	$w = 4$
Proposed ZCC	30	125	$w = 4$

4. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

The performance of the proposed ZCC code for OCDMA systems using direct detection is simulated in Opti System 7.0. In this simulation, a conventional signal mode fiber with reference wavelength = $\mathbf{1550\ nm}$ and attenuation of $\mathbf{0.2\ dB/km}$ is used. The chromatic dispersion parameter is $\mathbf{16.75\ ps/nm/\ km}$ and the polarization mode dispersion (PMD) coefficient is equal to $\mathbf{0.5\ ps/sqrt(k)}$. The tests were carried out atarate of $\mathbf{1\ GHz}$ for $\mathbf{50\ km}$ distance with standard single mode optical fiber. The wavelengths used are in the range of $\mathbf{1478.8\ nm}$ to $\mathbf{1485.2\ nm}$. The spectral width of each chip is0.8 nm. The performance of the system was described by the BER and the eye patterns analysis.

Fig.1 show the eye diagram and BER values of the proposed ZCC code in OCDMA system. The number of users was chosen to be equal to three and the code weight value $\boldsymbol{W = 4}$ at a data rate of $\mathbf{1\ GHz}$.



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From Fig. 1, the eyes diagram of the proposed Zech's ZCC code system gives a significant performance in terms of large eye opening which can be useful for practical BER applications. The height of the eye opening shows the noise margin or immunity to noise whiles the width, the difficulties to distinguish between ones and zeros in the signal. The BER values of $4.7 \times 10^{-31}\ dB$ was obtained for the code weight 4 and three users at a data rates of1 GHz. This BER value is much higher than the basic required 10^{-9} value for system performance. The corresponding fill factor (Q) is $11.529\ \ (> 6)$. Thus, the BER less than 10^{-9} or $(Q\ > 6)$ can also be obtained with more than three users or data rates less than 1GHz. Bit Error Rate (BER) using Gaussian approximation is given as [6]:

$$BER = \frac{1}{2}\sqrt{\frac{SNR}{8}} \quad (8)$$

For direct detection SAC-OCDMA systems, the SNR expression is given by:

$$SNR = \frac{\left(\frac{\mathcal{R}\ P_{sr} W}{L}\right)^2}{2\ eB\ \mathcal{R}\ P_{sr}(W/L\ ) + (4\ \frac{K_b T_n B}{R_L})} \quad (9)$$

Where $\mathcal{R}$: Responsivity of the photodiode, B : Electrical equivalent noise bandwidth receiver, K_b : Constant Boltzmann, R_L : Resistance load, T_b : Temperature of noise at the receiver, P_{rc} : Effective power at the receiver and e : Charge of Electron.

Performance comparison between MDW, Hadamard, ZCC [17] and our proposed ZCC codes using direct detection is illustrated in Fig. 2. The BER is evaluated taking into account thermal and shot noises only. BER against the total number of active users is simulated by Matlab software. Following Table 6 illustrates the parameters used in our analysis.

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Table 6: Table of parameters.

$\mathcal{R}$	Responsivity of the photodiode	1
B	Electrical equivalent noise bandwidth of the receiver	$311\ MHz$
K_b	Constant of the Boltzmann	$\lambda = 0$
R_L	Resistance load	$1030\ \Omega$
T_b	Temperature of noise at the receiver	$300\ K$
P_{rc}	Effective power at the receiver	$-10\ dBm$
e	Charge of Electron	$1.6\ 10^{-19}C$

Fig. 2 show the BER as a function of the number of active users when the effective receiver power is $P_{sr} = -10\ dBm$. As we can conclude from Fig. 2, our proposed ZCC construction method code gives the best performance and outperforms the others methods. 90 users can be supported with our method with approximately a BER of 9×10^{-10}. The ZCC method of reference [17] approaches our method for up to 120 simultaneous number of active users. These results are a consequence of the code cross correlation properties that eliminates the effect of MAI and PIIN that have an important impact in decreasing the system performance.

Fig. 3 shows the BER and the Fill Factor (Q) vs fiber length at different data rates, 1 GHz, 2 GHz and 3.5 GHz, respectively. As we can see, the BER value is increasing with the transmission distance. Moreover, at system performance BER 10^{-9}, the maximum transmission distance to be reached for the proposed scheme without amplification is approximately about of 66 Km at 1 GHz (exactly 65.7 Km with the corresponding values of BER and Q of $1.023 \times 10^{-9}\ dB$ and 5.99, respectively).

For user's data rate of 2 GHz, the maximum fiber distance is 62.5 Km with a Fill Factor Q value of 6 and a corresponding BER value of 9.64×10^{-10}. The maximum distance to be attained can be further reduced when the user data bit is increased. This later is about 57 Km for a user data bit of 3.5 GHz corresponding to a BER value of 4.195×10^{-10}. The large optical attenuation value of the longer fiber distance causes this reduction in performance.

As a result, the performance of the system is best with high BER values when the distance or user data rate used is well chosen. Using equation (10) below, the maximum fiber length is given as [19]:

$$P_{imax}(dB) = \alpha \times L + P_r(dB) \quad (10)$$

Where the maximum input power is denoted by P_{imax}, α denotes attenuation, P_r is the minimum receiver power and L denotes the transmission distance. For OCDMA system, the receiver dynamic range is between -7 dBm and -28 dBm [20]. From Fig. 3, the performance of

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the system is best with high BER values when the distance or user data rate used are well chosen.

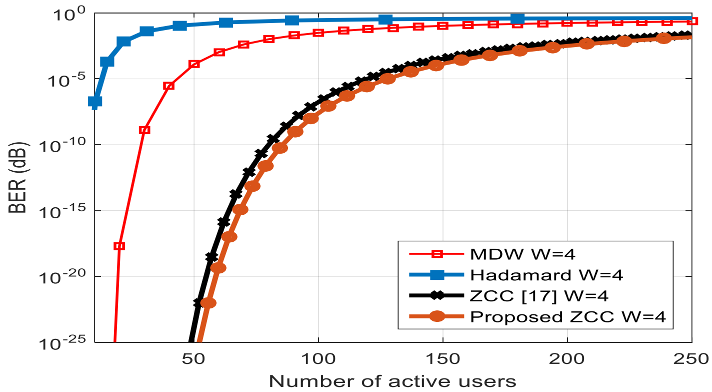

Figure 2. BER vs number of active users with $P_{sr} = -10$ dBm.

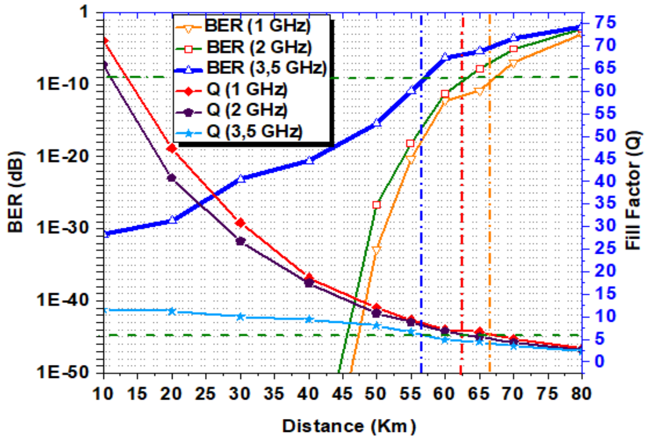

Figure 3. BER and Fill Factor (Q) versus fiber distance for the proposed Zech's ZCC code at different user's data rate 1GHz, 2 GHz

5. CONCLUSION

Minimum cross-correlation is one of the important properties in the code design. An efficient choice of ZCC code increases the system performance with simple code construction and high transmission quality. In this paper, we propose a new construction method of ZCC code for SAC-OCDMA systems. The key idea of this method is based on the use of Zech's logarithms in the construction of the codes and compares their performance with previously reported codes. Direct decoding technique is implemented at the receiver, which reduces the number of filters corresponding to one single branch of decoding.

Simulations and theoretical analysis show that our proposed construction code gives a significant improvement in system performances (BER, Q) as compared to other conventional coders. The minimum cross-correlation value (zero) not only eliminates efficiently the MAI and PIIN effects, but also improves the BER performance of the system. Furthermore, the proposed code can support more active users simultaneously at high data rates and longer distance with much more flexibility in the choice of number of users, weight and code length. The design of the code is made simple using a pre-stored table for codes.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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