

BEHAVIOR OF PASSIVE PILES IN SOFT CLAY FORMATIONS

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Abstract

Laterally loaded piles' behavior can be classified depending on how the lateral forces are transmitted to the piles into active or passive loaded piles. Active piles are subjected to direct lateral loads transferred directly from the superstructure to the pile head through the pile caps. On the other hand, passive piles are affected laterally as a result of squeezing and lateral movement of the surrounding soft soils under the effect of the additional vertical stresses). Down drag is defined as the downward settlement of a pile from dragging force exerted by the surrounding soft soil, while the drag load is the load transferred on a pile from negative skin friction. This frictional force reduces the pile structural capacity and decreases its serviceability, and must be considered in pile design. Typical examples of passive piles are deep foundations installed in soft clay and supporting bridges' abutments adjacent to the approach embankments, and piles used to stabilize earth slopes. The effect of lateral loading due to soil movement and squeezing on passive piles may lead to either structural distress of the deep foundations as a result of exceeding their lateral capacity, or serviceability problems as a result of exceeding the lateral displacement tolerable limits. Simulation of such a problem for a published field case study by using a 3D finite element program (Plax is 3D V2020) is investigated to depict the efficiency of using the numerical analysis in predicting the behavior of piles for such cases. The numerical analysis investigates the effect of lateral force on the horizontal soil displacement, lateral pile movement, and the affecting bending moment along the piles' lengths.

Keywords: soil-pile interaction, passive pile, deep soft clay, soil movement.

1) INTRODUCTION

Passive piles are subjected to lateral loads due to soil movement. Soil movement occurs when a pile is installed on an unstable slope, landslides, adjacent to deep excavation, and also in piles supporting bridge abutment approach embankment. The design of such piles is based on the assumption that forces from moving soil will act against the piles at the vertical loading side and soil tends to 'squeeze' at the other side of the piles. On the other hand, an active pile refers to a pile subjected to an external horizontal force.

Many researchers have conducted numerical studies (Finite Element modeling), field studies, empirical solutions, and centrifuge tests for piles cases that are subjected to lateral loads.

Broms (1965) used the analytical method to determine the magnitudes of lateral loads and lateral pile displacement at the ground surface and provided solutions for short and long piles embedded in cohesive and cohesion less soil and used the load method which considered that the horizontal loads due to embankment depended on the soil type, stiffness of the pile, and geometrical consideration. Techebotari off (1973), The authors considered that the lateral load on the pile could be described as a triangle when P_h reaches a maximum value of the horizontal load at the centre of the soft clay layer and the value of lateral load depends on values of vertical stress and coefficient of consolidation at the mid of soft clay layer. When the other authors like, De beer and Wallays (1972), considered that the lateral load on the pile could be rectangular acting on the pile along with the soil clay layer.

Poulos et al. (1995) discussed and investigated the behavior of single piles embedded in calcareous Sand examining the effect of changing piles' parameters such as fixed-head condition, pile diameter, and the ratio between the depth at which the soil movement's occur to the pile embedded length, and piles stiffness. Ellis used 3D centrifuge tests to study the behavior of pile foundation under abutment bridge approach embankment located above silty clay deposits. The fast test modelled a faster staged construction using wick drains in the clay layer, whereas the second test modelled a slower staged construction utilising wick drains in the clay layer.

Chen et al. (1997) proceeded to present model tests on pile groups subjected to lateral load due to soil movement with the same parameters utilized by Poulos et al. (1995). It was concluded that the lateral loads on piles were affected by the pile spacing, the number of piles, and the head fixity conditions. Pan et al. (2000) and Pan et al. (2002) presented as eries of laboratory model tests, where a 3D finite element model (ABAQUS) in soft Clay was performed to study the behavior of single and couple piles subjected to lateral soil movement and determine the ultimate soil pressure acting on the pile shaft. The analyses indicated that the behavior of piles under lateral load depends on the pile flexibility, the magnitude of soil movement, the pile head conditions, the shape of the soil movement profile, and the thickness of the moving soil mass. Kok et al. (2008) presented a numerical model for a single pile subject to lateral loads due to soil movement and compared the results from the finite element model to a numerical study. The design of passive piles was found to depend on several factors like soil movement mechanism, soil properties, pile head condition, and superstructure loading.

Bartlett et al. (2011), discussed the behavior of the embankment approach bridge overlying the soft soil when using the EPS geofoam to support a highway bridge without using a deep foundation. Those techniques are recommended by the Norwegian Public Roads Administration (NPRA) by using the Expanded Polystyrene (EPS) geofoam as super lightweight fill material in road embankments. Following the use of EPS geofoam for the I-15 Reconstruction Project, the performance of these fills is compared to other geotechnologies. Geofoam embankments were determined to have the best overall settlement performance of the technologies investigated. Gap closure and distortion of the geofoam embankment were roughly 1% of the embankment height due to the placement of the load distribution slab and overlying highway materials. For a 10-year post-construction period, the total postconstruction

settlement (base settlement and geof foam creep) is predicted to be about 50 mm at the wall face. The pattern of post-construction settlements suggests that geof foam embankments are behaving as intended and will meet the 1% creep strain 50-year post-construction deformation limit.

Dube et al. (2016) presented a numerical model of negative skin friction on driven piles located in soft clay soil and compared the results against the field recorded measurements, where the field program has studied the behavior of coated and uncoated pressurised concrete piles driven in soft clay soil. According to parametric and field studies, it is concluded that using of coated pile improves the behaviour of piles to decrease the negative skin friction and the coated shifted down the location of the neutral plane due to the coated pile being subjected to less drag load and hence less pile settlement compared to an uncoated pile. With increasing vertical load, the negative skin friction decreases over the length of piles. Moormann et al. (2016) focused their study on the numerical investigation of a single pile subjected to lateral loads and studied the effect of several factors on soil-pile interaction including time-dependent effects, a displacement between the pile and soil movement, cross-sectional, rough surface, and the velocity of soil movement. It is a conclusion that there was an increase in lateral load for around piles than for rough piles.

After studying the variety of different velocities of the moving around for round pile and square pile, an increase of 7% of the lateral thrust for a round pile with an increase of the lateral displacement of the soil by the factor of 10 was found. While an increase of 8% was found for a square pile. In general, the values for a square pile were larger by about 20% than for round piles.

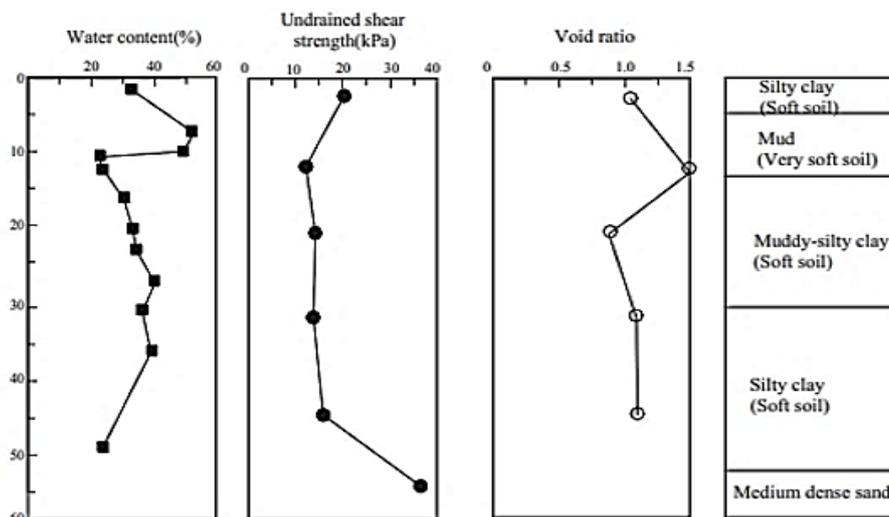
Xiao et al. (2018) discussed the behavior of pile foundation under an abutment bridge subjected to lateral loads using a series of 3D centrifuge tests performed on an approach embankment over a silty clay deposit improved by cement-fly ash-gravel (CFG) piles combined with geogrid. These piles (CFG) have four replacement ratios ($m=2.2\%$, 3.1% , 4.9% and 8.7%) of composite pile foundation and both piles are made from an aluminium alloy. Finite element analysis was utilized and then a comparison between calculated and measured results was carried out. It is concluded that the maximum horizontal displacement occurs at the pile head, with small displacement towards the embankment close to the pile tip. With the increasing replacement ratio, the horizontal displacement reduces due to the confining effect of the pile foundations. Using the higher ratio of replacement ratio (m) was referred to as a load of embankment transferring through the CFG piles to deeper soil layer and leading to less settlement and lower negative skin frictional forced-induced.

2) CASE STUDY

Different field experiments were carried out -in literature- to investigate the behavior of pile foundations in deep soft soils when subjected to adjacent surcharge loading. Under various surcharge loading conditions, the studies concentrated on investigating pile deflections, bending moments, and soil horizontal displacements.

The results of a series of instrumented field tests conducted by Li et al. (2019) aimed at investigating the pile behavior subjected to adjacent surcharge loading in deep soft soils. The Field test site was located in Ningbo North Railway Station, Ningbo, Zhejiang Province, and China. The test soil nature was composed of coastal alluvial plain and marine alluvial plain geomorphology, where the ground level ranges between 1.44 and 2.68 meters above the mean sea level. The soil stratification is composed of an upper layer of muddy silty Clay soft soil whose thickness extends down to 52.2m where the layer had slightly higher water content and very low untrained shear strength (C_u ranging between 10 and 20 kPa). That layer is followed by a medium dense Sand layer of higher shear strength (around 35 kPa). Figure (1) presents the available field data including the value of untrained shear strength, water content, and void ratio for the soil layers.

Figure (1) Soil Profile and its Properties in Field test (after Hong-quan Li, 2019)



During the field test, single row piles consisting of three bored piles of 1.0m diameter, 45.0m length, and 3.0m pile spacing, with a pile cap whose length is 7.4m, width is 1.4m, and thickness is 1.0 m, were tested. Figure (2) illustrates field test photographs showing (a) installation of measurement instruments, (b) bored piles construction, and (c) surcharge loading volume.

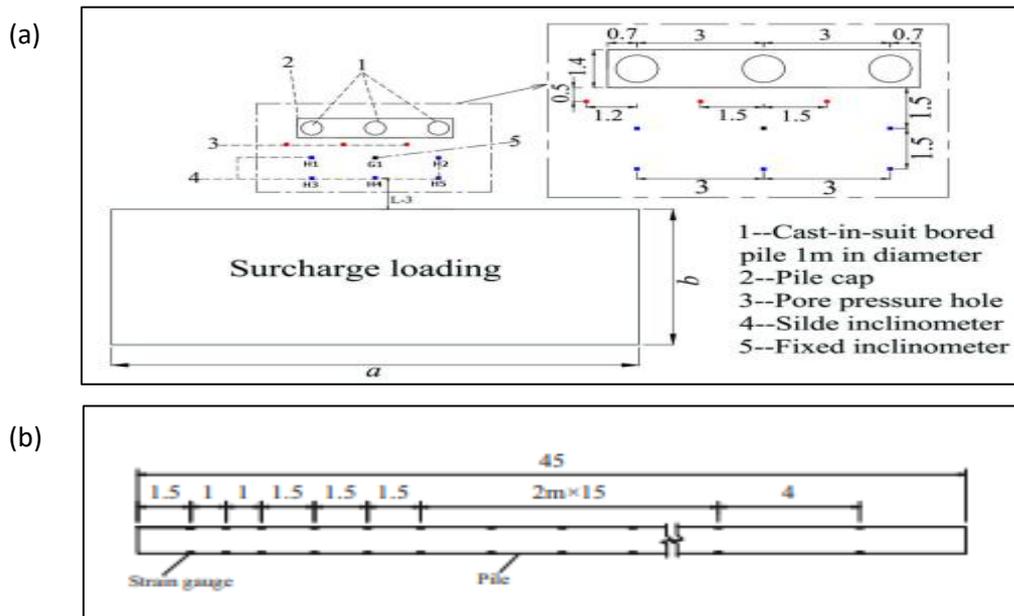
Figure (2) Field test photographs (after Hong-quan Li, 2019)



The surcharge was distributed beside the piles' zone at a distance of 5.0 m from the pile cap with an area covering dimensions of 25.0m in length and 15.0m in width. The loading process was controlled by increasing the surcharge volume to reach 55 kPa stresses. The time at which a definite height of surcharge is applied is referred to as construction time while the loading time is the time of applying constant surcharge loading. The construction time was about 1 day and loading time was also take about 1 day so the total operation took about 2 days.

To measure the strain of the pile shaft, 40 concrete strain gauges were inserted into the pile. The strain gauges were symmetrically positioned horizontally symmetrically of each test pile. Because of the large number of test sections on each pile, the test sections on three test piles were staggered vertically to ensure that the strain gauge wires could be successfully led out of the test pile without interference. One fixed inclinometer (G1) and five sliding inclinometers (H1–H5) were used to measure soil horizontal displacement, with the fixed inclinometer G1 and the sliding inclinometers H1–H2 positioned 1.5 m away from the pile and the sliding inclinometers H3–H5 positioned 3.0 m away from the pile as shown in Figure (3)

Figure 3 (a) fixed and sliding inclinometers to measure the lateral soil displacement. (b) Strain gauge fixed on pile length to measure bending moment and strains of piles (after Hong-quan Li, 2019).



Field test results are used for the validation of the capability of the Finite Element Model in simulating such problems. Such models will aid in further investigation of the behavior of piles under passive loading.

3) CASE STUDY NUMERICAL MODELLING

3.1 Geometry and Boundary Conditions

Three-dimensional numerical analyses of the case study are conducted using the finite element code, PLAXIS 3D 2020. The finite element geometry is 80.0m, 80.0m, and 70.0m in x, y, and z coordinate directions, respectively. The groundwater table is simulated at the ground surface according to the site conditions where the assigned hydraulic boundary conditions allowed for dissipation of the pore water through the horizontal boundaries and restricted it through the vertical boundaries.

3.2 Soil Layers Constitutive Model Parameters

The utilized volume element in the finite element model is the 10-noded (tetrahedral) elements. Hardening soil the constitutive model is adopted to simulate the soil behavior as it is compatible with studying the behavior of subsoil under the consolidation process by using the value of over consolidation ratio and overburden pressure in the constitutive model parametersto easily depict the nonlinear stress-strain behavior (Schanz et al. 1999). Triaxial loading stiffness (E_{50}), Triaxial unloading stiffness (E_{ur}) and Oedometer loading stiffness (E_{oed}) are assigned in the

parameters of the hardening soil model. Figure (4) illustrates the suggested soil stratigraphy utilized in the finite element analysis.

Figure (4) Soil Stratigraphy Utilized For the Case Study



In the literature of the case study, the field data doesn't specify if the subsurface conditions are in the normal or over consolidated state, and the value of cohesion and angle of friction is utilized to specify if the soil is in normal or over consolidated conditions.

Ladd and Foot (1974) developed a method named the SHANSEP method (stress history and normalized soil engineering properties) to express the normalized behavior of soils. SHANSEP testing was developed at MIT and was widely used to conclude the untrained shear strength of soil using the relationship between S_u/σ_{v0} and OCR expressed as follows.

Equation (1)

$$OCR = (S_u/\sigma_{v0} \times S)^{\frac{1}{m}}$$

Where:

OCR is the over-consolidation ratio,

S_u is the untrained shear strength,

σ_{v0} is the effective overburden pressure,

$m=0.8$, and $S=0.22$ depending on the type of soil.

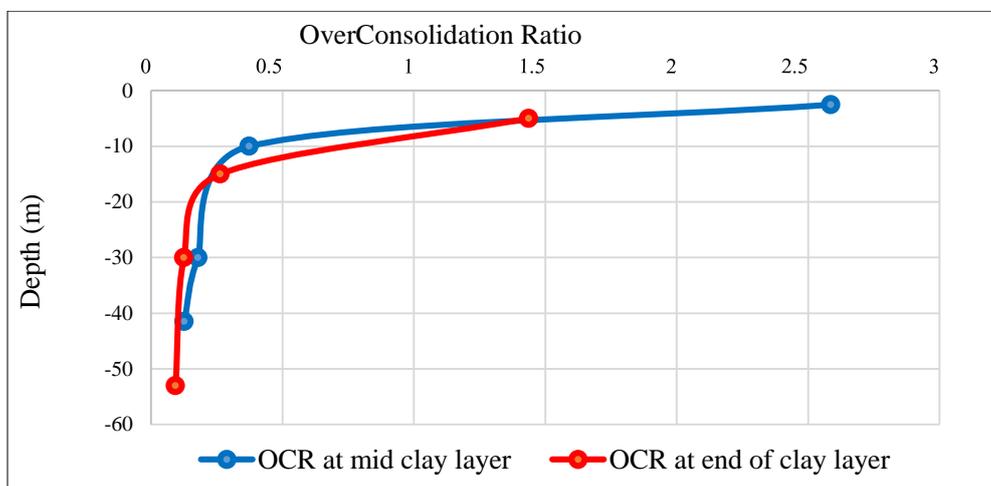
Figure (5) shows the value of the over-consolidation ratio (OCR) along with depth applying Equation (1).

Based on the calculations, it is found that the over-consolidation ratio is about 2.5 at mid-layer in the first layer (silty Clay layer) and then decreases to be under consolidated soil in Mud, Muddy, and silty Clay layer according to the field data shown in figure 2.

The water content value is less than 50% and the void ratio value is in the range of (0.98 to 1.43). At the under-consolidated soil layer(s), the value of deformation is expected to be higher and not match the value of pile deformation that appears in-field results.

Based on that, it is preferred to use normally consolidated or over consolidated conditions for the soil layers. Based on the value of untrained shear strength, water content, and void ratio, trials using two assumptions for the over consolidation ratio are utilized in the finite element model. The first assumed value of the OCR is equal to 1.0 (normally consolidated) while the second assumption is to use over-consolidated conditions considering different values of over-consolidation ratios such as 2.0, 1.6, and 1.4.

Figure (5) Value of the over-consolidation ratio (OCR) along the depth using the SHANSEP method



The soil parameters applied in the numerical model are presented in Table (1).

Table (1) Hardening soil model parameters

Soil Parameters	Silty Clay (1)	Mud	Muddy	Silty Clay (2)
Depth	0-5m	5m-15m	15m-30m	30m-53m
Soil model	HS-undrained A	HS-undrained A	HS- undrained A	HS-undrained A
Unit Weight (γ) (kN/m ³)	18.5	16.4	18.8	18.3
Void Ratio e_{init}	1	1.43	1.29	0.98
Permeability Coef. (cm/sec)	6E-7	1.5E-8	1.2E-7	1.3E-7
Cohesion (kPa)	10	10	10	10
Friction Angle(Φ)(degree)	12	10	11	12
E_{50} (KPa)	6,400	3,100	4,700	9,000
E_{oed} (KPa)	15,000	8,640	12,240	21,800
E_{ur} (Kpa)	32,000	15,500	23,500	45,000
OCR (assumption 1)	1	1	1	1
OCR (assumption 2)	(2-1.6-1.4)	(2-1.6-1.4)	(2-1.6-1.4)	(2-1.6-1.4)

3.3 Finite element simulation of Structural Elements

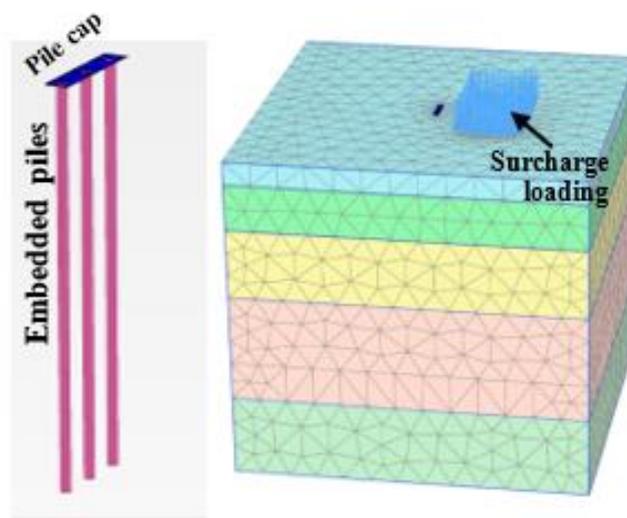
Linear Elastic embedded beam elements (embedded piles) aim at simulating the piles' behavior in the Finite element model. In Plax is 3D, an embedded pile consists of 3-noded beam elements with interface elements to describe the interaction between soil and pile along the pile length (Shaft Resistance) and at the pile tip (Tip Resistance).

The embedded pile can be considered as a volume pile where the soil volume elements surrounding the pile behave elastically in a zone identified by R_{eq} (embedded pile's equivalent radius). The embedded beam element parameters include a moment of inertia (I), and Young's modulus, and are across-section of the piles. The parameters of the structural element are summarized in Table (2). Figure (6) shows the simulation of the structural element in the finite element model (Plax is 3D).

Table (2) Parameters of Structure Element used in finite element model

Structure Element	Pile Cap (Plate Element)	Pile (Beam Element)
Constitutive model	Linear Elastic -Non-Porous	Linear Elastic -Non-Porous
γ (KN/m ³)	25	25
Young's modulus (KN/m ²)	33.6x 10 ⁶	33.6 x10 ⁶
Poisson's Ratio	0.17	0.17
Area (m ²)	10.36	0.785

Figure 6 Finite Element Model Mesh and simulation of the structural element



3.4 Finite Element stages of Calculation

In PLAXIS, different calculation types such as plastic, consolidation, and safety can be considered. Plastic calculations can be used to simulate elastic-plastic deformation analysis and are generally utilized in most practical geotechnical applications. No need to take the time effect into account except when needed as in the soft soil creep model. Consolidation calculations are used for simulating the drainage process and dissipation of pore water pressure in saturated soil (clay) considering the required time interval in the consolidation process to investigate the stresses and deformation at different degrees of consolidation. Safety calculations are performed especially in slope stability and unstable construction.

The finite element calculation process is divided into five stages. The first stage is calculating the initial stresses acting in the soil continuum before loading is applied where the at-rest earth pressure coefficient (K_0) ranges between (0.7-0.8) for different types of soil layers.

The second stage includes structural simulation of piles and pile cap by activating the corresponding structural elements with a time interval of 1 day. The third stage shows the loading construction simulation by activating the surface load with a time interval of 1 day. The last stage of calculation involves the loading time in which the constant surcharge loading is kept active for 1 day.

4) RESULTS AND DISCUSSION

4.1 Lateral pile Displacement

Figures (7) illustrate the comparison between the measured data along the pile length (using field results) and corresponding calculated results computed from the numerical model using over consolidation ratio (OCR) = 1 to represent the normally consolidated conditions, while Figure (8) shows the same for the other previously mentioned different OCR values to reflect the over consolidated conditions to study the pile behavior under the effect of different values for the over consolidation ratio.

From figures (7) and (8), a fair matching is recognized between the measured pile displacement and the calculated ones using OCR equal to 1.6 which shows that the soil layers in a field test located in Ningbo North Railway Station, Ningbo, Zhejiang Province, and China [39] were over consolidated.

Figure (9) shows a comparison between the measured field displacement and the corresponding numerical analysis results using OCR = 1.6, to easily discuss the findings. The maximum value of lateral pile displacement is read at the pile head and deflection decreases to reach zero value at the pile tip in numerical results whereas the zero value of pile displacement is reached at a depth of 30m for field measured results.

The results show that the measured lateral displacement at the pile head is 19.97 mm compared to 21.06 mm as resulted from the numerical model which shows that fair convergence is reached using numerical analysis.

Figure (7) Comparison between the measured Lateral pile displacement and corresponding finite element results using the different values of over consolidation ratio=1 (normally consolidated soil)

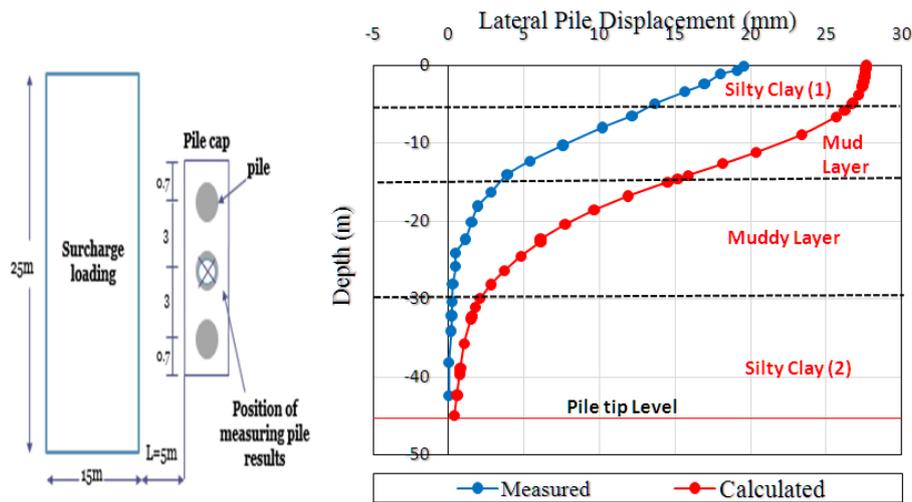


Figure (8) Comparison between the measured lateral pile displacement and corresponding finite element results using the different values of over consolidation ratio

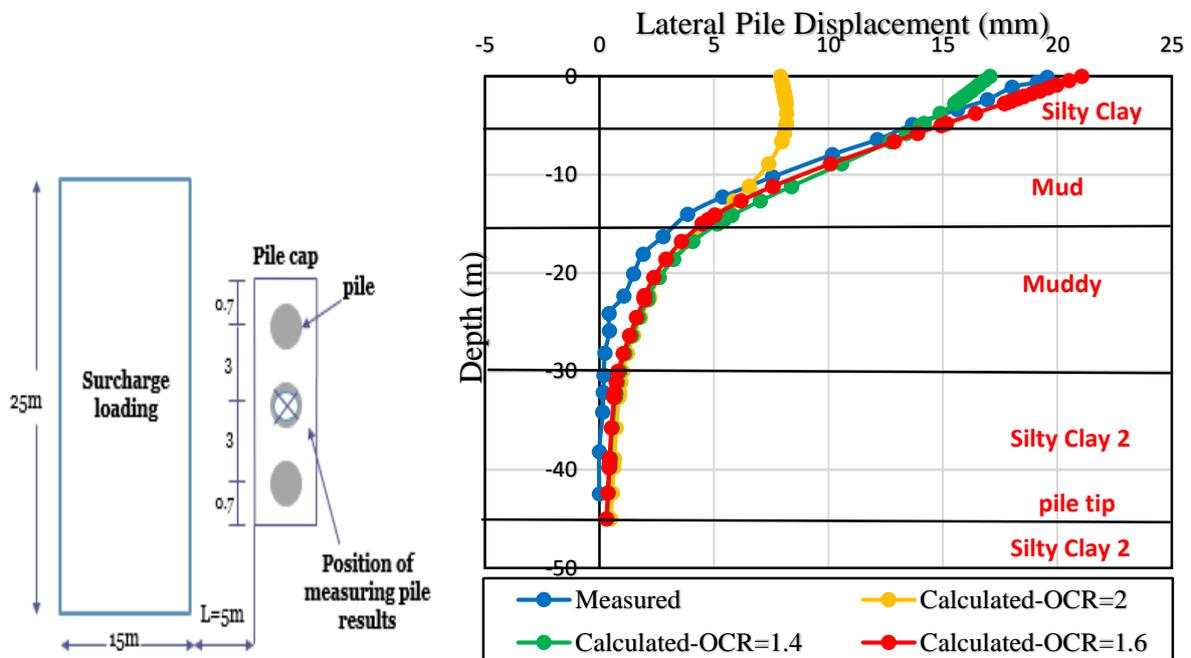
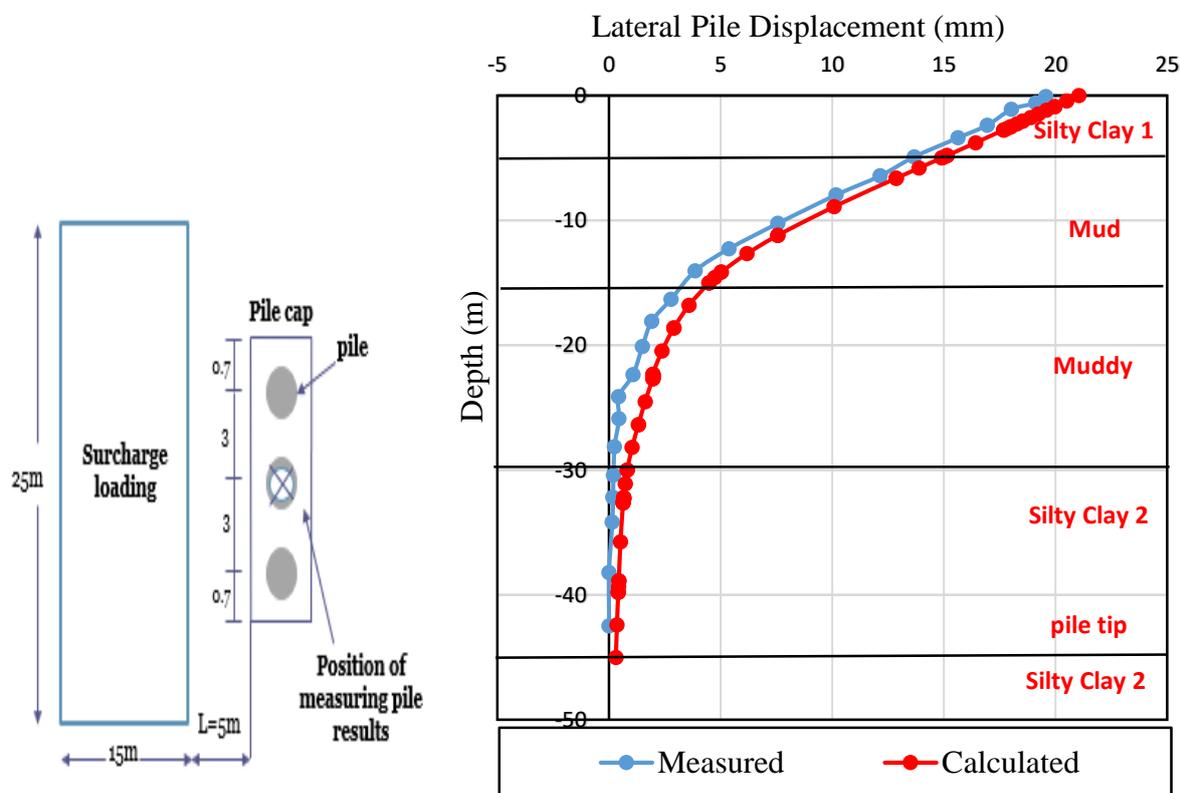


Figure (9) Comparison between the measured lateral pile displacement and corresponding finite element results using over consolidation ratio value of 1.6

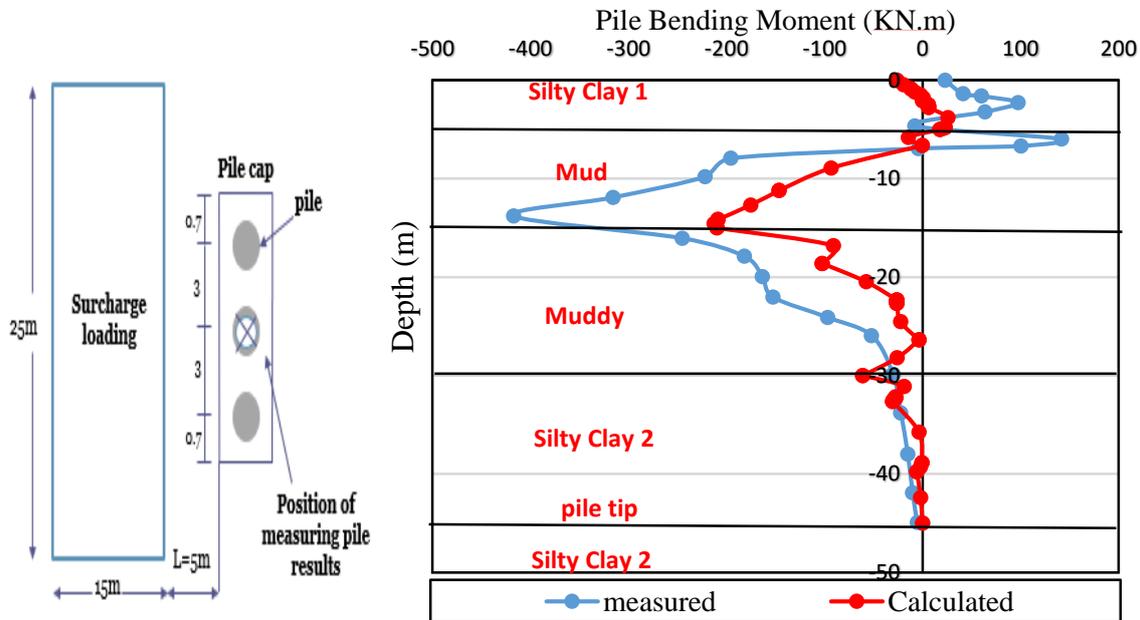


4.2 Pile Bending Moment

Figure (10) presents a comparison between the measured bending moment along the pile length and the corresponding finite element results using $OCR = 1.6$. The maximum bending moment of the pile was measured at a depth of 13.0 m (Mud Layer) and decreases to reach less than 20 kN.m below the depth of 30 m. This can be explained as a result of the existence of the muddy soft soil at a depth from 5 to 13 m under the effect of the surcharge loading where the depth of the maximum bending moment was dependent on the depth of the soft soil layer. The smaller values of bending moment on piles are due to the construction and loading times implemented in the short term.

The maximum bending moment of the pile from the field data (-418 kN.m) was higher than the corresponding computed using numerical analysis (209 kN.m) at -13 m depth (Mud Layer). The difference between the measured and calculated results is about 50 %. The difference between the measured and calculated bending moments can be attributed to the pile being rotated when installed in soil or the movement of the strain gauge in piles.

Figure 10: Comparison between the measure and computed Bending Moment values when using the value of Over Consolidation Ratio is 1.6



4.3 Lateral soil Displacement

Figures (12) and (13) illustrate the comparison between the measured and calculated results of lateral soil displacement at G1 (fixed inclinometer) and H4 (sliding inclinometer), respectively. The maximum value of soil displacement at the ground surface in the calculated results decreases to around zero at a depth of 35m (silty Clay layer 2).

The inclinometer (H4) provided a horizontal displacement reading of around 40mm which is greater than the reading of the inclinometer (G1) whose reading was about 37mm. The reason for this was because (H4) was located only 1.50m near surcharge loading while (G1) was at a larger distance (around 2.5 m) away from surcharge loading. Results of the numerical analysis show that the soil displacement value at the (H4) location is greater than the value of lateral soil displacement at (G1). The results of lateral soil displacement indicate that the closer the distance to the surcharge loading area, the greater the lateral soil displacement is.

The difference between the measured and the calculated results for (G1) (fixed inclinometer) was about 22.9% and around 12.75% for (H4) (sliding inclinometer) which shows fair agreement between the measured and calculated results.

Figure 11 Lateral Soil Displacement at H4 (Sliding inclinometer) for Measured and Calculated results.

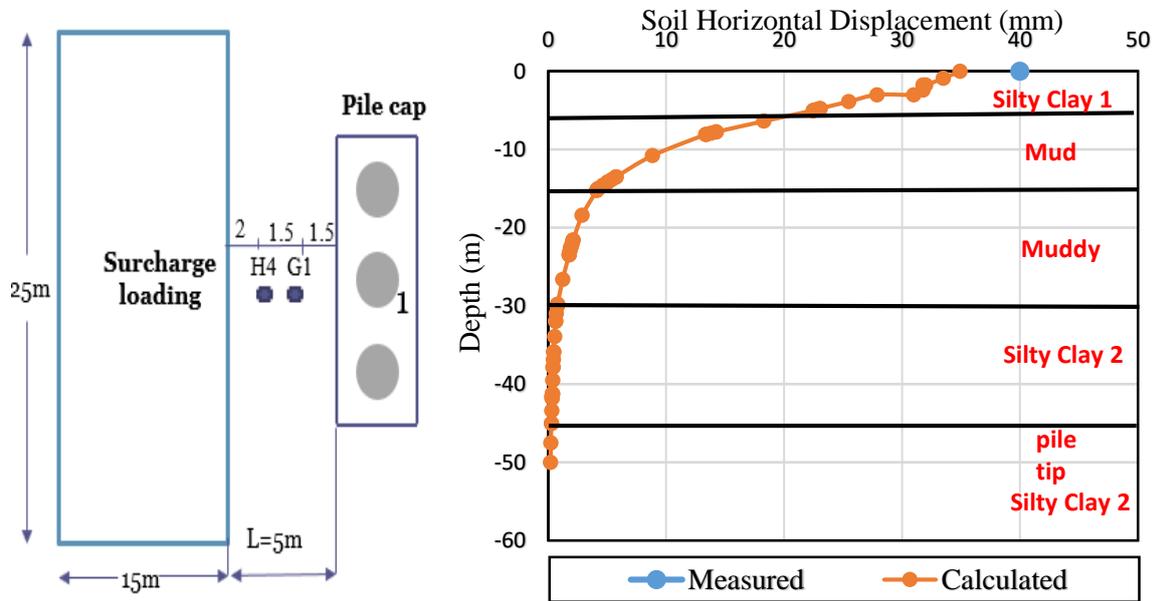
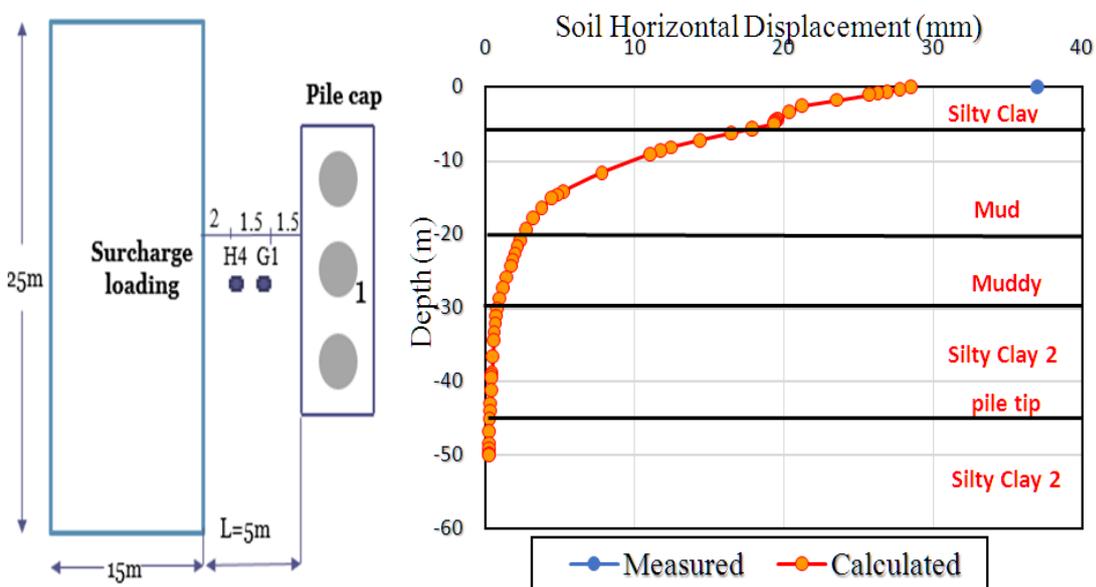


Figure 12 Lateral Soil Displacement at G1 (Sliding Inclinometer) for Measured and Calculated results



CONCLUSIONS

The behavior of piles subjected to adjacent surcharge loading in soft soil is investigated using field study and 3-D finite element numerical analysis. The study focused on the lateral pile displacement, bending moment of the pile, and horizontal soil displacement. Based on the study, the following can be concluded:

1. The performance of passive piles in a soft soil layer can be simulated with a non-linear elastoplastic model stress-dependent stiffness constitutive law.
2. Three-dimensional finite element modelling (3D FEM) can accurately depict the performance of passive piles in a deep soft layer under surcharge loading.
3. There is a fair agreement between measured and calculated results in comparing the lateral pile displacement and horizontal soil displacement values. However, there is a great difference between the measured and corresponding computed values of piles' bending moment. Such difference may be attributed to the pile being rotated when installed in soil or the movement of the strain gauge in piles.

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