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COGNITION TRAJECTORY IN TEACHING AND LEARNING MATHEMATICS

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Abstract

We explore the mutual influence between cognition trajectory and teaching and learning mathematics, include the content knowledge. That presents the two fields to affect the interplay due to theoretical constructs. Through discussion, the representations progressed the courses and developed professional knowledge. We move to both areas, and the exploration shows a trend toward subject knowledge.

Keywords: cognition trajectory, teaching and learning mathematics, content knowledge, re-representation, project-based learning

INTRODUCTION

The fields of cognition focus on the interaction between students and the teaching and learning, particularly mathematics. These two disciplines share a shared focus on the three perspectives of students, educators, and information processing. The features concern class interaction, ranging from selecting a strategy to evaluating pertinent knowledge. In contrast, information is also discovering unstructured content that is required for teaching and learning (Weston, 2018). To define these fields and evaluate their relationship, i.e., an information behavior based on mathematics teaching and learning, with a focus on the relationships between different mathematical content knowledge. However, the development of theories in both disciplines is restricted (Turner & Rowland, 2011), while mathematics is generally driven by practical problems and often evaluates knowledge via practical criteria (Turner, 2012), where the consideration of theoretical constructs drives mathematics education research.

This article aims to explore theoretical constructs in cognition, understand teaching and learning, and comprehend the relationship among cognition trajectories. To us, a theoretical construct is a body of knowledge that guides significant empirical and practical procedures. The two fields give rise to different paradigms for providing information in math instruction. First, the researchers place it in the field spectrum. Second, understanding the theoretical basis and the constructions promotes communication and analysis between the two disciplines. Providing chances for interdisciplinary research collaboration and enhancing the impact of research is the final objective.

METHOD

The literature review was chosen for this research method. With this method, an analysis of several kinds of literature is carried out by connecting thematic and coherent ideas about the

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trajectory of cognition in the teaching and learning of mathematics, arranging them so that they become the central issue (Lawrence, 2011; Selly, 2013). To achieve this research objective, we collected journals and books on the trajectory of cognition in mathematics education. Then we analyze and synthesize to design theoretical constructions about the existence of a wedge of cognitive ideas between teaching and learning to produce mathematical cognition trajectories.

LITERATURE REVIEW

Türnüklü & Gündogdu (2015) identified the theoretical constructs and focused on the information process. In mathematics education (particularly, the teaching and learning), we depict the academic construct in Table 1.

Table 1: Theoretical constructs of cognition trajectory in teaching and learning mathematics

Constructs	Dimensions		
	Transformation	Connection	Contingency
Pedagogy	Subject knowledge	Mathematical terminology	Insight
Representations	Materials	Purpose	Procedure
Complexity	Tools and resources	Conceptual	Principles
Subject knowledge	Pedagogic content knowledge	Instruction	Demonstration

We investigated cognition trajectory from the theoretical constructs to the dimensions. In mathematics education, no earlier research has provided insights into the constructs underpinning the cognitive process fields. Second, a great deal of earlier work on theoretical documentation has been on empirical analysis. None of the prior studies offered a structure for including contrast and comparison elements.

Table 1 aligns the constructs to the dimension of mathematics education research.The theoretical construct by groupings (i.e., pedagogy, representation, complexity, and teachers' & students' mathematics knowledge) sheds light on the primary focus of each. We defined the categories of dimension viewpoint as follows: (a) Transformation, the academic discipline dealing with information-specifically interaction with mathematics; (b) Connection, the area of academic study concerned with information retrieval-specifically mathematical terminology, the purpose of the teaching and learning, conceptual approach, and the instruction; and (c) Contingency refers to the concepts of procedure and principles for getting insight and the demonstration. We emphasized how to conceptualize information for mathematics, while Simon (2014) provided an overview of the concept in the broad information. Attempting to give structure to the idea of "information," Rowland (2013) presented three interpretations of information: "Information-as-process," "information-as-knowledge," and "information-as-thing," in addition to a characteristic use of information to designate informative things. Data are symbols representing the qualities of objects or occurrences; information is a description of that item; knowledge is the level of instruction; understanding is the level of explanation for specific situations; and wisdom is the level of evaluation or judgment (Mason, 2021). The information searching discipline focuses on data and information, occasionally mathematics knowledge, and integrated teaching and learning

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(Rif'at, 2017), placing database researchers in charge of data and teaching and learning researchers in charge of knowledge.

RESULTS

Identifying the 4 constructs was a teaching and learning process, beginning by identifying the constructs that spanned the cognition process and mathematical communication and literacy fields. In certain situations or contexts, pupils may not desire information even though it is readily available. For example, we did not need numeric information in visual representations and vice-versa (Rif'at, 2018). In addition, people will resist using an information system exactly because they are aware that it will offer the needed data in certain circumstances. For example, for literacy, from a linear equation could be made become a system. It develop mathematics situations than having and getting the solution. Nevertheless, the solution is more comprehensive; it is a visual representation based on mental imagery.

The advancement of the representation has been made possible by the information field's consistent belief that one can cultivate and establish a link to knowledge. In response to user input, generally a query, delineating information is a sort of lateral thinking that entails providing more thorough data. There is a set of answers for questions and answering models that address a topic. In cognitive systems, however, there is a collection of results associated with the degree to which they match a query or teaching level. Based on a metric, not all outcomes are of equal importance. When implemented, mathematics is not applied to the diminishing likelihood of relevance, but rather to the matching representation of the Query. In searching, information representation is likewise a widely acknowledged concept; nevertheless, the evaluation is typically determined by cognitive, emotive, or skill criteria. Literally, if a learner can identify a question with a subject, other representations with similar qualities will also be pertinent. For example is that the symbolic and numeric to visual. Fernandez, Llinares, & Rojas (2020) state that closely associated ideas tend to be relevant to the exact requests and implied result in more effective and efficient. Another example is that mathematics transformation understood by the graph than algebra. The appropriate alternative representation would be similar to the one already discovered; but, this new document would be of little utility to the user, as it may not include any new information. However, that limitation permeates a wide range of information, relevant feedback, and performance improvement in traditional representation.

The cross-cultural intellectual perspective of information is to find a concept that generates discussion. Particularly, teachers use textbooks mainly than opening discourses. Relevance plays an important, essential, and pivotal role in all elements of information representation, including theory, implementation, and evaluation (Yang et al., 2015), which concentrate on the relationship between information items and a submitted query. This pertains to the performance of information, with relevance serving as an underlying requirement for recall and improvement. According to Livy, Herbert, & Vale (2019), relevance depends on judgments between information and representation need situations. For example, mental arithmetic is more auditory than the visual. Wilson, Mojica, & Confrey (2013) and Rif'at (2018, p. 77)

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expanded the concept by emphasizing how the same perception may change over time. Someone mathematics perception tend to visually than conceptualization. In order to compare representations or information between two or more courses, one must determine how the idea is defined and treated in each discipline of mathematics. In other word, the system of information constructs a model designed by students' empirical situations.

In striving for the intellectual perspective orientation of teachers and students, we seek a method involving the most enduring tenets, including Internet research, library research, and classic information retrieval techniques or textbooks (Beilstein, et all., 2021). Instead of weighing all of the various algebraic representations available to them, they frequently make decisions based on a small subset. Statistically, the best one should be close to the option in mathematics (Arzarello & Taranto, 2021). For example, re-representation is based on cognition. The Principle is embedded in foraging theory of Wickstrom & Langrall (2020). That is the application of the Principle when taking actions they want or think with the expenditure of the rigor application (Rif'at, 2021). For example is manipulation and demonstration,

Information representation is a changing process that appears in a model (Xie, 2008). For example, teachers presented information further developed as a concept. The process is also the basic assumption for information searching models, including numerous information searching strategies, information seeking model finding in environment resources, and information seeking model seeking in environments (Petrou & Goulding, 2011). The method focuses on the change in the cognitive and emotional states of the user. During and between user-system interactions, a significant degree of search goal revision occurs as a learning resource.

Beginning with Cognition Trajectory, we stress the epistemological that should refer to the "genealogy of concepts" (see (Rif'at, 2001)) and "progressive conceptualizations" (see (Longo, 2010)), i.e., beginning with "human activity" and continuing through the origin of a concept. The crucial point is the "great stability and reliability" of Mathematics, for which we must account. In developing genealogical characteristics, students can make observations about the accuracy of the solutions. It can be the expected approach so that the settlement becomes the essence of what and how to make it. In relation to the theoretical principles of mathematics (Borba & Llinares, 2012), we might define it as a conceptual fragment of knowledge construction. Mathematics is the process of communicating and understanding mathematics, the concept of knowledge without absolutes. If not, it is prose, whether we recognize it or not. Mathematics is congruent with the development of figures since it is re-represented with its knowledge during its construction. It resonated with the objective nature of the objects. It is feasible, for instance, that complex numbers produced from equations and enhanced with the Argan-Gauss interpretation would render them understandable. However, natural languages provide tools that may be transferred from one field to another and which completely organize events and living forms. It will be able to do cognitive analysis as part of an epistemological study if we reconsider the maths problem. For example, the convention of writing the figures only on alternate squares is handy. The cognition moves from configuration (model and symbol) to behavior (operation and final model).

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Rarely do educators ask whether more knowledge is preferable in a given context or whether pupils desire informational fragments. This concept of always being good contrasts with that of information-searching teachers, who acknowledge that students may purposefully shun information in a given setting, based on class observations. This study shows that students might choose to disregard lessons they aren't interested in, and studies of math instruction show that the "information provisioning" assumption isn't always correct. Rif'at (2022) found, for instance, that while more information does increase confidence, it does not necessarily aid decision-making. For example, the solution of a limit function by substitution showed incorrect choices based on the reason. This is intuitive, and you should not assume that learners are always reasonable when weighing concepts and information representation approaches (Rif'at, 2001, p. 52). As the material unfolds, students develop a more specific emphasis on the ideas, going from feelings of uncertainty to controlling increased solutions. For example, in concepts of discontinuity functions (in calculus), the students envision a predicament that prevents them from proceeding without acquiring additional information and developing a revised representation. After modeling the notion based on the knowledge gained, the learner can go on to discovery and continue. For example, to get a result from a derivative chain needs an effort. The information stream supposes representation as an interactive process between the learning and teaching system. Ruthven (2011) notes that interactivity and students interact increasingly in an environment with various information systems. Regarding the concept of interaction, there is arguably greater agreement between the two domains than any other cognitive construct. The concept that pupils have preferred representations when searching for information appears as a construct. Some of these depictions are not even technological in nature. Depending on the required information and the circumstances, the student may consult several sources. In empirical work, fact-finding activities drive students to particular categories of content, whereas process tasks direct them to alternative sources. For instance, the visual depiction of the calculation is not the same as the algebraic computation. Understanding the term's preference and usage is crucial for answering students' underlying information needs from a search standpoint (Gumiero & Pazuch, 2021).

Mathematics is among our modes of communication and knowledge, which are also its limitations. We manage the most "dynamic" situations precisely when we perceive the global determination (rule, principle) that renders them understandable. Even geometric shapes have mathematical imitations (perception and belief). Nevertheless, the moment becomes challenging to grasp (it is possibly not “that which just counts”). Mathematics inadequately describes variance/invariance, order/disorder, and integration/separation (see (Francis & Jacobsen, 2013)). In other words, one cannot delve into the particulars of discourses, yet doing so is part of the epistemological endeavor (Sen, 2021). This analysis of its constitutive trips helps us to explain the emergence of new knowledge dynamics. In addition, it prevents us from applying the same mathematical tools everywhere, where new tools and observables (new invariants) are required. The study of teaching and learning mathematics has focused on cognition trajectory, although there is a misrepresentation that it could change the way of thinking. For example, changing procedural knowledge to axiomatic is because of the availability of cognition. In other words, the influence of the field in cognition construct is by

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the trajectory imperative. There also is a similar element of cognition determinism; however, it is less directly apparent. As an example, mathematics knowledge works and logic. Nevertheless, how these are doing could be studied and enhanced by mathematics through teaching and learning.

The more accessible knowledge, teachers and students will likely use information. Tanisli, Ayber, & Karakuzu (2018) stated that the development had concerned with making it easier to understand. The expression needs context and visualization in terms of interfaces. The cognition construct focuses on thinking skills pertinent to mathematics information and expands, controls, poses, and develops axiomatic systems. For example, a visual in the paradigm is also the thinking style; terminologies are in the set of geometric thinking, while the others are resources in the domain. And they are depicted in Table 2.

Table 2: The cognition trajectory with effect on information constructs

Information constructs	Effects	Results
Mathematics concept	Mathematics learning	Procedural
Relationship of Mathematics	Organizing knowledge	Procedural
Perception	Generally insignificant in solving problems	Belief
Representation	Algorithmic thinking, a minor area in mathematics	Image
Similarity	The central concept is a limited focus on a solution	Procedural
Principle	Expressions to determine information and resources	Belief
Interaction	Mathematical discourses	Belief
Preference	Information, impact on the cognition process	Procedural

DISCUSSION AND IMPLICATIONS

By examining the eight cognition constructs and their relationship to teaching and learning, we reflect on the effects between the two (cognition trajectory and the teaching and learning) associated disciplines in mathematics. Specifically, the impact of each cognition construct focuses on the status of procedural, belief, or image relative to a particularly given construct. This described the research as (1) Procedural: tending to approach the given construct; (2) Belief: changing in diverse directions in relation to the given construct; and (3) Image: demonstrating a shift in relation to the given construct.

Table 2 presents a recap of the procedural and the image existing between information constructs and the effect on the teaching and learning, along with areas of mathematics, i.e., belief. Four cognitions have formed both fields, whose viewpoints continue to converge (i.e., the procedural). The researchers accepted the construct of Procedural, Similarity, and Preference. There appears to be tremendous synergy between the fields regarding these constructs and the development of discourses in both domains. The fields strive to obtain information, view the interactions, and will follow a sufficing concept pattern. Regarding the meanings of these three constructs, there is a consensus across disciplines regarding their applicability. For example, a graph represented to solve a problem shows a development strategy of mathematics understanding. Students tend to update their knowledge and skills. These are a kind of project-based learning.

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Two constructs (Representation and Preference of Mathematics Concept) also show increasing similarities between representation and mathematics algorithmic thinking. For example, arising to visualize is also an expanding manifestation of information searching. Meta-search and other complicated information processing methods are receiving more attention as users demonstrate a preference for these methods. These two concepts allow for a more sophisticated understanding of search and users in the context of information retrieval. That is to improve the searching by discussing the method and its effects on the student.

There is a generic technique for constructing the relationship between information demands and mathematical objects, with both fields specifying this relationship. However, the two fields have long used it with different meanings, emphasizing either cognitive or computational concepts. The expanding manifestations are algorithmic, topicality, and situational recognized; the complexity is in searching and information retrieval. For example, to solve a division of fraction problem, not reverse the de-numerator, but take that and put it into the numerator. That is iterative thinking and using the information manageable.

There are seven results where the fields of cognition constructs are relatively stable. Teachers universally accept the Mathematics Concepts; in most cases, the construct remains questioned. For instance, mathematics objects are the foundational assumptions of the discipline and the development based on proof by merely the axiomatic systems. That is one of the problems in learning math. At the same time, Image is critical and paid little attention to (Rif'at, 2001, p. 72). For example, the definition of Operation is a function, then a division is the sum sometimes, but not in the role of learning activities. In contrast, the principle is crucial to numerous mathematical information-searching models and problem-solving techniques. The Mathematics Concept (Multiple Definition of Mathematics) is a construct dimensionally opposing the viewpoints, with a significant change from either field. For example, that is true in the system. At the same time, the development limits the making of connections, concerning the construct of Information as a primary expression in teaching and learning mathematics, concerning its importance, impact, and implementation. Teachers and students hold it more as the cognitive expression of mathematics needs in a more procedural (mechanical) way. For example, merely to solve, find, check, fill, show, and prove the truth, but not to design or to construct it for some enhancement.

Three constructs noted belief between cognition constructs (Perception, Principle, and Interaction) to the effect of mathematics instruction. With the Mathematics Relationship, the information field emphasized cognitive and emotive characteristics of higher order, such as teaching and learning. Conversely, the continuation of information retrieval focuses on algorithmic improvements and sometimes knowledge resources. The hunt for information shifted to the mathematics of organizational, cultural, and literacy factors that influence or moderate the advantages (see (Rif'at & Sudiansyah, 2022)). If teachers and students extend information into collaborative searching, there is some progress in algorithmic approaches. The third type of divergence motivates information to interact with its relevance through a variety of resources in order to provide mathematics content to teachers and students. In spite of the desire to provide specific details, the overall trend is that if there is mathematics knowledge,

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instructors can and should develop a system to provide it to students; however, they do not actively pursue this role in mathematics teaching and learning.

Implications

Overall, these constructs can aid new ideas in understanding the relationship between the cognition process and mathematics teaching and learning. Nonetheless, there are further special ramifications. First, there are tensions between the fields due to the competition for fundamental structures. The uncertainty caused by contradictory concepts might not be a problem. For instance, this resulted in an increase in mathematical modeling because it provides representation for study. These exist because of their individual contributions. Second, the similarities between the disciplines suggest the potential for sustained and expanded collaboration between the disciplines. For example, a division problem (particularly in fractions) is to build more relationships in basic operations and to get any of the principles. Third, the impact of constructs can make their unique contribution. For example, a straight line is also a triangle. However, a foundational understanding of each field's core is necessary for the objective evaluation and interpretation of the aims.

The theoretical and empirical constructs have one or more trajectories but are challenging to prove. Empirical observations were repeatable to refute when making incorrect estimations and predictions (see (Isiksal & Çakiroglu, 2011)). Reasoning is an alternative way to decide on different statements to confirm a result from a trajectory. We believe that the fields have constructed nets for managing the information. Other mathematics education researchers have concluded that the areas are due to problem-solving representation. For example, in implementing the constructs, asking provocative and guiding questions is crucial (see (Tanisli, 2013; Tanisli, Ayber & Kuzu, 2018)). The perspective has helped teachers understand how to conduct teaching that enables cognition trajectory (Francis & Jacobsen, M., 2013). In our case, that reflects the awareness within the scope of cognition in the teachers' professional knowledge. Teachers advanced within the context of the cognition trajectory, notably in the foundation, transformation, and connection component, which is in information processing (Livy et al., 2019).

This indicates how a combined perspective contributed to the teachers' understanding of instructional patterns by providing a temporal trajectory. Drawing from our empirical work, we analyzed the themes that underlie research streams in the cognition trajectory field. We believe that such a study will contribute to a deeper understanding of academic strengths and shortcomings, as well as the relationship between the subject matter and instruction. For example, teaching and learning culture promotes proof, i.e., logically manipulating mathematics objects. The behaviors stimulate mathematics learning based on experiences and matters presented in relatively bounding for other subjects (or students). But offering the content ability prioritizes prior experience (information retrieval) built solidly and contrary to the richness of scientific culture (Francis & Jacobsen, 2013). While, Ulusoy & Çakiroglu (2013) states that higher education students must also express mathematics in the information process. They can use an indirect object to assist the challenging context. In the micro didactical approach, the indirect object applies to broader mathematics terms for opening a meaning. For

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instance, class culture is a set of behaviors that could be constructed and handled during teaching and learning. At the same time, Yildirim (2013) states that the learning culture is naturalistic and solves problems. The way they teach math today is entirely confusing and unintuitive. That is to students with mathematical talent and reasoning skills. They'll learn math but aren't good at it. However, they could probably solve any problems on the homework, but getting a 0 for not solving them the way the books say a student should solve them. For instance, linear algebra makes a lot more sense for solving systems of equations than taught in pre-algebra. In mathematics, someone who comes up with a new way to solve an old problem that cuts out unnecessary steps is given awards for achievement. For example, students team up to build a model and minimize the necessary entities.

CONCLUSION

We provided a detailed description of the theoretical frameworks that support trajectory concepts, research efforts, and empirical findings. Our definitions and importance ratings were based on a thorough examination of the connections between and differences between the various concepts. The contribution of research to each topic can be better understood when theoretical structures are identified within a holistic framework. Further describing one or more of these concepts for future research could expand the implications of information searching and retrieval studies.

Research results have shown that understanding cognition trajectory distinguishes from teaching and learning itself given to teachers during the design and implementation process. In addition, the interrelation between cognition and teaching and learning mathematics could reveal a theoretical (or conceptual) framework contributing to mathematics classes. However, we need elaboration; for example, teaching geometry with a combination of the cognition trajectory and the information representation considering a direction for future research. In other words, mathematics ought to be learned in dynamic (image) information of problems, not merely in mathematical thinking and reasoning, but processing by the cognition trajectory. That develops teaching and learning systems to solve problems or to develop mathematics knowledge. As an illustration, one of the trajectories is cryptography information, which is practical and seems to have run out of steam, and string theory, which is un-provable but has run out of mathematics. We conclude that the concepts of teaching and learning mathematics basing on the characteristic of cognition built by the trajectory to get any mathematics education products.

References

Arzarello, F., & Taranto, E. (2021). Mathematics teacher educators within new technological environments: changing the perspective. In M. Goos & K. Beswick (Eds.), The Learning and Development of Mathematics Teacher Educators (pp. 383–400). Springer.

Beilstein, S.O., Henricks, G.M., Jay, V., Perry, M., Bates, M.S., Moran, C.G., & Cimpian, J.R. (2021). Teacher voices from an online elementary mathematics community: examining perceptions of professional learning. Journal of Mathematics Teacher Education, 24, 283–308.

ISSN 1533-9211
DOI 10.5281/zenodo.7118087

Borba, M.C. & Llinares, S. (2012). Online mathematics teacher education: overview of an emergent field of research. ZDM Mathematics Education, 44, 697–704.

Fernandez, C., Llinares, S., & Rojas, Y. (2020). Prospective mathematics teacher development of noticing in an online teacher education program. ZDM Mathematics Education, 52, 959–972.

Francis, K. & Jacobsen, M. (2013). Synchronous online collaborative professional development for elementary mathematics teachers. International Review of Research in Open and Distance Learning, 14(3), 319– 343.

Gumiero, B. S., & Pazuch, V. (2021). Teachers' knowledge mobilized in geometry lessons and contingency situations. International Electronic Journal of Mathematics Education, 16(1).

Isiksal, M., & Çakiroglu, E. (2011). The nature of prospective mathematics teachers' pedagogical content knowledge: The case of multiplication of fractions. Journal of Mathematics Teacher Education, 14(3), 213–230.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. Journal for Research in Mathematics Education, 41(2), 169–202.

Livy, S., Herbert, S., & Vale, C. (2019). Developing primary pre-service teacher' mathematical content knowledge: opportunities and influences. Mathematics Education Research Journal, 31, 279–299.

Longo (2010) Theorems as constructive visions. Invited lecture, ICMI 19 conference on Proof and Proving, Taipei, Taiwan, May 10 - 15, to appear.

Mason, J. (2021). Learning about noticing: by and through noticing. ZDM Mathematics Education, 53, 231–243.

Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), Mathematical Knowledge in Teaching (pp. 9–25). Springer.

Rif'at, M, (2001). The Effect of Visual Teaching and Learning for Enhancing Visual Deduction on the Problems (Pengaruh Pembelajaran Visual Dalam Rangka Meningkatkan Kemampuan Deduksi Visualistik pada Soal Berciri Visual), Unpublished doctor's dissertation, Bandung, Post Graduate Program, UPI Bandung, 2001.

Rif'at, M. (2017). Building Up Pre-Service Mathematics Education Students' Geometry Competency Based On Visual. 5th South East Asia Development Research (SEA-DR), International Conference. Advances in Social Science, Education and Humanities Research, 100, 164-173. Atlantis Press.

Rif'at, M. (2018). The Exploring of Visual Imagery: In Their Relation to the Students' Mathematical Identity. Higher Education Research, 3 (5), pp. 75-91.

Rif'at, M., Kusumastuti, N., & Siregar, N. (2022). Visual representation in solving mathematics problems. Proceedings of the Sixth SENATIK Conference for Research, AIP (pp. Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (pp. 2741–2748)

Rif'at, M & Sudiansyah. (2022). Development of Mathematics Learning Model Concept-Based Understanding. Budapest International Research and Critics Institute Journal (BIRCI Journal), 5 (3), pp. 99-109.

Rivera, F., & Becker, J.R. (2011). Formation of pattern generalization involving linear figural patterns among middle school students: Results of a three-year study. In J. Cai & E. Knuth (Eds.), Early Algebraic (pp. 323–363). Springer.

Rowland, T. (2013) The Knowledge Quartet: the genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. SISYPHUS Journal of Education, 1(3),15–43.

Ruthven, K. (2011). Conceptualizing mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), Mathematical Knowledge in Teaching (pp. 83–96). Springer.

Sen, C. (2021). Assessment of a middle school mathematics teacher's knowledge for teaching the 5th grade subject of fractions. Turkish Journal of Computer and Mathematics Education, 12(1), 96–138.

DOI 10.5281/zenodo.7118087

Simon, M.A. (2014). Hypothetical learning trajectories in mathematics education. In S. Lerman (Ed.), Encyclopaedia of Mathematics Education, (pp. 272–275). Springer.

Tanisli, D. (2013). Preservice primary school mathematics teachers' questioning skills and knowledge of students in terms of pedagogical content knowledge. Education and Science, 38(169), 80–95.

Tanisli, D., Ayber, G. & Karakuzu, B. (2018). Lesson design integration of the middle school mathematics teachers into teaching. Anadolu Journal of Educational Sciences International, 8(2), 514–567.

Tanisli, D., Köse, N., & Turgut, M. (2019). Professional development through a web-based portal: The progress of mathematics teachers teaching algebra based on hypothetical learning trajectories. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (pp. 2741–2748). Utrecht University and ERME.

Turner, F. (2012). Using the knowledge quartet to develop mathematics content knowledge: the role of reflection on professional development. Research in Mathematics Education, 14(3), 253–271.

Turner, F., & Rowland, T. (2011). The knowledge quartet as an organising framework for developing and deepening teachers' mathematical knowledge. In T. Rowland & K. Ruthven (Eds.), Mathematical Knowledge in Teaching (pp. 195–212). Springer.

Türnüklü, E., Akkas, E.N., & Gündogdu Alayli, F. (2015). Mathematics teachers' perceptions of quadrilaterals and understanding the inclusion relations. In B. Ubuz, Ç. Haser & M.A. Mariotti (Eds.), Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education (pp. 705–714). Middle East Technical University and ERME.

Ulusoy, F. & Çakiroglu, E. (2013). In-service elementary mathematics teachers' conceptions of histogram and difficulties about its teaching process. Elementary Education Online, 12(4), 1141–1156.

Weston, T. L. (2018). Using the knowledge quartet to support prospective teacher development during methods coursework. In S. E. Kastberg, A. M. Tyminski, A. E. Lischka, & W. B. Sanchez (Eds.), Building Support for Scholarly Practices in Mathematics Methods (pp. 69–83). Information Age Publishing.

Wickstrom, M.H., & Langrall, C.W. (2020). The case of Mrs Purl: Using a learning trajectory as a tool for teaching. Journal of Mathematics Teacher Education, 23, 97–125.

Wilson P. H., Mojica, G. F., & Confrey, J. (2013). Learning trajectories in teacher education: Supporting teachers' understandings of students' mathematical thinking. Journal of Mathematical Behaviour, 32, 103–121.

Yang, K.-L., Hsu, H.-Y., Lin, F.-L., Chen, J.-C., & Cheng, Y.-H. (2015). Exploring the educative power of an experienced mathematics teacher educator-researcher. Educational Studies in Mathematics, 89, 19– 39.

Yildirim, A. (2013). Teacher education research in Turkey: Trends, issues and priority areas. Education and Science, 38(169), 175–191.