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STABILITY ANALYSIS OF GANG HIERARCHY MODEL: A GRAPHICAL APPROACH

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Abstract:

Crime is a purposeful action or exclusion infringing upon regulation. A non linear mathematical model has been proposed and investigated where concentration is on reducing crime in the society. In modelling process, total population is divided into four compartments based on gang hierarchy namely, potential, fringe, core member and jailed. The basic reproductive number (R_0) is calculated using next generation matrix and found to be greater than 1 which is an ideal situation. Both local and global stability analysis is derived to understand the rate of criminals committing crime in society. Graph-theory method based on Kirchhoff's matrix theorem is used to prove the global stability of the endemic equilibrium which is globally asymptotically stable. The model is validated by numerical simulation to authenticate theoretical results.

Keywords: Mathematical model, gang hierarchy, Lyapunov function, global stability, equilibria, simulation.

1. Introduction

Mathematical modelling is the method of sketching a physical world problem in terms of equations and using these equations for both understanding and finding the solution to the problems. Members committing crime are criminals and scientific study of crime as social phenomenon is called criminology. Crime is one of the major and neglected problem in our society. Social problems will quite often foster when we become careless and neglect to see the issues that are created. Social problems need not affect people directly, but often affect all of us indirectly. When a person commits crime not just he gets affects but also his family members and community at large. The best model to describe crime is infectious disease model as infection spreads from person to person when in contact; similarly, non-criminals also have high chances of becoming criminal. Crime not only spreads in society because of interactions between criminals and non-criminals but also due to various factors like poverty, unequal opportunity, malnutrition, employment discrimination and substance abuse. The main aim of punishment is to deprive the power of criminals from the society by sending them to rehabilitation or imprisonment. By sending criminals to rehabilitation, the rate of driving back to criminal activities after punishment will decrease which is extremely useful in controlling crime in society. Crime can also be caused with the help of technology. (Steinmetz, 2022) talks about computer related crimes and how much of a necessity technology are. (Gordon et al., 2022) offer new theoretical insights into how the relationship between technology and damage might be rethought and problematized.

Many researcher have done studies on infectious disease using mathematical model over the past few years. The effects of the BCG vaccination on COVID-19 transmission, severity, and death have been modelled by (Shah et al., 2021). The results obtained shows that BCG vaccination has lower chances of getting COVID-19 infection, limited hospital stays and increase rate of recovery. (Shah, Sheoran and Shah, 2020) have developed HIV-TB coinfections disease model taking HIV-contaminated person as the initial stage. Reproductive number is found to be greater than 1 which implies that HIV-TB cannot vanish completely from the society. (Hategekimana, Saha and Chaturvedi, 2017) used a SEICRS epidemiological model to study amoebiasis. It is established that both disease-free and disease-endemic equilibria are asymptotically stable locally and globally. In this paper crime has been studied using the same epidemiological approach as mentioned in all the previous studies. The same infectious disease model is used to study crime by (McMillon, Simon and Morenoff, 2014; González-Parra, Chen-Charpentier and Kojouharov, 2018; Raimundo, Yang and Massad, 2018; Roslan et al., 2018). In particular, (Shukla et al., 2013) has researched how technology may reduce crime in society. According to the established model, the equilibrium density of the burden of crime diminishes as technology advances. In addition, (Misra, 2014) have proposed and analyzed a dynamical model to investigate how the use of force by the police affects societal crime control. It has also been shown that crime in society can be entirely under control if criminal immigrants are deported and there are enough police personnel on the streets. Further, (Goyal et al., 2015) analyzed government efforts to control criminal activity in a society using a mathematical model. The analysis demonstrates that the propagation of criminals ideologies can be restrained if the appropriate level of government pressure is exerted. Not just infectious disease or crime, many of the social problem faced by society can be studied using this epidemiological approach.

(Yeolekar and Shukla, 2015) have studied the problem of liquor transmission. Using numerical simulation it is shown that 44% of the population in infectious class is getting liquored and almost 15% gets chain liquor that is those who cannot live with liquor. (Shah et al., 2018) have formulated mathematical model to study obesity which causes infertility in women. Analysis shows that system formulated is both locally and globally stable. Numerical simulation shows that 17% of women's affect by infertility are due to intake of high calorie food. (Chinnadurai and Athithan, 2022) constructed model using ordinary differential equation to study unemployment during COVID-19. Particularly, middle-income countries were severely impacted. In the study they say by decreasing the effectiveness of COVID-19 dissemination, unemployment decreases.

In most of the studies cited above, global stability is not done using graph-theoretical method which is considered in this paper. The paper is organized as follows, model description, reproductive number, local stability analysis, global stability analysis and numerical simulation.

2. Mathematical Modeling

In modelling process, total population T at time $t > 0$ is divided into four compartments or disjoint groups (P, C, F, J) who vary in their crime lifestyle(Glosser et al., no date). P represents the class who are currently not vulnerable to crime life, yet are potential to criminal activities.

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Criminals are represented by F and C where F represents fringe membership and C represents the core membership. Core members are people who have been with the group for long time and are probably going to be associated with the group forever and are most likely to be the most threatening members. The fringe members engage in low-level criminal activity and socialize with the gang. J represents members who are jailed.

The notation and parameters used in modelling process are given in the following table 1.

Table 1: Notations and parameters

Notation	
Λ	Recruitment rate in potential class.
β	Rate at which potential class members convert to fringe members.
ϵ	Rate at which fringe class member become core member.
γ	Imprisonment rate.
η	Relapse rate.
d	Natural mortality rate.
α	Crime related mortality rate
δ	Intervention parameter for fringe members.

Using a few assumptions and parameters mentioned as above, the mathematical model is formulated. The formulated model is described by the following system of non-linear differential equations:

$$\frac{dP}{dt} = \Lambda - \frac{\beta PC}{T} + \delta F - dP$$

$$\frac{dF}{dt} = \frac{\beta PC}{T} - (d + \epsilon + \delta)F$$

$$\frac{dC}{dt} = \epsilon F - (d + \gamma + \alpha)C + \eta J$$

$$\frac{dJ}{dt} = \gamma C - (d + \eta)J$$

……………(1)

Where $P + F + C + J = T$. Also, $P > 0; F, C, J \geq 0$.

Adding all the above differential equations, we get

$$\frac{d}{dt}(P + F + C + J) = \frac{dT}{dt} = \Lambda - (\alpha C + dT) \geq 0$$

……………(2)

Therefore, the formulated model has the following feasible region

The Seybold Report
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$$\Gamma = \left\{(P,F,C,J) \in R^4 : P + F + C + J \leq \frac{\Lambda - \alpha C}{d}\right\}. \quad \text{................(3)}$$

3. Modelling Analysis

3.1 Calculation of R_0

The basic reproductive number is the measure of transmissibility of crime which is treated as infectious disease. It provides us with a straightforward explanation of how crime develops or declines in society. Theoretically, if R_0 is more than the threshold value, crime endures in society; if R_0 is less than the threshold value, crime dies out (Diekmann, Heesterbeek and Roberts, 2010). There are three main approaches to find the basic reproductive number, that is statistical(Zhao et al., 2020), stochastic(Riou and Althaus, 2020) and mathematical(Fosu, Akweittey and Adu-Sackey, 2020). Here mathematical approach has been used to obtain R_0 using Next Generation Matrix (NGM).

The subsystem is deduced from (1) as

$$\frac{dF}{dt} = \frac{\beta PC}{T} - (d + \epsilon + \delta)F$$

$$\frac{dC}{dt} = \epsilon F - (d + \gamma + \alpha)C + \eta J$$

$$\frac{dJ}{dt} = \gamma C - (d + \eta)J \quad \text{................(4)}$$

(4) Is considered as infectious compartment and applying NGM method,

$$F = \begin{bmatrix} 0 & \frac{\beta P^*}{T^*} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} (d + \epsilon + \delta) & 0 & 0 \\ -\epsilon & (d + \gamma + \alpha) & -\eta \\ 0 & -\gamma & (d + \eta) \end{bmatrix}$$

Thus the R_0 can be calculated as $\rho(FV^{-1})$

$$\Rightarrow R_0 = \frac{\beta P^*}{T^*} \frac{\epsilon(d + \eta)}{(d + \epsilon + \delta)((d + \alpha)(d + \eta) + d\gamma)}$$

3.2 Equilibrium

This section will determine the equilibrium of the local stability and global stability of the potential crime model. There are two equilibrium points for the constructed model, crime free equilibrium $E_1 = \left(\frac{\Lambda}{d}, 0, 0,0\right)$ and crime persistent equilibrium E_2. In this paper, we find local

The Seybold Report
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and global stability for E_2, that is crime persistent equilibrium as is it realistic as people committing no crime is not possible in the society.

3.2.1 Local Stability

The right hand sides of each equation in model (1) can be set to unity to yield the crime persistent equilibrium point E_2 and taking $T = T^*$, we get

$$C^* = \frac{\Lambda - dT^*}{\alpha}, J^* = \frac{\gamma(\Lambda - dT^*)}{\alpha(d+\eta)}, F^* = \frac{(\Lambda - dT^*)[(d+\gamma+\alpha)(d+\eta) - \eta\gamma]}{\alpha\epsilon(d+\eta)}$$

$$and\ P^* = \frac{(d+\epsilon+\delta)[(d+\gamma+\alpha)(d+\eta) - \eta\gamma]}{\beta\epsilon(d+\eta)} T^*.$$

Therefore, $E_2 = (P^*, F^*,\ C^*,\ J^*)$ be the unique positive equilibrium point of the system (1).

The stability of the equilibria E_2 is discussed in details and to determine local stability of E_2, the Jacobian matrix for model (1) is given as follows:

$$J = \begin{bmatrix} -\frac{\beta C^*}{T^*} - d & \delta & -\frac{\beta P^*}{T^*} & 0 \\ \frac{\beta C^*}{T^*} & -(d+\epsilon+\delta) & \frac{\beta P^*}{T^*} & 0 \\ 0 & \epsilon & -(d+\gamma+\alpha) & \eta \\ 0 & 0 & \gamma & -(d+\eta) \end{bmatrix}$$

For the Jacobian matrix $J(P^*, F^*,\ C^*,\ J^*)$, $|J - \lambda I| = 0$ is the characteristic equation and the roots of following quadratic equation gives us the eigenvalues:

$$\lambda^4 + \tilde{a}\lambda^3 + \tilde{b}\lambda^2 + \tilde{c}\lambda + \tilde{d} = 0$$

Where,

$$\tilde{a} = (4d + \alpha + \gamma + \delta + \epsilon + \eta) + \frac{\beta C^*}{T^*}$$

$$\tilde{b} = (d+\eta)\left(3d + \alpha + \delta + \epsilon + \frac{\beta C^*}{T^*}\right) + (d+\epsilon)\frac{\beta C^*}{T^*} + (d+\epsilon+\delta)(2d+\alpha+\gamma) + d\gamma + (d+\gamma+\alpha)\left(d + \frac{\beta C^*}{T^*}\right) + \frac{\beta\epsilon P^*}{T^*}$$

The Seybold REPORT

ISSN 1533-9211

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$$\tilde{c} = (d+\gamma+\alpha)\left(\frac{\beta C^*}{T^*} + d + 2d^2 + d\eta + 2d\epsilon + \epsilon\eta + 2d\delta + \delta\eta + \frac{\beta C^*}{T^*}(d+\epsilon)\right) + (d+\epsilon+\delta)\frac{\beta C^*}{T^*} + (d+\epsilon+\delta)d - \frac{\beta d\delta C^*}{T^*} - \frac{\beta\delta\eta C^*}{T^*} + \frac{\beta\epsilon\eta P^*}{T^*} - 2d\gamma\eta - \gamma\epsilon\eta - \gamma\delta\eta - \frac{\beta\gamma\eta C^*}{T^*}$$

$$\tilde{d} = (d+\eta)\left[d(d+\epsilon+\delta)(d+\gamma+\alpha) - \frac{\beta\epsilon d P^*}{T^*} - \frac{\beta\delta C^*}{T^*}(d+\gamma+\alpha)\right] - d\gamma\delta\eta - d\gamma\epsilon\eta - d^2\gamma\eta + d\left(\frac{\beta d C^*}{T^*} + \frac{\beta\epsilon C^*}{T^*} + \frac{\beta\delta C^*}{T^*}\right)(d+\gamma+\alpha) + [\eta(d+\epsilon+\delta)(d+\alpha) + \gamma\delta\eta]\frac{\beta C^*}{T^*}$$

It follows from above that $\tilde{a},\ \tilde{b},\tilde{c},\tilde{d} > 0$ and $\left(\tilde{a}\tilde{b}-\tilde{c}\right)\tilde{c} > \ \tilde{a}^2\tilde{d}$. Therefore by the Rowth-Hurwitz criterion, the unique positive equilibrium point E_2 of the model (1) is locally asymptotically stable.

3.2.2. Global Stability

Global stability is studied for crime persistence equilibrium using Kirchhoff's matrix tree theorem and two new identities to derive graph theoretical conclusions, the same is discussed by (Shuai and van den Driessche, 2013; Shah, Satia and Yeolekar, 2018).

Proposition (Kirchhoff's matrix tree theorem). Assume $n \geq 2$ and let c_i be the cofactor of $l_{ii}\ in\ L$ (The Laplacian matrix). Then

$$c_i = \sum\nolimits_{\tau\in\mathbb{T}_i} \omega(\tau), \qquad i = 1,2,\ldots\ldots,n$$

Where $\mathbb{T}_i$ is the set of all spanning trees $\tau\ of\ (G,A)$ that are rooted at vertex i, and $\omega(\tau)$ is the weight of τ. If (G,A) is strongly connected, then $c_i > 0\ for\ 1 \leq i \leq n.$

Theorem 1: Let c_i be as given in the Kirchhoff's matrix tree theorem. If $a_{ij} > 0$ and $d^+(j) = 1$ for some $i,j\ then\ c_i a_{ij} = \sum_{k=1}^{n} c_k a_{ki}$.

Theorem 2: Let c_i be as given in the Kirchhoff's matrix tree theorem. If $a_{ij} > 0$ and $d^-(j) = 1$ for some $i,j\ then\ c_i a_{ij} = \sum_{k=1}^{n} c_j a_{jk}$.

Theorem 3: Let U be an open set in $\mathbb{R}^m$. Suppose that the following assumptions are satisfied:

i. There exist function $L_i : U \to \mathbb{R}, G_{ij}: U \to \mathbb{R}$ and constants $a_{ij} \geq 0$ such that for every $1 \leq i \leq n, L_i' \leq \sum_{j=1}^{n} a_{ij} G_{ij}(z)\ for\ z \in U.$

ii. For $A = \left[a_{ij}\right]$, each directed cycle $C\ of\ (G,A)$ has $\sum_{(s,r)\in\varepsilon(C)} G_{rs}(z) \leq 0\ for\ z \in U,\ where\ \varepsilon(C)$ denotes the arc set of the directed cycle C.

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Then, the function $L(z) = \sum_{i=1}^{n} c_i L_i(z)$, with constant $c_i \geq 0$ as given in the proposition of Kirchhoff's matrix tree theorem, satisfies $L' \leq 0$ $then$ V is a Lyapunov function for the system.

Lyapunov function will be created in order to determine the specific positive equilibrium point E_2.

Let $L_1 = P - P^* - P^* ln\frac{P}{P^*}$, $L_2 = F - F^* - F^* ln\frac{F}{F^*}$, $L_3 = C - C^* - C^* ln\frac{C}{C^*}$ and $L_4 = J - J^* - J^* ln\frac{J}{J^*}$.

Now differentiating L_1 with respect to t, we get

$$L_1' = \left(1 - \frac{P^*}{P}\right) P'$$

$$= \left(1 - \frac{P^*}{P}\right)\left(\Lambda - \frac{\beta PC}{T} - dP + \delta F\right)$$

$$= \left(1 - \frac{P^*}{P}\right)\left(\frac{\beta P^* C^*}{T^*} + dP^* - \delta F^* - \frac{\beta PC}{T} - dP + \delta F\right)$$

$$\leq \beta\left(1 - \frac{P^*}{P}\right)\left(\frac{P^* C^*}{T^*} - \frac{PC}{T}\right)$$

$$= a_{13} G_{13}$$

Similarly, we get

$$L_2' = \left(1 - \frac{F^*}{F}\right) F'$$

$$= \left(1 - \frac{F^*}{F}\right)\left(\frac{\beta PC}{T} - dF - \epsilon F - \delta F\right)$$

$$\leq \frac{\beta P^* C^*}{T^*}\left(1 - \frac{F^*}{F}\right)\left(1 - \frac{PCT^*}{P^* C^* T}\right)$$

$$= a_{21} G_{21}$$

$$L_3' = \left(1 - \frac{C^*}{C}\right) C'$$

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$$= \left(1 - \frac{C^*}{C}\right)(\epsilon F - dC - \gamma C - \alpha C + \eta J)$$

$$\leq \epsilon F^* \left(1 - \frac{C^*}{C}\right)\left(1 - \frac{F}{F^*}\right) + \eta J^* \left(1 - \frac{C^*}{C}\right)\left(1 - \frac{J}{J^*}\right)$$

$$= a_{32}G_{32} + a_{34}G_{34}$$

and

$$L_4' = \left(1 - \frac{J^*}{J}\right)J'$$

$$= \left(1 - \frac{J^*}{J}\right)(\gamma C - dJ - \eta J)$$

$$\leq \gamma C^* \left(1 - \frac{J^*}{J}\right)\left(1 - \frac{C}{C^*}\right)$$

$$= a_{43}G_{43}$$

With the set of four vertices and the above results obtained, the weighted graph is constructed and shown in figure 1.

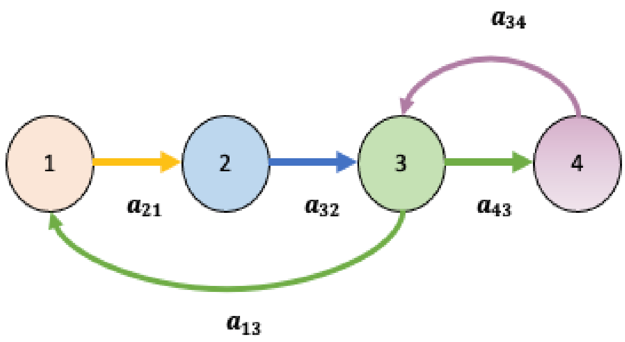

Figure 1: For the transmission of crime model the weighted graph (G,A) is constructed

With $a_{13} = a_{21} = \frac{\beta P^* C^*}{T^*}, a_{32} = \epsilon F^*, a_{34} = \eta J^*, a_{43} = \gamma C^*$ and all other $a_{ij} = 0$. From figure 2 we can see that the weighted graph (G, A) has four and two, vertices and cycles respectively. Where each cycle $G_{21} + G_{32} + G_{13} = 0$ *and* $G_{43} + G_{34} = 0$. Then, there exists $c_i, 1 \leq i \leq 4$, such that $L = \sum_{i=1}^{4} c_i L_i$ is Lyapunov function for (1) by theorem 3.

The relationship between $c_i's$ can be found from theorem 1 and theorem 2,

$d^+(1) = 1 \Rightarrow c_2 a_{21} = c_1 a_{13} \Rightarrow c_1 = c_2 = u\ (as\ a_{13} = a_{21})$,

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$$d^+(2) = 1 \Rightarrow c_3 a_{32} = c_2 a_{21} \Rightarrow c_3 = \frac{c_2 a_{21}}{a_{32}} = \frac{u\beta P^* C^*}{\epsilon F^* T^*} = v$$

And

$$d^-(4) = 1 \Rightarrow c_4 a_{43} = c_3 a_{34} \Rightarrow c_4 = \frac{c_3 a_{34}}{a_{43}} = \frac{v\eta J^*}{\gamma C^*}.$$

Therefore,

$$L = \sum_{i=1}^{4} c_i L_i$$

$$= c_1 L_1 + c_2 L_2 + c_3 L_3 + c_4 L_4$$

$$= uL_1 + uL_2 + vL_3 + \frac{v\eta J^*}{\gamma C^*} L_4$$

Where u and v are arbitrary constants.

This verifies that $\{E_2\}$ is the only invariant set in $int(\Gamma)$, where $L' = 0$. Hence E^* is globally asymptotically stable in $int(\Gamma)$ and thus unique.

4. Numerical Simulation

Numerical simulation is performed in this section to study the behavior of model (1) formulated. This is done using MATLAB tool box. The values of the parameters considered is given in Table 2 (Comissiong, Sooknanan and Bhatt, 2012; Sooknanan, Bhatt and Comissiong, 2013). This is due to the reason that, obtaining real time data from the system is a daunting task but will make an effort to collect the same in the future work.

Table 2: Parameter and their values used in model simulation

Parameter	Λ	α	β	γ	δ	ϵ	η	d
Value	1000	0.0503	0.71	0.115	0.1265	0.21	0.56	0.00258

The parameter values listed above indicate, the value of R_0 is greater than 1 and the components of the equilibrium are obtained as follows

$$P^* = 3736.0, F^* = 4709.3, C^* = 18238.0, J^* = 3722.5$$

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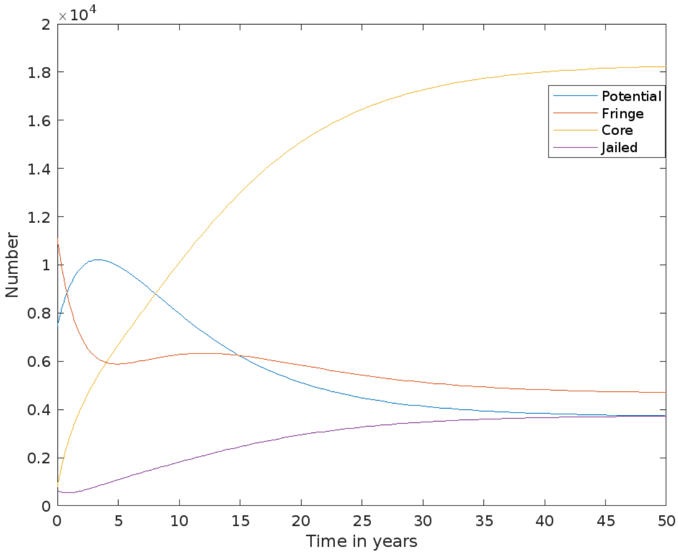

Figure 2: Time series plot for model (1)

Figure 2 depicts the coexistence of persistent equilibrium for Model (1) with the set of parameter values taken into account in Table 2. The advantage of simulation is that the changes happening to all the classes simultaneously with time progression can been seen and it can also be seen in figure 2 that the core members will continuously increase with time.

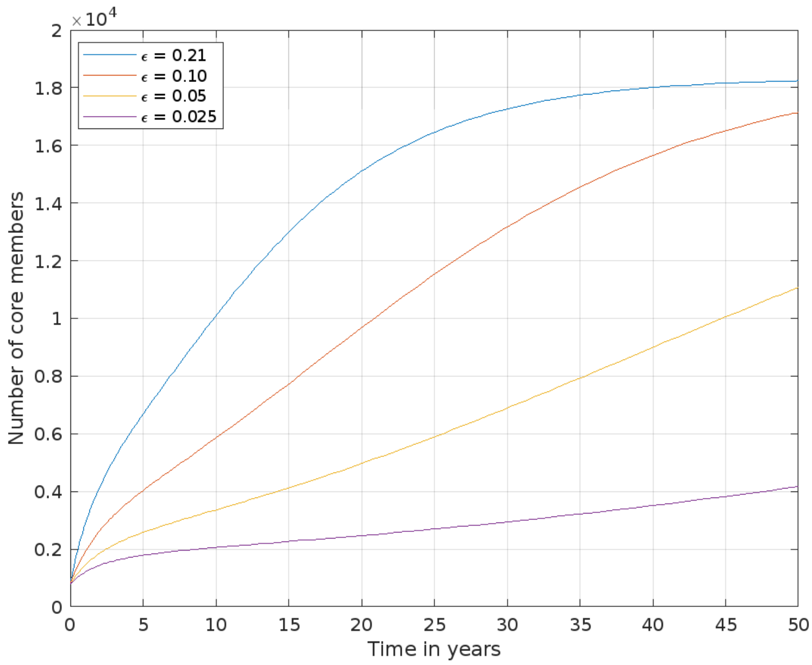

Figure 3: Change in number of core member for different rate of fringe member converting to core member, ϵ.

ISSN 1533-9211
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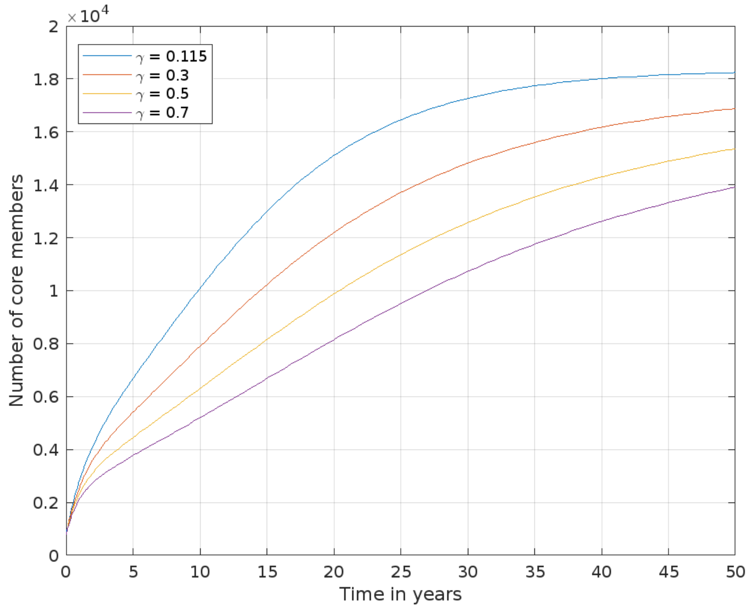

Figure 4: Effect on the core member for different rate of imprisonment, γ.

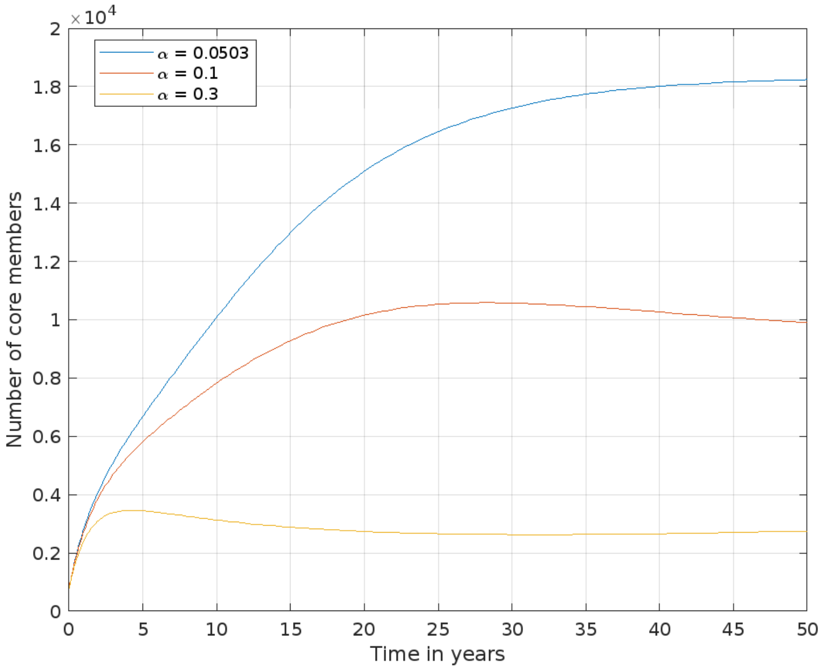

Figure 5: Effect on the core member for different rate of crime related death, α.

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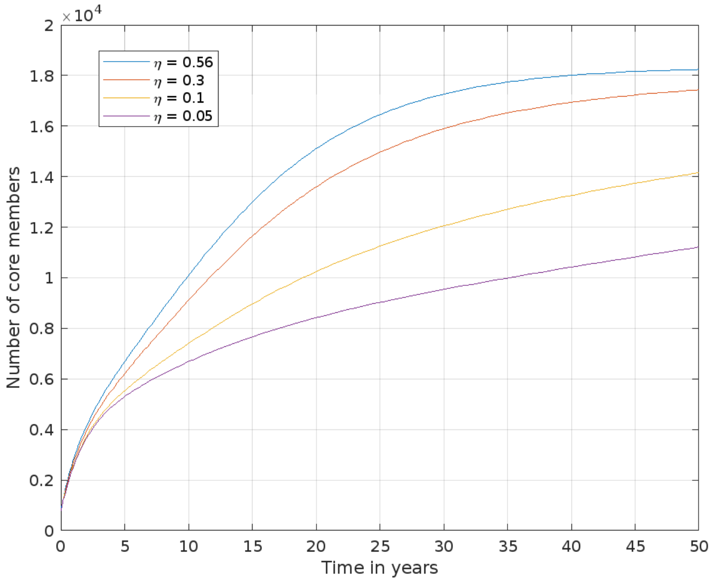

Figure 6: Variation in number of core member for different relapse rate, η.

It is captivating to study the impact of ϵ, α, γ and η on the core member population. As a result, we create several graphs that show how these four factors value affect the variation of the core members (see Figure 3, 4, 5 and 6). Figure 3 shows that as the value of ϵ declines, the population of core members also does. The best policy to decrease ϵ is to council them so they stop all the criminal activity. On the other side, as the value of γ and α rises, the population of core members declines (see Figure 4 and 5).

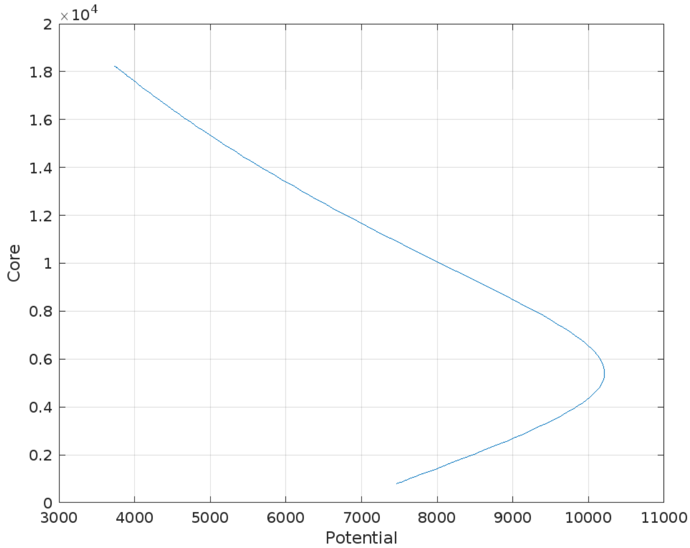

Figure 7: Interaction between groups (Potential and Core) for model developed

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Similarly, Figure 6 shows that as the value of η falls, the number of core members also reduces. η value can be decreased by providing jobs after imprisonment to lead their life so that they will not relapse to commit crime. Figure 7 shows the interaction between different groups. Potential verses core member graph is generated using the above set of parameters. When interaction between potentials and core member increase, core member population increases in society.

5. Discussion

In the previous work done by the author, the concentration was on local stability analysis and the numerical simulation. In the present paper more emphasis has been given to the global stability analysis. This has been achieved through graph theoretical approach based on Kirchhoff's matrix tree theorem and two new combinatorial identities which was mentioned in the work of (Shuai and van den Driessche, 2013) for disease endemic equilibrium. But in this work author has not done any simulation and it is for endemic diseases. In the present model same method has been used for crime model and numerical simulation has been performed to authenticate theoretical results. One more compartment (fringe) has been introduced in comparison with previous work, in which the significance is given to gang hierarchy approach. It is observed that the core members can be minimized by minimizing the contact of entry level criminals with the core members who have entered into hard core criminal activities and is shown in figure 3.

(Comissiong, Sooknanan and Bhatt, 2012) have used same gang hierarchy approach used in the present work but they have not shown how the interaction is between compartments which is shown in this paper which gives clear picture when interaction between potentials and core member increase, core population increase in society and global stability analysis is also done using two new methods in our work which is not done by them. (Shukla et al., 2013) have done study on what is the role of technology in reducing crime in society. They have done stability analysis for the formulated model and threshold parameter (R_0) is not found which is calculated in the present work. The threshold parameter (R_0) gives a clear picture on whether crime persist or dies out in society. If $R_0 > 1$ crime persist in society and $R_0 < 1$ crime diminishes. (Comissiong, Sooknanan and Bhatt, 2012) have calculated threshold parameter for the model formulated and it is found to be more than 5 which means crime exists in society even after decades but in the present work R_0 is very close to 1 and it can be reduced by varying the parameter values to reduce crime in society. In future varying the parameter will be done to check the effect on model.

6. Conclusion

A crime is an act that is against the law. As discussed in Section 1 crime is one of the major and neglected social problem and possibly compared to an epidemic's spread. The compartmental epidemic modelling approach is therefore ideal for researching crime in society. Four compartments are created from the total population namely, potential, fringe, core member and jailed. Reproductive number R_0 is obtained and this shows us crime persist in

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society as R_0 obtained in greater than 1. Two equilibrium points, crime-free equilibrium E_1 and crime persistence equilibrium E_2 are obtained by the equilibrium analysis performed in Section 3.2. The analysis in section 3.2.1 shows that E_2 is locally asymptotically stable. The conditions for global stability for the crime persistence equilibrium point E_2 are obtained using Lyapunov function and graph theoretical results based on Kirchhoff's matrix tree theorem. Various parameters effect on core population has been studied numerically in section 4. From figure 3 and 6, it is clear that decrease in $\eta\ and\ \epsilon$ decreases core member population and in turn decreases crime in society. Increase in $\alpha\ and\ \gamma$ decreases core member population and this can been seen from figure 4 and 5. Also the interaction between different compartments can be seen from figure 7.

The following points are highlighted in current study

- Interaction between potential and core member plays an important role for conversion from potential to fringe which can be one of the reason to increase crime in the society.
- The parameter δ in the current work represents the fact that the fringe member may depart the class for a variety of causes. Counseling, family pressure, and self-realization are a few of the causes.
- Graph theoretical results based on the construction of the Lyapunov function and Kirchhoff's matrix tree theorem are used to determine the global stability of the crime persistent equilibrium. It is discovered to be globally asymptotically stable.
- In section 5, the impact of $\epsilon, \alpha, \gamma\ and\ \eta$ on the population of core member is shown.

In future work some more parameters and state variables can be incorporated to make the model more realistic. Also case studies will be considered in future to study the effect of this method on real time data.

7. References

Chinnadurai, K. and Athithan, S. (2022) 'Mathematical modelling of unemployment as the effect of COVID-19 pandemic in middle-income countries', The European Physical Journal Special Topics 2022, pp. 1–8. Available at: https://doi.org/10.1140/EPJS/S11734-022-00620-8.

Comissiong, D.M.G., Sooknanan, J. and Bhatt, B. (2012) 'Life and Death in a Gang-A Mathematical Model of Gang Membership', Journal of Mathematics Research, 4(4), p. p10. Available at: https://doi.org/10.5539/JMR.V4N4P10.

Diekmann, O., Heesterbeek, J.A.P. and Roberts, M.G. (2010) 'The construction of next-generation matrices for compartmental epidemic models', Journal of the Royal Society Interface, 7(47), p. 873. Available at: https://doi.org/10.1098/RSIF.2009.0386.

Fosu, G.O., Akweittey, E. and Adu-Sackey, A. (2020) 'Next-generation matrices and basic reproductive numbers for all phases of the Coronavirus disease', Open Journal of Mathematical Sciences, 4(1), pp. 261–272. Available at: https://doi.org/10.30538/OMS2020.0117.

Glosser, A.M. et al. (no date) 'Gangs in a rural town: An analysis of community perceptions of crime, gangs, and the new in-migrant population'.

ISSN 1533-9211

DOI 10.5281/zenodo.7128838

González-Parra, G., Chen-Charpentier, B. and Kojouharov, H. v. (2018) 'Mathematical modeling of crime as a social epidemic', Journal of Interdisciplinary Mathematics, 21(3), pp. 623–643. Available at: https://doi.org/10.1080/09720502.2015.1132574.

Gordon, F. et al. (2022) 'Beyond Cybercrime: New Perspectives on Crime, Harm and Digital Technologies', International Journal for Crime, Justice and Social Democracy, 11(1), pp. I–VIII. Available at: https://doi.org/10.5204/IJCJSD.2215.

Goyal, A. et al. (2015) 'Modeling the role of government efforts in controlling extremism in a society', Mathematical Methods in the Applied Sciences, 38(17), pp. 4300–4316. Available at: https://doi.org/10.1002/MMA.3368.

Hategekimana, F., Saha, S. and Chaturvedi, A. (2017) 'Dynamics of Amoebiasis Transmission: Stability and Sensitivity Analysis', Mathematics 2017, Vol. 5, Page 58, 5(4), p. 58. Available at: https://doi.org/10.3390/MATH5040058.

McMillon, D., Simon, C.P. and Morenoff, J. (2014) 'Modeling the Underlying Dynamics of the Spread of Crime', PLOS ONE, 9(4), p. e88923. Available at: https://doi.org/10.1371/JOURNAL.PONE.0088923.

Misra, A.K. (2014) 'Modeling the effect of police deterrence on the prevalence of crime in the society', Applied Mathematics and Computation, 237, pp. 531–545. Available at: https://doi.org/10.1016/J.AMC.2014.03.136.

Raimundo, S.M., Yang, H.M. and Massad, E. (2018) 'Contagious Criminal Career Models Showing Backward Bifurcations: Implications for Crime Control Policies', Journal of Applied Mathematics, 2018. Available at: https://doi.org/10.1155/2018/1582159.

Riou, J. and Althaus, C.L. (2020) 'Pattern of early human-to-human transmission of Wuhan 2019 novel coronavirus (2019-nCoV), December 2019 to January 2020', Euro surveillance : bulletin Europeen sur les maladies transmissibles = European communicable disease bulletin, 25(4). Available at: https://doi.org/10.2807/1560-7917.ES.2020.25.4.2000058.

Roslan, U.A.M. et al. (2018) 'A MATHEMATICAL MODEL ON THE DYNAMICS OF POVERTY, POOR AND CRIME IN WEST MALAYSIA', Far East Journal of Mathematical Sciences (FJMS), 107(2), pp. 309–319. Available at: https://doi.org/10.17654/MS107020309.

Shah, N.H. et al. (2018) 'Mathematical Model of Infertility Due to Obesity in Female', pp. 38–49. Available at: http://www.newtheory.org (Accessed: 17 August 2022).

Shah, N.H. et al. (2021) 'Modelling the Impact of Nationwide BCG Vaccine Recommendations on COVID-19 Transmission, Severity and Mortality', Mathematical Engineering, pp. 21–37. Available at: https://doi.org/10.1007/978-981-33-6264-2_2.

Shah, N.H., Satia, M.H. and Yeolekar, B.M. (2018) 'Stability of 'GO-CLEAN' Model Through Graphs', Journal of Computer and Mathematical Sciences, 9(2), pp. 79–93. Available at: https://doi.org/10.29055/JCMS/734.

Shah, N.H., Sheoran, N. and Shah, Y. (2020) 'Dynamics of HIV-TB Co-Infection Model', Axioms 2020, Vol. 9, Page 29, 9(1), p. 29. Available at: https://doi.org/10.3390/AXIOMS9010029.

Shuai, Z. and van den Driessche, P. (2013) 'Global Stability of Infectious Disease Models Using Lyapunov Functions', http://dx.doi.org/10.1137/120876642, 73(4), pp. 1513–1532. Available at: https://doi.org/10.1137/120876642.

Shukla, J.B. et al. (2013) 'Role of technology in combating social crimes: A modeling study', European Journal of Applied Mathematics, 24(4), pp. 501–514. Available at: https://doi.org/10.1017/S0956792513000065.

Sooknanan, J., Bhatt, B. and Comissiong, D.M.G. (2013) 'CATCHING A GANG - A MATHEMATICAL MODEL OF THE SPREAD OF GANGS IN A POPULATION TREATED AS AN INFECTIOUS DISEASE',

ISSN 1533-9211
DOI 10.5281/zenodo.7128838

International journal of pure and applied mathematics, 83(1), pp. 25–43. Available at: https://doi.org/10.12732/IJPAM.V83I1.4.

Steinmetz, K.F. (2022) 'Crime in the Age of the Smart Machine: A Zuboffian Approach to Computers and Crime', International Journal for Crime, Justice and Social Democracy, 11(1), pp. 225–238. Available at: https://doi.org/10.5204/IJCJSD.2136.

Yeolekar, N.H. and Shukla, B.M. (2015) 'Liquor Habit Transmission Model', Applied Ma-thematics, 6, pp. 1208–1213. Available at: https://doi.org/10.4236/am.2015.68112.

Zhao, S. et al. (2020) 'Preliminary estimation of the basic reproduction number of novel coronavirus (2019-nCoV) in China, from 2019 to 2020: A data-driven analysis in the early phase of the outbreak', International Journal of Infectious Diseases, 92, pp. 214–217. Available at: https://doi.org/10.1016/J.IJID.2020.01.050