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OPTIMIZATION OF INSOLVENCY RISK OF PENSION FUND BASED ON NEUTROSOPHIC FUZZY APPROACH

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Abstract:

Pension funds needs to achieve a high-income return to able to pay an actuarial expectations of different kinds of benefits of pension plan in future. The model of asset allocation management of a pension fund must consider a wide planning horizon because of its long-term liabilities. The problem of uncertainty and vagueness environment of stock market and scarcity of data on the probability of carrying out investment planes in the capital market can be solved by using neutrosophic fuzzy numbers which lets taking into consideration all the possible designs of financial assets achievement indicators when do the model for their achievements especially in the situation perspective of realization that refers to investment plans with vague probability and investment plans without the degree of vagueness.

Keywords: Insolvency risk; Mean and variance; Neutrosophic set theory; Pension fund.

1. INTRODUCTION

The structure of financial and economic indicators is very serious for the minimizing of financial crises that will supply timely caution of potential risks. There are a large institutions are consider the second largest segment of the financial system are pension funds where it based on constant inflows provided by pension insurance it has a great potential for further growth where widely significance of it for the financial stability.

In the banking and other kinds of the financial markets, financial assets of household's pension funds are play as significant debtor and creditor of it. For ownership links between the companies that manage pension funds and local banks, so pension funds may be play as investors in it. Reputational risk is therefore difficult to measure and extremely significant (Taleski & Bogdanovski, 2015).

Haberman and Sung presented defined benefits plans have been usually modeled as quadratic optimal control problems. As clear in Merton’s model, the reality of the fund's dynamics is supposed nonlinear, and that it is actually that managers’ objectives should be related with the minimization (optimization) of contribution risk and insolvency risk. The concepts of the tow risks are defined as quadratic deviations of amortization rates and fund reserve with respect to normal cost and benefits.

There are two types of risks concerning determine the stability and security of funding of pension plan called the contribution rate risk and the insolvency risk. The main objective of the system associated with these two previous risks is to obtain the optimal contributions of system

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(plan) through dynamic optimization subject to group of constraints. The target of plan's financing status can be attained by adjust the funding policy based on the optimal funding policy (Haberman & Sung, 1994).

Actuaries confront vagueness and uncertainty because unpredictability of different parameters influencing premiums (contributions), investments cash flows, claims and benefits.

The investments of pension funds are mostly in classical assets such as bonds, bills and shares. The shape of obligations the participants of defined pension plan are benefits paid at contingencies that plan's rules have it.

By minimizing some convex combination of contribution risk and insolvency risk, the promoter of pension plan can be control the stability and security of the pension plan(Josa-Fombellida et al., 2018)

Pension funds need to make high scales of income return to match to actuarial liabilities and to pay different kind of benefits. Because of its long term obligations, an Asset and Liability Management (ALM) pattern of a pension fund should consider a great planning horizon. ALM controls of the solvency of the fund by agreeable investments strategy andcontribution policies.

Actuaries must be evaluate pension fund periodically and predict annual cash-flow of income and liabilities. The task of pension fund is paymentthe benefit to the participants.Ibrahim, R. I. et al. presents study analyzes nine scenarios under assumptions in the actuarial model, namely death rate, retirement age and rate of salary increase. The result of this study clarify that the implied death rates experience and rate of salary increase rate assumptions have an important effect on pension benefits. The funding of pension fund'sliabilities bytwo main sources: thereturns from its asset portfolio and regular contributions made by members or the sponsor of the fund (Ibrahim RI et al., 2021).

A pension fund has long term payments may be up to decades because long-term liabilities for participants and their beneficiaries, and therefore its planning horizon is very huge, the target of ALM is to find acceptable contribution and investment policies that secure that the solvency of the fund is enough during pension fund' planning horizon (Elsebaey S.S. et al. , 2021).

Nowadays, because of widely significance of pension funds for the financial stability then it has essential tools in financial markets, where the investments of it represent a large percentage of financial market process.

Since the 1980s, the use of the planning under uncertainty by stochastic programming becoming more popularity and it provides a more realistic tool for dynamic financial analysis.

Many researchers widely applied uncertain programming in management and financial problems where they proposed uncertain optimization approaches, such as approaches using interval numbers, stochastic and fuzzy logics, or uncertain variables and indeterminate methods.

The objective functions or constrained functions of uncertain programming operations are usually transformed into a crisp or deterministic programming problem to get the optimal crisp

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value of the objective function and the optimal feasible deterministic solution of the decision variables.

Indeed, an undefined optimal solution for the decision variables and the undefined optimal value of the objective function may yield by indeterminate programming problems which appropriate for real problems with indeterminate environments. So, it is essential to know how to treat indeterminate programming problems with undefined solutions. So for this reason a neutrosophic fuzzy numbers is suitable approach for determine of the optimal asset portfolios, indicators of financial performance and presenting to the investors in the stock market additional information.

Neutrosophy means the research of ideas and notions that are not true, nor false, but in the midest (i.e. ambiguous, unclear, neutral ,indeterminate, vague, incomplete, contradictory, etc.).

With an overview of fuzzy counterparts, neutrosophy drives to an entire family of not traditional only but to new mathematical theories. The uncertainty is representing in fuzzy set by using the single-valued membership which exists in its attributes. So, one cannot represent when loss, win, and draw match independently. To perform this, we need to describe them design in membership-values of truth, falsity, and indeterminacy (Smarandache F. et al., 2019).

Hussein I. H. et al. used trapezoidal fuzzy weighted membership function for solve the fuzzy multiple objective function but in this paper the optimization of neutrosophic portfolios is find portfolios consisting of more than one of financial assets, this portfolio modeled using neutrosophic fuzzy triangular numbers, where the neutrosophic return for each financial asset can be attained and hence determination of the portfolio return, the neutrosophic risk, the neutrosophic covariance for each financial asset and hence determination of the portfolio risk (Hussein I. H. & Mitlif R. J., 2021).

The investors established two main conditions on the financial market to get an optimal portfolio of financial assets. Firstly, predefined financial return as well as minimizing the risk of the financial assets neutrosophic portfolio, secondly, predefined rate of risk as well as maximization the return the financial assets neutrosophic portfolio. In the two cases should calculate the weight financial assets in all portfolios, for which the return of portfolio attains the level set by pension fund planer and the minimum value of insolvency risk.

The important question of rational investors in the capital market, what is the magnitude of money they must invest in compliance with the optimal conditions concerning the return and risk?, so the share of financial assets answers for this question by determine the optimal portfolio that satisfy investor's conviction.

Bolos et al. showed neutrosophic fuzzy theory can help portfolio management teams analyzes an investment decisions and trading history. With such real time feedback, portfolio managers may be able to avoid sub optimal decisions and improve their results over time (Boloș MI et al. , 2021).

The invention of this paper is the use of neutrosophic fuzzy triangular numbers for the performance indicators of the financial asset portfolio of pension fund. Hence the structure of

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the neutrosophic portfolio is attained and insolvency risk of pension fund. Optimal financial asset neutrosophic portfolios consists of groups of financial assets that form a portfolio for which the portfolio return has a defined value (satisfy that save pension fund towards insolvency risk) and insolvency risk of the fund is minimal. The paper thus presents, through invention, package of data for pension fund planner created by using neutrosophic fuzzy triangular numbers.

The paper organized as follows. In Section tow, shows literature review for the problem. In Section three, shows Pre-Requisites are useful for the problem. In section four, shows proposed model for the problem. In section five, shows numerical example to illustrate the model and finally, Section 6 presents a conclusion.

Some of hundreds if not thousands of papers based on Markowitz model have been published on portfolio selection used fuzzy probabilities, where extending probability into fuzzy probability. So authors say about these papers "it is a normal extension of Markowitz's model".

Liu in a research performed on Taiwan's stock market is discussed the fuzzy portfolio optimization problem. He took about advantage of the fuzzy data representation and fuzzy numbers and presented that more return equal more risk (Liu ST., 2011).

Karpenko et al. show that there are some features of the fuzzy sets can aid the prediction of investment decisions for corporations in terms of uncertainty and risk investor is taking (Karpenko L et al., 2020).

Boloșet al. showed in their paper that an applications of neutrosophic theory are used in different area such as data mining, information systems, artificial intelligence, decision making, soft computing, computational modeling, image processing, robotics, medical diagnosis, biomedical engineering, economic forecasting, investment problems, social science, etc. (Boloș MI et al., 2021).

There are only few studies that use the advantages of the neutrosophic theory for the portfolio theory. Islam and Ray used a neutrosophic optimization approach for a multi-objective portfolio selection. They studied a novel objective function in their model based on entropy and the problem of portfolio selection is generalized with diversification, stating that, the neutrosophic theory can easily be applied to different areas of operations research and engineering sciences (Islam S & Ray P., 2018).

Pamucar et al. used the linguistic neutrosophic numbers for eliminating the personality which derives from the qualitative estimation and the assumptions made by the decision-making in hard situations in which use a multi criteria decision making model (Pamuˇcar D et al., 2018).

Elsebaey et al. presented in their paper that used approach ofrough set theory for minimizing insolvency risk of pension fund. They proposed method uses a rough set theory approach for optimizing the mean-variance model defined by Markowitz. Rough interval quadratic programming approach for portfolio selection problem considered as minimize the total variability in the future returns (Elsebaey S.S. et al. , 2021).

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2. METHODOLOGY

Every investor has main aim in portfolio selection is to get portfolio return as high as possible such that have minimum variation or dispersion in the portfolio return. Risk of portfolio measures by the variance i.e. measures the variation from the expected return.

2.1. Pre-Requisites

Definition1: (Elsebaey, S. S. et al. 2021) Minimizing of insolvency risk of pension fund

The model states as below,

Let the portfolio consisting of $\boldsymbol{n}$ assets and operating over next time of one period

$$\text{Min} \quad Var(R_P) = \sum_{i=1}^{n} var(R_i)\, x^2{}_i + 2\sum_{j\neq i=1}^{n} cov(R_{i\,;}\, R_j\ )\, x_i x_j$$

Subject to

$$E(R_P) = \sum_{i=1}^{n} E(R_i) * \ x_i \geq \boldsymbol{K};$$
$$(1 + E(R_P)) * (M) + C - P \geq \ 0; \quad \text{(insolvency risk)}$$
$$\sum_{i=1}^{n} x_i = 1\ ;$$
$$\forall x_i \geq 0; \qquad i = 1,2,\ldots,n \text{(Short sell not allowed)} \qquad (1)$$

Where

$E(R_P)$: Expectation of return of portfolio;

$V(R_P)$: Variance of return (risk) of portfolio;

K: Required return satisfied the balance in pension plan and secures the fund against insolvency;

$cov(R_{i\,;}R_j)$: Covariance betben returns of assetsi, j;

M: Fund’s reserve of pension plan;

C: Contributions paid by scheme’s participants;

P: Grantees benefits paid by pension scheme plus all administrative expenses;

x_i: Proportion at assets i.

In order to construct a model using neutrosophic fuzzy numbers of the following definitions, let there is financial asset (A_i) can be written with neutrosophic fuzzy numbers and it has different elements described by the average return, risk, and covariance as below

Definition 2: (Bolos et al., 2019) the financial asset returns by using the neutrosophic fuzzy number has the form

$\widetilde{R_{A_i}} = \langle\left(\widetilde{R_{A_{a_i}}}, \widetilde{R_{A_{b_i}}}, \widetilde{R_{A_{c_i}}}\right); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$ is given by:

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$$\langle E_f(\widetilde{R_A}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle = \langle(\tfrac{1}{6}(\widetilde{Ra}_{a_1} + \widetilde{Ra}_{c_1}) + \tfrac{2}{3}\widetilde{Ra}_{b_1}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle \quad (2)$$

Definition 3: (Bolos et al.) The financial asset variance of a neutrosophic fuzzy number[13] $\widetilde{R_{A_i}} = \langle(\widetilde{R_{A_{a_i}}}, \widetilde{R_{A_{b_i}}}, \widetilde{R_{A_{c_i}}}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$ is given by:

$$\langle\widetilde{\sigma f^2_{A_i}}; w\widetilde{\sigma_A}, u\widetilde{\sigma_A}, y\widetilde{\sigma_A}\rangle =$$

$$\langle\tfrac{1}{4}\left[\left(\widetilde{R_{A_{b_1}}} - \widetilde{R_{A_{a_1}}}\right)^2 + \left(\widetilde{R_{A_{c_1}}} - \widetilde{R_{A_{b_1}}}\right)^2\right] + \tfrac{2}{3}\left[\widetilde{R_{A_{a_1}}}\left(\widetilde{R_{A_{b_1}}} - \widetilde{R_{A_{a_1}}}\right) - \widetilde{R_{A_{c_1}}}\left(\widetilde{R_{A_{c_1}}} - \widetilde{R_{A_{b_1}}}\right)\right] + \tfrac{1}{2}\left(\widetilde{R_{A_{a_1}}}^2 + \widetilde{R_{A_{c_1}}}^2\right) - \tfrac{1}{2}E_f{}^2(\widetilde{Ra_i}); w\widetilde{Ra}, u\widetilde{Ra}, y\widetilde{Ra}\rangle \quad (3)$$

Definition 4: (Bolos et al., 2019) by using the triangular neutrosophic fuzzy numbers covariance between two financial assets modeled is given by:

$$cov(\widetilde{Ra}_1, \widetilde{Ra}_2) = \langle(\tfrac{1}{4}[(\widetilde{Ra}_{b_{11}} - \widetilde{Ra}_{a_{11}})(\widetilde{Ra}_{b_{21}} - \widetilde{Ra}_{a_{21}}) + (\widetilde{Ra}_{c_{11}} - \widetilde{Ra}_{b_{11}})(\widetilde{Ra}_{c_{21}} - \widetilde{Ra}_{b_{21}})] + \tfrac{1}{3}\{[\widetilde{Ra}_{a_{21}}(\widetilde{Ra}_{b_{11}} - \widetilde{Ra}_{a_1}) - \widetilde{Ra}_{a_{11}}(\widetilde{Ra}_{b_{21}} - \widetilde{Ra}_{a_{21}})] - [\widetilde{Ra}_{c_{11}}(\widetilde{Ra}_{c_{21}} - \widetilde{Ra}_{b_{21}}) + \widetilde{Ra}_{c_{21}}(\widetilde{Ra}_{c_{11}} - \widetilde{Ra}_{b_{11}})] + \tfrac{1}{2}(\widetilde{Ra}_{a_{11}}\widetilde{Ra}_{a_{21}} + \widetilde{Ra}_{c_{11}}\widetilde{Ra}_{c_{21}}) + \tfrac{1}{2}E_f(\widetilde{Ra}_1)E_f(\widetilde{Ra}_2)); w\widetilde{Ra}_1 \wedge w\widetilde{Ra}_2, u\widetilde{Ra}_1 \vee u\widetilde{Ra}_2, y\widetilde{Ra}_1 \vee y\widetilde{Ra}_2\rangle \quad (4)$$

Definition 5: (Bolos et al., 2021) the neutrosophic portfolio returns

It denoted byform$\langle\widetilde{R_p}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle$ can be written by using fuzzy triangular neutrosophic numbers of theform:

$\tilde{R}_{A_i} = \langle(\tilde{R}_{A_{ai}}, \tilde{R}_{A_{bi}}, \tilde{R}_{A_{ci}}); w\widetilde{R_A}, u\widetilde{R_A}, y\widetilde{R_A}\rangle$ Which an element of the neutrosophic portfolio and is formulated as:

$$\langle\widetilde{R_p}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle = \textstyle\sum_{i=1}^{n}\langle x_{A_i}(\tfrac{1}{6}\left(\widetilde{R_{A_{a_i}}} + \widetilde{R_{A_{c_i}}}\right) + \tfrac{2}{3}\widetilde{R_{A_{b_i}}}); w\tilde{R}_{A_i}, u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle \quad (5)$$

Where $x_{A_i}$is the weight of the asset A shared in the portfolio.

Definition 6: (Bolos et al., 2021) The neutrosophic portfolio risk

For portfolio$\langle\tilde{P}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle$, which written using neutrosophic triangular fuzzy numbers, the neutrosophic portfolio risk has the form: $\langle\widetilde{\sigma_p^2}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle$, which written using neutrosophic fuzzy numbers as:$\tilde{\sigma}_{A_i} = \langle(\tilde{\sigma}_{A_{ai}}, \tilde{\sigma}_{A_{bi}}, \tilde{\sigma}_{A_{ci}}); w\widetilde{\sigma_A}, u\widetilde{\sigma_A}, y\widetilde{\sigma_A}\rangle$This is a basic parameter of the neutrosophic portfolio that formulated as:

$$\langle\widetilde{\sigma_p^2}; w\widetilde{\sigma_p}, u\widetilde{\sigma_p}, y\widetilde{\sigma_p}\rangle = \textstyle\sum_{i=1}^{n} x_{A_i}^2\langle\tfrac{1}{4}[\left(\widetilde{R_{A_{b_i}}} - \widetilde{R_{A_{a_i}}}\right)^2 + \left(\widetilde{R_{A_{c_i}}} - \widetilde{R_{A_{b_i}}}\right)^2]; w\tilde{R}_{A_i}, u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle + \langle\tfrac{2}{3}\left[\widetilde{R_{A_{a_i}}}\left(\widetilde{R_{A_{b_i}}} - \widetilde{R_{A_{a_i}}}\right) - \widetilde{R_{A_{c_i}}}\left(\widetilde{R_{A_{c_i}}} - \widetilde{R_{A_{b_i}}}\right)\right]; w\tilde{R}_{A_i}, u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle + \langle\tfrac{1}{2}\left(\widetilde{R_{A_{a_1}}}^2 + \widetilde{R_{A_{c_1}}}^2\right); w\tilde{R}_{A_i}, u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle - \langle\tfrac{1}{2}E_f{}^2(\widetilde{R_{A_i}}), w\tilde{R}_{A_i}, u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle +$$

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$$2\sum_{i=1}^{n}\sum_{j=1}^{n}x_{A_i}x_{A_j}\langle\Bigg(\frac{1}{4}\Big[\Big(\widetilde{R_{A_{b_i}}}-\widetilde{R_{A_{a_i}}}\Big)\Big(\widetilde{R_{A_{b_j}}}-\widetilde{R_{A_{a_j}}}\Big)+\Big(\widetilde{R_{A_{b_c}}}-\widetilde{R_{A_{b_i}}}\Big)\Big(\widetilde{R_{A_{c_j}}}-\widetilde{R_{A_{b_j}}}\Big)\Big]+$$
$$\frac{1}{3}\Big\{\Big[\widetilde{R_{A_{a_j}}}\Big(\widetilde{R_{A_{b_i}}}-\widetilde{R_{A_{a_i}}}\Big)+\widetilde{R_{A_{a_i}}}\Big(\widetilde{R_{A_{b_j}}}-\widetilde{R_{A_{a_j}}}\Big)\Big]-\Big[\widetilde{R_{A_{c_i}}}\Big(\widetilde{R_{A_{c_j}}}-\widetilde{R_{A_{b_j}}}\Big)+\widetilde{R_{A_{c_j}}}\Big(\widetilde{R_{A_{c_i}}}-$$
$$\widetilde{R_{A_{b_i}}}\Big)\Big]\Big\}+\frac{1}{2}\Big(\widetilde{R_{A_{a_i}}}\widetilde{R_{A_{a_j}}}+\widetilde{R_{A_{c_i}}}\widetilde{R_{A_{c_j}}}\Big)+\frac{1}{2}E_f\big(\widetilde{R_{A_i}}\big)E_f\Big(\widetilde{R_{A_j}}\Big)\Bigg);w\tilde{R}_{A_i}\wedge w\tilde{R}_{A_j},\ u\tilde{R}_{A_i}\vee$$
$$u\tilde{R}_{A_j},y\tilde{R}_{A_i}\vee y\tilde{R}_{A_j}\rangle=\langle\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{x}_i\tilde{x}_j\widetilde{\sigma_{ij}};w\widetilde{\sigma_{ij}},u\widetilde{\sigma_{ij}},y\widetilde{\sigma_{ij}}\rangle \qquad (6)$$

Then for any financial asset neutrosophic portfolio has the form $\langle\tilde{P};w\widetilde{R_p},u\widetilde{R_p},y\widetilde{R_p}\rangle$ there are two fundamental operators $\langle\widetilde{R_p};w\widetilde{R_p},u\widetilde{R_p},y\widetilde{R_p}\rangle$and$\langle\widetilde{\sigma_p^2};w\widetilde{\sigma_p},u\widetilde{\sigma_p},y\widetilde{\sigma_p}\rangle$

2.2. Insolvency risk of pension fund problem by neutrosophic fuzzy parameters:

$$\mathrm{Min}\langle\widetilde{\sigma_p^2};w\widetilde{\sigma_p},u\widetilde{\sigma_p},y\widetilde{\sigma_p}\rangle=\langle\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{x}_i\tilde{x}_j\widetilde{\sigma_{ij}};w\widetilde{\sigma_{ij}},u\widetilde{\sigma_{ij}},y\widetilde{\sigma_{ij}}\rangle$$

Subject to

$$\langle\widetilde{R_p};w\widetilde{R_p},u\widetilde{R_p},y\widetilde{R_p}\rangle=\sum_{i=1}^{n}\langle x_{A_i}\Big(\frac{1}{6}\Big(\widetilde{R_{A_{a_i}}}+\widetilde{R_{A_{c_i}}}\Big)+\frac{2}{3}\widetilde{R_{A_{b_i}}}\Big);w\tilde{R}_{A_i},\ u\tilde{R}_{A_i},y\tilde{R}_{A_i}\rangle\geq\boldsymbol{K};$$

$(1+\langle\widetilde{R_p};w\widetilde{R_p},u\widetilde{R_p},y\widetilde{R_p}\rangle)*(M)+C-P\geq 0$;(Insolvency risk)

$\sum_{i=1}^{n}x_{A_i}=1$;

$x_{A_i}\geq 0; i=1,2,\dots,n$(Short sell not allowed)

Where

$\langle\widetilde{R_p};w\widetilde{R_p},u\widetilde{R_p},y\widetilde{R_p}\rangle$: Neutrosophic portfolio return

$\langle\widetilde{\sigma_p^2};w\widetilde{\sigma_p},u\widetilde{\sigma_p},y\widetilde{\sigma_p}\rangle$: Neutrosophic portfolio variance

K: Required return satisfied the balance in pension plan and secures the fund against insolvency crisis.

M: Fund's reserve of pension plan.

C: Total Contributions.

P: Grantees benefits paid by pension scheme plus all administrative expenses

x_{A_i}: Proportion at assets i

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3. RESULTS AND DEDUCTION

Numerical Example:

To illustrate the Insolvency Risk Pension Fund problem with triangular neutrosophic numbers coefficients the numerical example as following. (This example is taken from (Boloș MI et al., 2021) and used it in the investment strategy of pension fund by certain numbers.

Consider there are3 assets: asset one A_1 asset tow A_2 and asset three A_3 to determine three neutrosophic triangular numbers are specific for the return of financial assets:

$\tilde{R}_{A_1} = \langle(.15,.2,.28);\ .5,.2,.3\rangle$ For$\widetilde{R_A} \in [.15;\ .28]$;

$\tilde{R}_2 = \langle(.2,.3,.4);\ .6,.3,.2\rangle$ For$\widetilde{R_A} \in [,2;\ ,4]$;

$\tilde{R}_3 = \langle(.15,.25,.3);.4,.3,.3\rangle$ For$\widetilde{R_A} \in [,15;\ ,3]$;

Thus by use Eq.2, the result will beas:

$E_f\left(\widetilde{R_{A_1}}\right) = \langle 0{,}205;\ 0.5, 0.2, 0.3\rangle$

$E_f\left(\widetilde{R_{A_2}}\right) = \langle 0{,}300;\ 0.6, 0.3, 0.2\rangle$;

$E_f\left(\widetilde{R_{A_3}}\right) = \langle 0{,}242;\ 0.4, 0.3, 0.3\rangle$;

Thus by use Eq.3, the result will beas:

$\widetilde{\sigma f^2_{A_1}} = \langle 0.022;\ 0.5, 0.2, 0.3\rangle$

$\widetilde{\sigma f_{A_1}} = \langle 0{,}147;\ 0.5, 0.2, 0.3\rangle$

$\widetilde{\sigma f^2_{A_2}} = \langle 0.047;\ 0.6, 0.3, 0.2\rangle$

$\widetilde{\sigma f_{A_2}} = \langle 0{,}216;\ 0.6, 0.3, 0.2\rangle$

$\widetilde{\sigma f^2_{A_3}} = \langle 0.030;\ 0.4, 0.3, 0.3\rangle$

$\widetilde{\sigma f_{A_3}} = \langle 0{,}174;\ 0.4, 0.3, 0.3\rangle$

Thus by use Eq.4, the result will beas:

$cov(\tilde{R}_{A_1}, \tilde{R}_{A_2}) = \langle 0.093;\ 0.5, 0.3, 0.3\rangle$

$cov(\tilde{R}_{A_1}, \tilde{R}_{A_3}) = \langle 0.87;\ 0.4, 0.3, 0.3\rangle$

$cov(\tilde{R}_{A_2}, \tilde{R}_{A_2}) = \langle 0.110;\ 0.4, 0.3, 0.3\rangle$

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This example evaluates participants of some Egyptian funds at 30/9/2021 and its data and results as following steps:

The required rate of return secure the fund and collecting data as:

M (reserve of pension fund at start year) = 378600936 L.E.;

K(The required rate of return secure the fund)=0.25

P (All expected liabilities at first year) = 10910079L.E.;

C (expected contribution at first year) = 47797294 L.E.;

The actuary's results see Table 1:

Table 1: Cash flow for the expected liabilities and contributions

Years	Income	All liabilities
0	47 797 294	100 910 079
1	47 449 742	107 330 383
2	48 089 326	82 210 994

Table 1 illustrate the promised benefits for the participants and their future contributions, from the evaluation's results sponsor's decisions should be invest the surplus to attain the required rate of return to satisfy fund's balance.

So the proposed model for this problem as:

$$\text{Min}\langle\widetilde{\sigma_p^2}; w\widetilde{\sigma_p}, u\widetilde{\sigma_p}, y\widetilde{\sigma_p}\rangle = \langle\tfrac{1}{2}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{x}_i\tilde{x}_j\widetilde{\sigma_{ij}}; w\widetilde{\sigma_{ij}}, u\widetilde{\sigma_{ij}}, y\widetilde{\sigma_{ij}}\rangle$$

Subject to

$$\textstyle\sum_{i=1}^{n}\langle x_{A_i}(\tfrac{1}{6}\left(\widetilde{R_{A_{a_i}}} + \widetilde{R_{A_{c_i}}}\right) + \tfrac{2}{3}\widetilde{R_{A_{b_i}}}); w\tilde{R}_{A_i},\ u\tilde{R}_{A_i}, y\tilde{R}_{A_i}\rangle \geq \mathbf{0.25};$$

$$(1 + \langle\widetilde{R_p}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle) * (378600936) + 47797294 - 100910079 \geq 0;$$

$$\textstyle\sum_{i=1}^{n} x_{A_i} = 1;$$

$x_{A_i} \geq 0; i = 1,2,\dots,n$ (Short sell not allowed)

By replacing in the formula where$x_{A_i} = x_i$then:

$$\text{Min}\langle 0.022x_1^2; 0.5, 0.2, 0.3\rangle + \langle 0.047x_2^2; 0.6, 0.3, 0.2\rangle + \langle 0.030x_3^2; 0.4, 0.3, 0.3\rangle + 2 * \langle 0.093x_1x_2;\ 0.5, 0.3, 0.3\rangle + 2 * \langle 0.087x_1x_3;\ 0.4, 0.3, 0.3\rangle + 2 * \langle 0.110x_2x_3;\ 0.4, 0.3, 0.3\rangle$$

Subject to

$$\langle 0{,}205x_1; 0.5{,}0.2{,}0.3\rangle + \langle 0{,}30x_2; 0.6{,}0.3{,}0.2\rangle + \langle 0{,}242x_3; 0.4{,}0.3{,}0.3\rangle \geq 0.25;$$

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$(1 + \langle 0{,}205x_1;\ 0.5, 0.2, 0.3\rangle + \langle 0{,}30x_2;\ 0.6, 0.3, 0.2\rangle + \langle 0{,}242x_3;\ 0.4, 0.3, 0.3\rangle) * (378600936)$

$+47797294 - 100910079 \geq 0;$

$x_1 + x_2 + x_3 = 1\ ;$

$x_{A_i} \geq 0; \qquad i = 1,2,3$

Then the result is:

$\text{Min}\langle 0.022x_1^2 + 0.047x_2^2 + 0.030x_3^2 + 0.187x_1x_2 + 0.173x_1x_3 + 0.220x_2x_3;\ .4, .3, .3\rangle$

Subject to

$\langle 0.205x_1 + 0.30\, x_2 + 0.242\, x_3\ ;\ 0.4, 0.3, 0.3\rangle \geq \mathbf{0.25};$

$(325488151 + \langle 77613192x_1 + 113580281x_2 + 91495226x_3;\ .4, .3, .3\rangle) \geq 0;$

$\sum_{i=1}^{3} x_i = 1\ ;$

$x_i \geq 0; i = 1,2,3$

By using Winqsb program to solve the previous problem the results as below:

The asset A_1has $E_f\left(\widetilde{R_{A_1}}\right)$ as

$$E_f\left(\widetilde{R_{A_1}}\right) = \langle .205;\ .5, .2, .3\rangle$$

And the weight of investment in A_1 will be$x_1 = \langle 0.53;\ 0.4, 0.3, 0.3\rangle$; and the asset $A_2$and hence $E_f\left(\widetilde{R_{A_2}}\right) = \langle .30;\ .6, .3, .2\rangle$and a share' weight of $x_2 = \langle .47;\ .4, .3, .3\rangle$ and the asset $A_2 and\ hence E_f\left(\widetilde{R_{A_3}}\right) = \langle .242;\ .4, .3, .3\rangle$and a share' weight of$x_3 = 0.000$. Thus, establishing the portfolio return formed with the structure of those three assets will be:

$\langle \widetilde{R_p}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle = \langle 0.25;\ 0.4, 0.3, 0.3\rangle$

And hence, the portfolio risk, it will beset as follows:

$\langle \widetilde{\sigma_p^2}; w\widetilde{\sigma_p}, u\widetilde{\sigma_p}, y\widetilde{\sigma_p}\rangle = \langle 0.06;\ 0.4, 0.3, 0.3\rangle$

$\langle \widetilde{\sigma_p}\ ; w\widetilde{\sigma_p}\ , u\widetilde{\sigma_p}\ , y\widetilde{\sigma_p}\rangle = \langle 0.245; 0.4, 0.3, 0.3\rangle$

The portfolio risk $\langle \widetilde{\sigma_p^2}; w\widetilde{\sigma_p}, u\widetilde{\sigma_p}, y\widetilde{\sigma_p}\rangle = \langle 0.06;\ 0.4, 0.3, 0.3\rangle$ is proportional to the value of the required portfolio return: $\langle \widetilde{R_p}; w\widetilde{R_p}, u\widetilde{R_p}, y\widetilde{R_p}\rangle = \langle .25;\ .4, .2, .3\rangle$.

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4. CONCLUSION

The study under vagueness makes the strategies of investment portfolio of pension fund more realistic and practice to describe the rate of expected return and risk rate. The usefulness of paper for the sponsors of the pension plan is: the ability for choosing the risk to achieve higher expected returns, and on other hand determining strategies for selecting the portfolios which saving the pension plan against insolvency risk.

The paper clarified that the optimal neutrosophic portfolios are modeled using neutrosophic triangular numbers that allow for the definition of the return and the risk of portfolio by determination of financial performance indicators for all financial asset.

The determination of the weights of financial assets in the total value of the portfolio consisting of N financial assets under minimizing risk at specific return rate designed using neutrosophic fuzzy triangular numbers.

Using this approach for financial asset portfolios has the feature of introducing a class of more data for decision makers on the stock market, namely the degree of vagueness/ realization. The contribution of this research paper is given by the theoretical proof with the help of approach of the achievement parameters to the optimal portfolios for any pension fund, namely, the portfolio structure for pension fund, Insolvency risk at given required portfolio return.

A case study of the pension fund with all required data is considered to show some properties of the model. The decision maker would check the fund's finance status every short certain period. A portfolio optimization method which minimizes the portfolio risk is introduced and this method is applied to the 3 well known stocks of Stock Market

The consist of practical implications of the research paper are set the investment decisions on the stock market by using neutrosophic performance parameters, namely the structure, the risk and the return of neutrosophic portfolio. These parameters apply to optimal financial asset portfolios that secure the pension plan against insolvency risk.

The future work directions are the following: use another artificial intelligence approaches to obtain the portfolios satisfy all constraints and objective function of the system such as evolutionary algorithms and swarm intelligence approaches, in extending, with also the help of neutrosophic fuzzy numbers.

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Authors' contributions statement:

S. S. E., H. Z. , N. R. S. and H. S. contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript. All authors discussed the results to the final manuscript.

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