ISSN 1533-9211

DOI 10.5281/zenodo.7427505

FREE OSCILLATIONS OF DISSIPATIVELY INHOMOGENEOUS SYSTEMS

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Annotation

The article deals with oscillations of a cylindrical shell located in an infinite environment. In a homogeneous system, the cylindrical shell and the infinite medium are elastic or viscoelastic. If one of them is elastic, the other is viscoelastic, a heterogeneous system is obtained. The task is reduced to compiling the dispersion equation and determining the values of the complex frequencies

Keywords: Continuity, mechanical system, cylindrical shell, dissipative system, mechanical properties, free vibration, mathematical equations, viscoelasticity, mechanical engineering, radius of curvature.

INTRODUCTION:

Most of the shells used in mechanical engineering are thin shells, but are based on the use of a rather complex mathematical apparatus. Their theory is built on the assumption that the material is isotropic, obeys Hooke's law, and the displacements of the points of the shell are small compared to its thickness.

The article deals with the interaction of a cylindrical shell with various mechanical systems covering it. The influence of the dissipative properties of the medium on systems is studied.

A cylindrical shell and a mechanical system are elastic or viscoelastic, a dissipative homogeneous system is obtained, if one of them is elastic, the other is viscoelastic, a dissipative inhomogeneous system is obtained.

METHODS:

The task is reduced to drawing up the dispersion equation, determining the values of the complex frequencies.

ISSN 1533-9211
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The differential equation for the motion of an infinite medium in potentials is written [1]:

$$\nabla^2\varphi_j - \frac{1}{C_{bj}^2}\frac{\partial^2\varphi_j}{\partial t^2} = 0; \qquad (1)$$

$$\nabla^2\psi_{rj} - \frac{1}{C_{kj}^2}\frac{\partial^2\psi_{rj}}{\partial t^2} = 0;$$

$$C_{bj} = \sqrt{\frac{\tilde{\lambda}_j + 2\tilde{\mu}_j}{\rho_j}} -$$

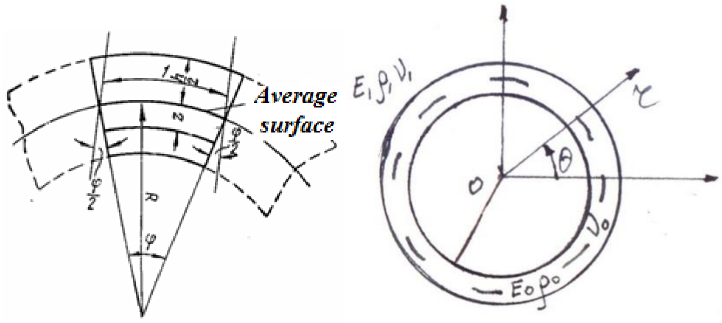
$$C_{kj} = \sqrt{\frac{\tilde{\mu}_j}{\rho_j}} -$$

$\nabla -$ Laplace operator,

j – number of layers, j=1.

The elastic Lamé coefficients are related to the elastic modulus E and Poisson's ratio as follows.

$$\tilde{\lambda}_j = \frac{E_j\nu_j}{(1+\nu_j)(1-2\nu_j)}; \tilde{\mu}_j = \frac{E_j}{2(1+\nu_j)} \qquad (2)$$



1-fig.

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ISSN 1533-9211
DOI 10.5281/zenodo.7427505

RESULTS:

Let us write down the solutions of the wave equation (1) for an unbounded medium.

$$\varphi_1 = \sum_{n=0}^{\infty} A_n H_n(\alpha_1 r)\cos(n\theta)\cdot \ell^{-i\omega t};$$

$$\psi_1 = \sum_{n=0}^{\infty} B_n H_n(\beta_1 r)\sin(n\theta)\cdot \ell^{-i\omega t};$$

(3)

$$\alpha_j = \frac{\omega}{c_{bj}} - \gamma;\ \beta_j = \frac{\omega}{c_{zj}} - \gamma;\ \gamma = 0{,}577216.$$

Displacement U_r, U_θ with a flat problem through potentials φ,ψ are written as follows [3,4].

$$U_r = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta};$$

$$U_\theta = -\frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta}; \qquad (4)$$

Formulas for stresses in a plane problem are written as follows [3, 4].

$$\sigma_{rr} = -2\mu[(a\alpha^2 + D_1)\varphi + D_2\psi];$$

$$\sigma_{r\theta} = 2\mu[-D_2\varphi + \frac{1}{2}(\beta^2 + 2D_1)\psi];$$

$$\sigma_{\theta\theta} = 2\mu[(\frac{\lambda}{2\mu}\alpha^2 + D_1)\varphi + D_2\psi];$$

$$D_1 = \frac{1}{r}(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial\theta^2}); \qquad (5)$$

$$D_2 = \frac{1}{r}\frac{\partial}{\partial\theta}(\frac{1}{r} - \frac{\partial^2}{\partial r^2});$$

$$a = \frac{\lambda + 2\mu}{2\mu};$$

The displacement and stresses of the medium, taking into account (4, 5), takes the form

$$U_{r1} = \sum_{n=0}^{\infty}[A_n H_n^{(1)'}(\alpha_1 r) + B_n H_n^{(1)}(\beta_1 r)]\cdot\cos(n\theta)\cdot \ell^{-i\omega t};$$

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ISSN 1533-9211
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$$U_{\theta 1} = -\sum_{n=0}^{\infty} [\frac{n}{r} A_n H_n^{(1)}(\alpha_1 r) + B_n H_n^{(1)'}(\beta_1 r)] \cdot \sin(n\theta) \cdot \ell^{-i\omega t};$$

$$\sigma_{rr1} = \frac{1}{r^2} \sum_{n=0}^{\infty} \{\alpha_1^2 r^2 [-\lambda_1 H_n^{(1)'}(\alpha_1 r) + 2\mu_1 H_n^{(1)''}(\alpha_1 r)] A_n + 2\mu_1 n[\beta_1 r H_n^{(1)'}{}_n(\beta_1 r) -$$
$$H_n^{(1)'}(\beta_1 r)] B_n\} \cdot \cos(n\theta) \cdot \ell^{-i\omega t};$$

$$\sigma_{r\theta} = -\frac{\mu_1}{r^2} \sum_{n=0}^{\infty} \{\beta_1^2 r^2 [-\lambda_1 H_n^{(1)'}(\beta_1 r) + H_n^{(1)''}(\beta_1 r)] B_n + 2n[\alpha_1 r H_n^{(1)'}{}_n(\alpha_1 r) -$$
$$H_n^{(1)}(\alpha_1 r)] A_n\} \cdot \sin(n\theta) \cdot \ell^{-i\omega t};$$

$$\sigma_{\theta\theta} = \frac{1}{r^2} \sum_{n=0}^{\infty} \{[-\lambda_1 \alpha_1^2 r^2{}_1 H_n^{(1)'}(\alpha_1 r) + 2\mu_1 H_n^{(1)''}(\alpha_1 r) - n^2 H_n^{(1)}(\alpha_1 r)] A_n + 2\mu_1 [n H_n^{(1)'}(\beta_1 r) - \tag{6}$$
$$\beta_1 r H_n^{(1)'}(\beta_1 r)] B_n\} \cdot \cos(n\theta) \cdot \ell^{-i\omega t};$$

The differential equation of a thin-walled cylindrical shell is obtained in the form [2, 3]

$$\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial W}{\partial \theta} + \frac{h^2}{12R^2}(\frac{\partial^2 V}{\partial \theta^2} - \frac{\partial^3 V}{\partial \theta^3}) = \frac{R^2}{c^2}(\frac{\partial^2 V}{\partial t^2} - \frac{1}{\rho_0 h}\sigma_{r\theta n/R});$$

$$\frac{\partial V}{\partial \theta} + W + \frac{h^2}{12R^2}(\frac{\partial^4 V}{\partial \theta^4} - \frac{\partial^3 W}{\partial \theta^3}) = -\frac{R^2}{c^2}(\frac{\partial^2 W}{\partial t^2} - \frac{1}{\rho_0 h}\sigma_{rrn/R}); \tag{7}$$

In formula (7), R,h is the radius and wall thickness of the cylindrical shell;

R_0, ρ_0, ν_0 – Modulus of elasticity, density and Poisson's ratio of the cylindrical shell;

$\sigma_{r\theta n/R}, \sigma_{rrn/R}$ – Stress appearing on the surface of a cylindrical shell.

The solution of the differential equation (7) is sought in the form of a series

$$W = \sum_{n=0}^{\infty} W_n \cos(n\theta) \cdot \ell^{-i\omega t};$$

$$V = \sum_{n=0}^{\infty} V_n \sin(n\theta) \cdot \ell^{-i\omega t}; \tag{8}$$

Substituting the chosen solution (8) into the differential equation (7), we determine V_n, W_n.

$$W_n = \frac{\eta}{\Lambda}(Y_{12}\sigma_{r\theta n/R} + \sigma_{rrn/R} Y_{11});$$

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ISSN 1533-9211
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$$V_n = \frac{\eta}{\Lambda}(Y_{12}\sigma_{rrn/R} + \sigma_{r\theta n/R}Y_{22}); \qquad (9)$$

In formula (9),

$$\Lambda = Y_{12}^2 - Y_{11}Y_{12};\ \eta = \frac{R^2}{c^2\rho_0 h};$$

$$Y_{11} = n^2(1 + b - \frac{\omega^2 R^2}{c^2});\ Y_{12} = Y_{21} = n(1 + n^2 b);\ Y_{22} = 1 + n^4 b - \frac{\omega^2 R^2}{c^2};$$

$$b = \frac{h^2}{12R^2};$$

$$\sigma_{rrn/R} = R_{1n}A_n + Q_{1n}B_{1n};$$

$$\sigma_{r\theta n/R} = R_{2n}A_n + Q_{2n}B_{2n}; \qquad (10)$$

in (10)

$$R_{1n} = \alpha_1^2[-\lambda_1 H_n^{(1)}(\alpha_1 R) + 2\mu_1 H_n^{(1)''}(\alpha_1 R)];$$

$$R_{2n} = -\frac{2\mu_1}{R^2}n[\alpha_1^2 RH_n^{(1)}(\alpha_1 R) - H_n^{(1)}(\alpha_1 R)];$$

$$Q_{1n} = \frac{2\mu_1}{R^2}[\beta_1 RH_n^{(1)'}(\beta_1 R) - H_n^{(1)}(\beta_1 R)]; \qquad (11)$$

$$Q_{2n} = -\mu_1\beta_1^2[H_n^{(1)'}(\beta_1 R) + H_n^{(1)''}(\beta_1 R)];$$

To determine the constants An, Bn from the boundary conditions at $-r = R$, $U_{r1} = W$, $U_{\theta 1} = V$; we obtain algebraic systems of equations:

$$Z_{1n}A_n + Z_{2n}B_n = 0;$$

$$Z_{3n}A_n + Z_{4n}B_n = 0; \qquad (12)$$

The dispersion equation of the algebraic systems of equations (12) has the form:

$$[C] = |Z_{1n} \cdot Z_{4n} - Z_{2n} \cdot Z_{3n}| = 0; \qquad (13)$$

Where,

$$Z_{1n} = \frac{\eta}{\Delta}(Y_{12}R_{2n} + Y_{11}R_{1n}) - H_n^{(1)}(\alpha_1 R)$$

$$Z_{2m} = [\ ]$$

$$Z_{2n} = [\ \frac{\eta}{\Delta}(Y_{12}Q_{2n} + Y_{11}Q_{1n}) - \frac{n}{R}H_n^{(1)}(\beta_1 R)\ ];$$

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$$Z_{3n} = \frac{\eta}{\Delta}(Y_{12} R_{2n} - Y_{22} R_{2n}) + \frac{n}{R} H_n^{(1)}(\alpha_1 R); \qquad (14)$$

$$Z_{4n} = \frac{\eta}{\Delta}(Y_{12} Q_{2n} + Y_{22} Q_{2n}) + H_n^{(1)}(\beta_1 R);$$

DISCUSSION:

The dispersion equation (13) is solved by the Muller method. The dissipative property of a viscoelastic system is determined by the integral equations of Boltzmann – Voltaire [5]:

$$\lambda_n \varphi = \tilde{\lambda}_n [\varphi(t) - \int_0^t R_{\lambda_n}(t-\tau)\varphi(\tau)];$$

$$\mu_n \varphi = \tilde{\mu}_n [\varphi(t) - \int_0^t R_{\mu_n}(t-\tau)\varphi(\tau)]; \qquad (15)$$

In equation (15) R_{λ_n}, R_{μ_n} – Relaxation kernel; λ_n, μ_n – Lame operators, $\varphi(t)$ –-Arbitrary function, t-time.

Formula (15) is written as follows [6].

$$\lambda_n \varphi = \tilde{\lambda}_n [1 - \Gamma^c_{\lambda_m}(\omega_R) - i\Gamma^s_{\lambda_n}(\omega_R)]\varphi;$$

$$\mu_n \varphi = \tilde{\mu}_n [1 - \Gamma^c_{\mu_n}(\omega_R) - i\Gamma^s_{\mu_n}(\omega_R)]\varphi; \qquad (16)$$

Cosines, sines, the core of relaxation is written as follows [6].

$$\Gamma^c_{\lambda_n}(\omega_R) = \Gamma^c_{\mu_n}(\omega_R) = \frac{A\Gamma(\alpha)}{(\omega_R^2+\beta^2)^{1/2}} \cos(\propto \operatorname{arctg}\frac{\omega_R}{\beta});$$

$$\Gamma^s_{\lambda_n}(\omega_R) = \Gamma^s_{\mu_n}(\omega_R) = \frac{A\Gamma(\alpha)}{(\omega_R^2+\beta^2)^{1/2}} \sin(\propto \operatorname{arctg}\frac{\omega_R}{\beta}); \qquad (17)$$

$$\Gamma(\alpha) - \text{Gamma function};$$

A,α , β – Constant number.

A=0,048,$\alpha = 0{,}1,\ \beta = 0{,}05.$ In low viscosity;

A=0,078,$\alpha = 0{,}1,\ \beta = 0{,}05.$ In high viscosity.

The problem was studied in two versions. The ratio of elastic modul - $E=E_1/E_o$ varied in the interval (0, 1- 0,2); (n=2),$\nu_1 = 0{,}2;\ \nu_0 = 0{,}25;$

$$\rho = {}^{\rho_1}/_{\rho_0} = 0{,}6$$

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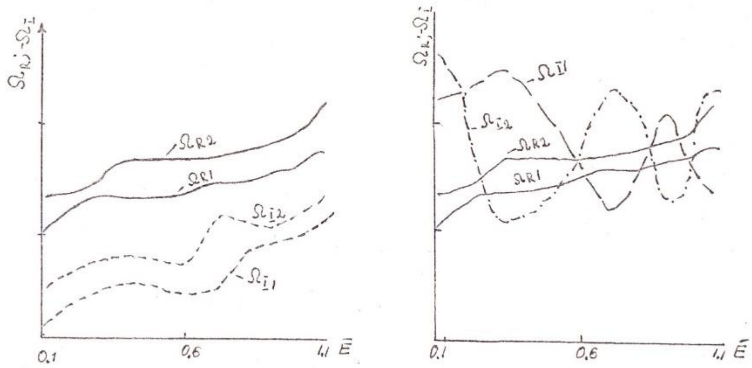

2-fig. Dependence of complex natural frequencies

CONCLUSION

1. The system is dissipative homogeneous, viscoelastic. The imaginary and real parts of the complex frequencies increase monotonically.
2. The system is dissipative inhomogeneous, the cylindrical shell is elastic, $R_{\lambda_n} = 0$; $R_{\mu_n} = 0$.

The imaginary and real parts of the complex frequencies change no monotonically. When the real parts of the complex frequencies approach, the imaginary parts intersect.

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