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ANALYZING AND PROJECTION OF FORECASTING POPULATION OF BANGLADESH USING EXPONENTIAL MODEL, LOGISTIC MODEL, AND DISCRETE LOGISTIC MODEL

HASAN AL MAMUN[1], MD. EAQUB ALI[2], KANAK CHANDRA ROY[3], REZAUL KARIM[1], NASIR UDDIN[1], and PINAKEE DEY[1*]

[1]Department of Mathematics, Mawlana Bhashani Science and Technology University, Tangail, Bangladesh.
[2]Department of Mechanical Engineering, Sonargaon University, Green Road, Dhaka, Bangladesh.
[3]Department of Applied Mathematics, Gono Bishwabidyalay, Savar, Dhaka, Bangladesh.
*Corresponding Author: Email: pinakeedey68@gmail.com

Abstract

Bangladesh has an excessively high population. With the eighth-highest population in the world, it is located in South Asia. The extreme population is currently one of Bangladesh's most significant issues. Therefore, the nation is greatly threatened by the growing population, which is why it is crucial to project Bangladesh's population growth. To forecast the future population, this study's models and designs are important for Bangladesh's population forecasting. Using actual data from 2001 to 2022 and U.N. forecasts from 2023 to 2100, the exponential, logistic growth, and discrete logistic models are used to anticipate Bangladesh's population from 2001 to 2100. By the exponential model, the expected population of Bangladesh is 863.30 million in 2100, and the growth rate is anticipated to be around 2%. Thus, based on the logistic model, Bangladesh's population growth rate has been anticipated to be around 4%, and in 2100, the country is expected to have a total population of 261.43 million. Finally, the discrete logistic model estimates Bangladesh's population growth rate to be around 4%, and the total predicted population of Bangladesh will be 179.798 million in 2100.

Keywords: Absolute Percentage Error, Carrying Capacity, Discrete Logistic Model, Exponential Model, Forecasting Population, Growth Rate, Logistic Model, Mean Absolute Percentage Error (MAPE).

1. INTRODUCTION

Bangladesh is a small country in South Asia. The biggest contributor to the nation's poverty, illiteracy, unemployment, and starvation is overpopulation. Therefore, the populace of Bangladesh is very alarmed by the population's cumulative tendency to rise. Therefore, when formulating a national strategy, population control should be given top attention. At this point, it can be useful to use and make use of a workable method for estimating and projecting the country's future population. The population model is essentially used in population dynamics. Many methods help to create reliable population models. Among these, the Verhulst logistic model and the exponential model by Thomas Robert Malthus are well recognized (J.D., 2002). It offers an easy calculation for forecasting populations and is unable to capture the exact future population figure. The Exponential model is unable to determine the environmental limit point. As a result, the population will continue to grow indefinitely, which is unrealistic. The primary flaw in Malthus's theory is that the population doubles every 25 years or more and is growing exponentially (Malthus, 1798). Because it doesn't account for environmental dynamics, this

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model is suitable for a relatively brief period and a tiny population. The logistic model predicts population expansion that eventually leads to population decline if carrying capacity is reached. It makes no mention of the resource limitations and instead theorizes the excess of births over deaths in a certain amount of time. Once the population has reached its peak, it starts to decline in an S-shaped curve. Comparatively speaking, the exponential model does not offer an estimation that is more precise than a logistic one. In Bangladesh, the connection between population increase and food production was examined by Kabir and Chowdhury (Kabir & Chowdhury, 1982). They advised giving population strategy for population reduction priority because they recognized the challenges of feeding the expanding population despite a significant rise in the food supply. Using the cohort-component technique and the supposition of no migration, Rabbani and Hossain predicted Bangladesh's population for the years 1975–2025. The medium variation predicted that there will be 178 million people living in the world in 2025 (NRR=1 for the period 1995–2000) (Rabbani et al., 1981).

To anticipate the population accurately, 1991–2011 population census data from Bangladesh were used. Bangladesh's population has been increasing, with a strictly declining growth rate, and will get to ease until a 0 (zero) growth rate is reached. When the growth rate is zero, the population of the nation will be at its highest. It is discovered that the logistic model is applied to census data of populations from various countries(Verhulst, 1845). Obaidullah (1976) introduced a model known as the "Expo-linear Model," which is said to describe population expansion over time better than either an exponential or a linear model(Obaidullah, 1976). However, he acknowledged that, in contrast to exponential or linear models, his model's parameter interpretation was challenging. Mallik (1980) examined the population movement in Bangladesh and predicted that there would be no further population growth in the following century (2080). The exponential growth model was refined by Beekman in 1981. This model used a Markov chain to replicate the decline in birthrate brought on by rural-urban migration (Beekman, 1981). Their inability to predict the year Bangladesh's population will peak is their work's primary drawback. Even if some scholars predict Bangladesh's population, someone has used data from the UN or the global meter, which is only marginally different from Bangladesh's real census data. However, we continue working with authentic Bangladeshi population census data. A student assessment of mathematical modeling at the primary school level has been successfully exploited by Turner et al. (Turner et al., 1969). Mathews (Mathews, 1992) have discussed bounded population growth: a curve fitting lesson. Tsoularis (Tsoularis, 2002) has discussed the analysis of Logistic Growth Models. Res et al. (Res et al., 2005) have discussed the advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance. Shair et al. (Shair et al., 2017) have evaluated Extensions to Coherent Mortality Forecasting Models. Zabadi et al. (Zabadi et al., 2017) have discussed a Mathematical and Statistical Approach for Predicting the Population Growth. Uddin et al. (Nizam Uddin et al., 2019) have Predicted for Future Population Growth of Bangladesh by Using Exponential & Logistic models. Ullah et al. (Ullah et al., 2019) have analyzed and Projected of Future Bangladesh Population Using Logistic Growth Model. Sardain et al. (Sardain et al., 2019) have discussed global forecasts of shipping traffic and biological invasions to 2050. Mahabub and Habib (Mahabub & Habib, 2019) have discussed

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an Overview of Weather Forecasting for Bangladesh Using Machine Learning Techniques. Voumik et al. (Voumik et al., 2019) have discussed forecasting GDP growth rates of Bangladesh : an empirical study. Abassian et al. (Abassian et al., 2019) have investigated in Mathematics Learning Five different perspectives on mathematical modeling in mathematics education. Rogers and Thrall (Rogers & Thrall, 2020) have discussed the Web Book of Regional Science Sponsored by Regional Population Projection Models By Series Editor. U.S. Bureau (Bureau, 2020) have discussed Demographic Turning Points for the United States : Population Projections for 2020 to 2060. Mac-camy et al.(Mac-camy et al., 2020) have investigated future population projection of Bangladesh with the help of Investigate future population projection of Bangladesh with the help of Malthusian model, Sharpe-lotka model and Gurtin Mac-Camy model. Biological Sciences et al. (Sciences et al., 2021) have discussed ecological forecasts reveal limitations of common model selection methods: predicting changes in beaver colony densities. Anam (Anam, 2022) has determined Future Population Projection of Bangladesh by Growth Rate Modeling Using Future Population Projection of Bangladesh by Growth Rate Modeling Using Logistic Population Model. Karim et al.(Karim et al., 2022) have studied Forecasting Bangladesh by Using Verhulst Logistic Growth Model and Population Model. For the Exponential growth model, the growth rate has been estimated at approximately 2%, and Bangladesh's population has been predicted to be 863.30 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 105.92%. According to the logistic model, the growth rate has been estimated at approximately 4%, and the total population of Bangladesh has been predicted to be 261.43 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 16.70%. By the Discrete Logistic model, the population of Bangladesh is calculated to be 179.798 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 0.84%. In this article, we determine the forecasting population using growth models (exponential model, logistic model, and discrete logistic model) for the Bangladesh census data.

2. METHODOLOGY

2.1. Model of Exponential Growth

Consider the population of a certain species at time t as p=p (t). The variation of the population is the rate of change of p is proportional to the current value of p, that is

$$\frac{dp}{dt} \propto p \implies \frac{dp}{dt} = rp \qquad (1)$$

Where r the proportionality is constant, called the rate of growth, depending on whether it is positive or negative. Let $r > 0$, so the population is growing.

From (1) $\frac{dp}{p} = rdt$

Integrating,

$$\implies \log p = rt + \log c$$

$$\implies p(t) = ce^{rt}$$

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If $p(0) = p_0$ then

$$p_0 = ce^0 = c$$

$$\therefore p = p(t) = p_0 e^{rt} \qquad (2)$$

2.2. Logistic Growth:

The growth rate depends on the population $\frac{dp}{dt} = rp$. We replace the constant r with a function $h(p)$ and thus we obtain the modified equation $\frac{dp}{dt} = h(p)p$. We now choose $h(p)$ so that $h(p) \cong r > 0$ when p is small, and $h(p)$ decreases as p grows larger. Having properties, the simplest function is $h(p) = r - ap$, where a is a positive constant. Thus, the modified equation becomes

$$\frac{dp}{dt} = (r - ap)p \qquad (3)$$

Which is known as the logistic equation or Verhulst equation.

2.3. Discrete Logistic Growth Models:

Let N_t and N_{t+1} be the population at time t and t+1 respectively. This leads us to study different equations of the form

$$N_{t+1} = N_t F(N_t) = f(N_t) \qquad (4)$$

Where $f(N_t)$ is generally a nonlinear function of N_t. In practice, it is simple to evaluate $N_{t+1}$and succeeding generations by applying (4) recursively if we are aware of the form of$f(N_t)$.

Assuming that $F(N_t) = r > 0$, and then from (1),

$$N_{t+1} = rN_t \quad \Rightarrow \quad N_t = r^t N_0. \qquad (5)$$

The population one step later is simply proportionate to the present population.

So, the population grows or decays geometrically according to whether $r > 1$ or $r < 1$ respectively. This simple model is not very realistic for most populations or for long times.

The Malthus model in this case is discrete. A slight modification of (5) could be

$$N_{t+1} = rN_S, \ \ N_S = N_t^{1-b}, \ \ \text{b constant}$$

Where N_s is the population that survives to breed. There must be restrictions so that $N_S \leq N_t$.

A variety of $f(N_t)$ has been used in Practical Biological situations by May and Oster [1976].

One such model sometimes referred to as the Verhulst process, is

$$N_{t+1} = rN_t\left(1 - \frac{N_t}{K}\right), \ \ r > 0, \ \ K > 0, \qquad (6)$$

Which is a kind of discrete analog of the continuous logistic growth model. For this model, if $N_t > K$ then $N_{t+1} < 0$.

One such frequently used model is

$N_{t+1} = N_t\, exp\left[r\left(1-\frac{N_t}{K}\right)\right],\quad r > o,\quad K > 0$ (7)

Where $exp(-rN_t/K)$ is a mortality factor that is more severe the l0061rger N_t. If $N_0 > 0$, then $N_t > 0$ for all t.

Let us rescale the nonlinear logistic model (6) by writing $u_t = N_t/K$ so that the carrying capacity K is now 1.

Then, let us consider the nonlinear logistic-type model is

$u_{t+1} = ru_t(1-u_t), r > 0,$ (8)

Where we assume $0 < u_0 < 1$ and we are interested in solutions $u_t \geq 0$.

Let u^* be the steady state.

Then

$u^* = ru^*(1-u^*)$

$\Rightarrow u^*[r(1-u^*)-1] = 0$

$\Rightarrow u^* = 0, r(1-u^*)-1 = 0$

$\Rightarrow u^* = 0, u^* = 1-\frac{1}{r} = \frac{r-1}{r}$ (9)

Here $u_{t+1} = ru_t(1-u_t) = f(u_t)$, say

$\therefore\ f'(u_t) = r(1-2u_t)$

$\therefore$ The corresponding eigenvalues λ are

$\lambda = f'(0) = r(1-2.0) = r$

and $\lambda = f'\left(\frac{r-1}{r}\right) = 2-r$ (10)

As r increases from zero to 1 the only realistic equilibrium is $u^* = 0$ which is stable. It is also clear from equation (4) that $u_1 < u_0 < 1$ and $u_{t+1} < u_t$ for all t, which implies that $u_t \to 0$ as $t \to +\infty$.

The first bifurcation is at r = 1 since $u^* = 0$ becomes unstable since its Eigen value $\lambda > 1$ for $r > 1$.

The positive steady state $u^* = \frac{r-1}{r} > 0$ is stable since for $1 < r < 3$ we have $-1 < \lambda < 1$. The second bifurcation is at $r = 3$, where $\lambda = -1$.

We introduce the iterative procedure

$$u_1 = f(u_0)$$

$u_2 = f\big(f(u_0)\big) = f^2(u_0)$ (11)

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$$\vdots$$

$$u_t = f^t(u_0)$$

to see what is happening when r passes through the bifurcation value r = 3.

Now the second iterate is

$$u_{t+2} = f^2(u_t) = r[ru_t(1-u_t)][1-ru_t(1-u_t)] \quad (12)$$

If it has an equilibrium, say $u_{t+2} = u_t = u_2^*$, then

$$u_2^* = r[ru_2^*(1-u_2^*)][1-ru_2^*(1-u_2^*)]$$

$$\Rightarrow u_2^*\left[r^2(1-u_2^*)\left(1-ru_2^*+ru_2^{*2}\right)\right] = u_2^*$$

$$\Rightarrow u_2^*\left[r^3u_2^{*3} - 2r^3u_2^{*2} + r^3u_2^* + r^2u_2^* - r^2 + 1\right] = 0$$

$$\Rightarrow u_2^*\left[r^2u_2^{*2}(ru_2^*-r+1) - r^2u_2^*(ru_2^*-r+1) - ru_2^*(ru_2^*-r+1) + r(ru_2^*-r+1) + 1(ru_2^*-r+1)\right] = 0$$

$$\Rightarrow u_2^*\left[(ru_2^*-r+1)\left(r^2u_2^{*2} - r^2u_2^* - ru_2^* + r + 1\right)\right] = 0$$

$$\Rightarrow u_2^*[ru_2^* - (r-1)][r^2u_2^{*2} - r(r+1)u_2^* + (r+1)] = 0 \quad (13)$$

The solutions are

$u_2^* = 0$, or $u_2^* = \frac{r-1}{r} > 0$ if $r > 0$

$$u_2^* = \frac{r(r+1) \pm \sqrt{\{r(r+1)\}^2 - 4r^2(r+1)}}{2r^2}$$

$$\Rightarrow u_2^* = \frac{(r+1) \pm \sqrt{(r+1)(r-3)}}{2r} > 0 \text{ if } r > 3$$

Thus, we see that there are two more real steady states of $u_{t+2} = f^2(u_t)$ if r>3.

Other properties of equation (6), such as chaos, stability, periodic solutions, and bifurcations, may be found in Murray (J.D., 2002).

3. DETERMINATION OF CARRYING CAPACITY AND GROWTH RATE

To predict the future population of Bangladesh, we need to determine the growth rate of Bangladesh using the exponential growth model. Using the actual population of Bangladesh in Table 1 with t = 0 i.e. t_0 corresponding to the year 2001, we have$P(0) = 131.6$.

Again t = 1 i.e. t_1 corresponding to the year 2002, we have$P(1) = 134.1$.

We know the Exponential Model is

$$P(t)=P_0e^{rt} \quad (14)$$

After simplifying this, we get

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$$P_1 = P_0 e^{r(t_1 - t_0)} \quad (15)$$

$$134.1 = 131.6 e^{r(1-0)}$$

$$r = ln \frac{134.1}{131.6} = 0.019$$

Hence, the general solution is

$$P(t) = 131.6 e^{0.019t} \quad (16)$$

This suggests that the prediction rate of population growth is 2% in Bangladesh with this we project the population of Bangladesh from 2023 to 2100. Again, based on table 1, let t=0, 1, and 2 correspond to the year 2001, 2002, and 2003 respectively. Then $P_0, P_1,$ and P_2 also corresponds 131.6, 134.1 and 136.6 (in millions).

From Logistic Model, the carrying capacity K is

$$K = \frac{P_0 P_1^2 + P_1^2 P_2 - 2P_0 P_1 P_2}{P_1^2 - P_0 P_2} \quad (17)$$

Putting the value of $P_0, P_1,$ and P_2 in (17), we get

= 268.2 (Million)

This is the predicted carrying capacity of the population of Bangladesh.

Again, the growth rate r is

$$r = \frac{1}{T} \ln \left(\frac{\frac{1}{P_0} - \frac{1}{K}}{\frac{1}{P_1} - \frac{1}{K}} \right) \quad (18)$$

Here, $T = 1, P_0 = 131.6, P_1 = 134.1,$ and $K = 268.2$ putting these in (18), we get

$$r = 0.03728992825$$

Now the Logistic Model is

$$P(t) = \frac{K}{1 + \left(\frac{K}{P_0} - 1\right) e^{-rt}}$$

Putting the value of $K, P_0,$ and r, we get

$$P(t) = \frac{268.2}{1 + \left(\frac{268.2}{131.6} - 1\right) e^{-(0.03728992825)t}} \quad (19)$$

Here, we know the Discrete Logistic Model is

$$P_{t+1} = P_t + rP_t \left(1 - \frac{P_t}{K}\right)$$

Initially, for $t = 0$, we get

$$P_1 = P_0 + rP_0 \left(1 - \frac{P_0}{K}\right)$$

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Putting the value of P_0, P_1, and K, we get

$$134.1 = 131.6 + 131.6r\left(1 - \frac{131.6}{268.2}\right)$$

$$r = \frac{134.1 - 131.6}{67.0266965} = 0.03729857103 \quad (20)$$

The general solution of the Discrete Logistic Model is

$$P_{t+1} = P_t + 0.03729857103P_t\left(1 - \frac{P_t}{268.2}\right)$$

Table 1: Projecting Population

Year	**Actual Population (In Millions) (United Nations, 2022)**	**Projecting Population (In Millions)**					
		1.Population by Exponential Model	**1. Absolute Error %**	**2. Population by Logistic Model**	**2. Absolute Error %**	**3. Population by Discrete Logistic Model**	**3. Absolute Error %**
2001	131.6	131.6	0.0%	131.6	0.0%	131.6	0.0%
2002	134.1	134.124305	0.01%	134.1	0.0%	134.1	0.0%
2003	136.6	136.6970302	0.07%	136.6	0.0%	136.6008692	0.0%
2004	139	139.3191046	0.22%	139.0982628	0.07%	139.1	0.07%
2005	141.3	141.9914748	0.48%	141.5930561	0.21%	141.4975301	0.1%
2006	143.4	144.7151053	0.91%	144.0826572	0.48%	143.7936598	0.3%
2007	145.4	147.4909796	1.44%	146.5653577	0.80%	145.888841	0.3%
2008	147.1	150.3200998	2.19%	149.0394682	1.3%	147.8831113	0.5%
2009	148.8	153.2034871	2.96%	151.5033231	1.8%	149.5773664	0.5%
2010	150.5	156.1421826	3.75%	153.9552844	2.3%	151.2708176	0.5%
2011	152.2	159.1372471	4.56%	156.3937467	2.8%	152.9634649	0.5%
2012	153.9	162.1897618	5.39%	158.8171405	3.2%	154.6553085	0.49%
2013	155.7	165.3008288	6.17%	161.2239367	3.5%	156.3463482	0.42%
2014	157.6	168.4715712	6.89%	163.61265	3.8%	158.1359847	0.34%
2015	159.4	171.7031337	7.72%	165.981842	4.1%	160.0240678	0.39%
2016	161.2	174.9966829	8.56%	168.330125	4.4%	161.8118519	0.38%
2017	163	178.3534078	9.42%	170.6561641	4.7%	163.5987348	0.37%
2018	164.7	181.7745202	10.37%	172.9586804	5.01%	165.3847165	0.42%
2019	166.4	185.2612552	11.33%	175.2364531	5.3%	167.0706496	0.40%
2020	168.1	188.8148715	12.32%	177.4883215	5.6%	168.7557789	0.39%
2021	169.4	192.436652	13.59%	179.7131866	6.09%	170.4401043	0.61%
2022	171.2	196.1279042	14.56%	181.9100127	6.3%	171.7275755	0.31%

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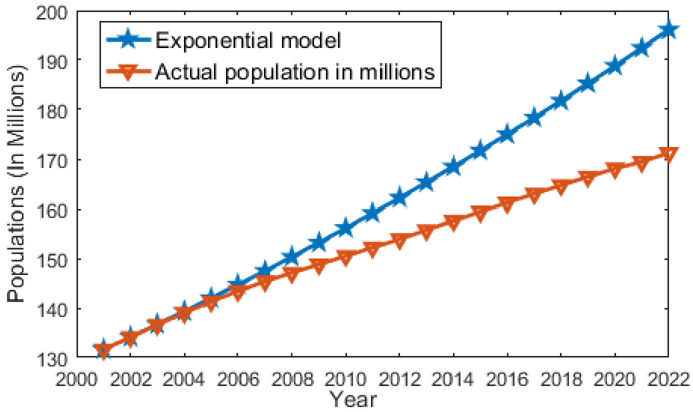

Figure 2: The actual (blue) and predicted (red) populations of Bangladesh using the Malthusian model (15) from 2001 to 2022

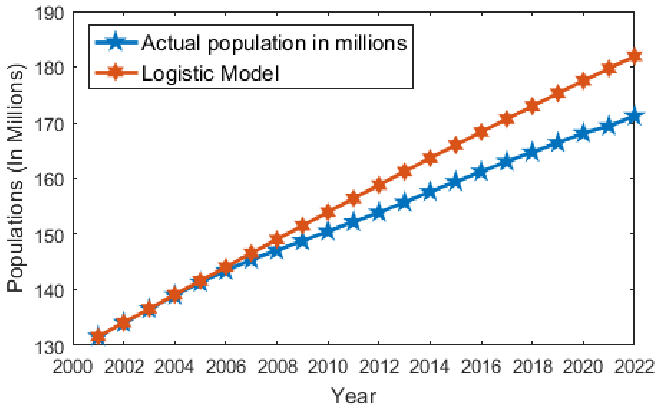

Figure 3: The actual (blue) and predicted (red) populations of Bangladesh using the Logistic model (17) from 2001 to 2022

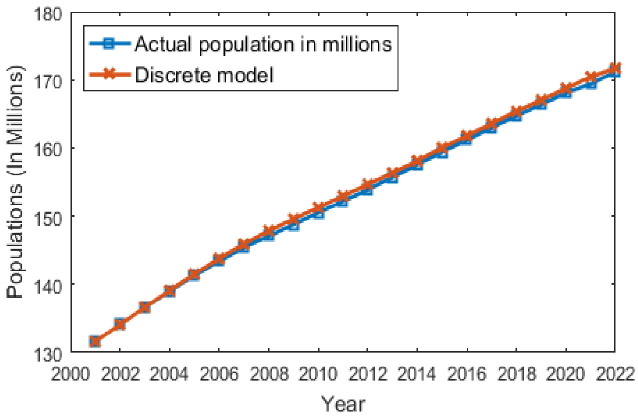

Figure 4: The actual (red) and predicted (blue) populations of Bangladesh using the Discrete Logistic model (17) from 2001 to 2022

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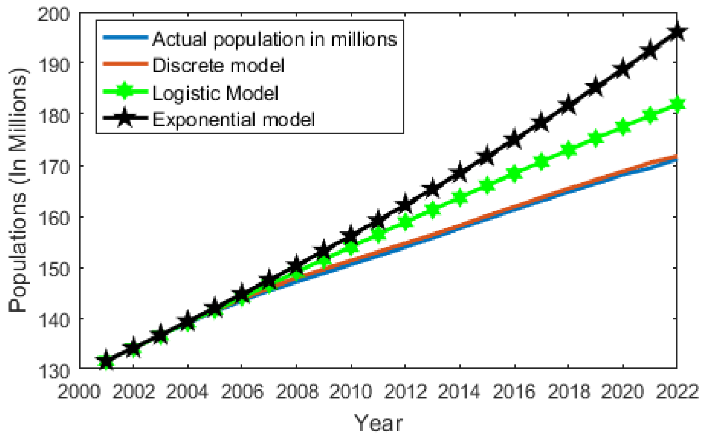

Figure 5: Comparison of population projection among Exponential (Malthusian), Logistic model, and Discrete Logistic model with actual data from 2001 to 2022

Table 2: Forecasting population of Bangladesh using Growth Models

Year	Actual Population and U.N. Projections (In Millions) (United Nations, 2022)	Projecting Population (In Millions)					
		Population by Exponential Model	Absolute Error %	Population by Logistic Model	Absolute Error %	Population by Discrete Logistic Model	Absolute Error %
2001	131.6	131.6	0.0%	131.6	0.0%	131.6	0.0%
2002	134.1	134.124305	0.01%	134.1	0.0%	134.1	0.0%
2003	136.6	136.6970302	0.07%	136.6	0.0%	136.6008692	0.0%
2004	139	139.3191046	0.22%	139.0982628	0.07%	139.1	0.07%
2005	141.3	141.9914748	0.48%	141.5930562	0.21%	141.4975301	0.1%
2006	143.4	144.7151053	0.91%	144.0826573	0.48%	143.7936598	0.3%
2007	145.4	147.4909796	1.44%	146.5653578	0.80%	145.888841	0.3%
2008	147.1	150.3200998	2.19%	149.0394683	1.3%	147.8831113	0.5%
2009	148.8	153.2034871	2.96%	151.5033232	1.8%	149.5773664	0.5%
2010	150.5	156.1421826	3.75%	153.9552846	2.3%	151.2708176	0.5%
2011	152.2	159.1372471	4.56%	156.3937468	2.8%	152.9634649	0.5%
2012	153.9	162.1897618	5.39%	158.8171406	3.2%	154.6553085	0.49%
2013	155.7	165.3008288	6.17%	161.2239369	3.6%	156.3463482	0.42%
2014	157.6	168.4715712	6.89%	163.6126502	3.8%	158.1359847	0.34%
2015	159.4	171.7031337	7.72%	165.9818422	4.1%	160.0240678	0.39%
2016	161.2	174.9966829	8.56%	168.3301252	4.4%	161.8118519	0.38%
2017	163	178.3534078	9.42%	170.6561643	4.7%	163.5987348	0.37%
2018	164.7	181.7745202	10.37%	172.9586807	5.01%	165.3847165	0.42%
2019	166.4	185.2612552	11.33%	175.2364534	5.3%	167.0706496	0.40%
2020	168.1	188.8148715	12.32%	177.4883218	5.6%	168.7557789	0.39%
2021	169.4	192.436652	13.59%	179.7131869	6.09%	170.4401043	0.61%
2022	171.2	196.1279042	14.56%	181.910013	6.3%	171.7275755	0.31%
2023	172.954	199.8899608	15.57%	184.077829	6.4%	173.5094519	0.32%

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2024	174.701	203.7241797	16.61%	186.2157289	6.59%	175.2449245	0.31%
2025	176.42	207.6319454	17.69%	188.3228725	6.75%	176.9726205	0.31%
2026	178.1	211.6146684	18.82%	190.3984859	6.91%	178.6717973	0.32%
2027	179.75	215.6737866	19.99%	192.4418612	7.06%	180.3316297	0.32%
2028	181.3	219.8107654	21.2%	194.4523564	7.25%	181.9610581	0.36%
2029	182.9	224.0270983	22.48%	196.429395	7.4%	183.4910435	0.32%
2030	184.4	228.3243073	23.82%	198.3724654	7.58%	185.0696824	0.36%
2031	185.8	232.703944	25.24%	200.2811196	7.8%	186.5490096	0.40%
2032	187.2	237.1675893	26.69%	202.1549724	7.989%	187.9291504	0.39%
2033	188.6	241.7168546	28.16%	203.9937001	8.16%	189.3087461	0.38%
2034	189.9	246.3533824	29.73%	205.7970387	8.37%	190.6877966	0.41%
2035	191.1	251.0788464	31.39%	207.5647829	8.6%	191.9678553	0.45%
2036	192.3	255.8949526	33.07%	209.2967834	8.84%	193.1490308	0.44%
2037	193.47	260.8034397	34.8%	210.992946	9.06%	194.3298058	0.44%
2038	194.5	265.8060796	36.66%	212.6532289	9.3%	195.4806757	0.50%
2039	195.5	270.9046784	38.57%	214.277641	9.6%	196.4935196	0.51%
2040	196.5	276.1010768	40.5%	215.8662394	9.86%	197.4765809	0.49%
2041	197.4	281.3971506	42.55%	217.4191277	10.14%	198.459364	0.54%
2042	198.3	286.7948119	44.6%	218.9364533	10.41%	199.343631	0.53%
2043	199.1	292.2960092	46.8%	220.418405	10.71%	200.2276728	0.57%
2044	199.95	297.9027285	48.99%	221.8652113	10.96%	201.0132985	0.53%
2045	200.7	303.616994	51.28%	223.2771374	11.25%	201.8478308	0.57%
2046	201.4	309.4408685	53.64%	224.6544834	11.55%	202.5840159	0.59%
2047	202.1	315.3764545	56.05%	225.9975816	11.82%	203.2709809	0.58%
2048	202.75	321.4258948	58.53%	227.3067945	12.1%	203.9578096	0.60%
2049	203.3	327.5913734	61.14%	228.5825126	12.44%	204.595457	0.64%
2050	203.9	333.875116	63.74%	229.8251517	12.71%	205.1349131	0.61%
2051	204.4	340.2793912	66.48%	231.0351514	13.03%	205.7233146	0.65%
2052	204.88	346.8065109	69.27%	232.2129726	13.34%	206.2135728	0.65%
2053	205.3	353.4588315	72.17%	233.3590956	13.67%	206.6841552	0.67%
2054	205.67	360.2387546	75.15%	234.4740181	14.004%	207.0958622	0.69%
2055	206	367.1487278	78.23%	235.5582532	14.35%	207.4585159	0.71%
2056	206.2	374.1912457	81.47%	236.6123277	14.75%	207.7819316	0.77%
2057	206.5	381.3688507	84.68%	237.6367801	15.07%	207.9779264	0.72%
2058	206.69	388.684134	88.05%	238.6321595	15.45%	208.2718977	0.77%
2059	206.8	396.1397365	91.56%	239.599023	15.86%	208.4580666	0.80%
2060	206.88	403.7383497	95.16%	240.5379352	16.27%	208.565844	0.81%
2061	206.9	411.4827169	98.88%	241.449466	16.69%	208.6442254	0.84%
2062	206.87	419.3756337	102.72%	242.3341898	17.14%	208.6638205	0.87%
2063	206.78	427.4199498	106.70%	243.1926835	17.61%	208.6344278	0.90%
2064	206.6	435.6185691	110.85%	244.025526	18.11%	208.5462483	0.94%
2065	206.4	443.9744514	115.1%	244.8332966	18.62%	208.3698826	0.95%
2066	206.18	452.4906133	119.46%	245.6165741	19.13%	208.17391	0.97%
2067	205.89	461.1701293	123.99%	246.3759357	19.66%	207.9583274	1.01%
2068	205.5	470.0161326	128.72%	247.1119562	20.25%	207.6741297	1.06%
2069	205.1	479.0318169	133.56%	247.8252071	20.83%	207.2918959	1.07%
2070	204.7	488.2204369	138.51%	248.5162556	21.41%	206.8998174	1.07%
2071	204.2	497.5853098	143.68%	249.1856642	22.03%	206.5076943	1.13%
2072	203.7	507.1298164	148.96%	249.8339899	22.65%	206.0174779	1.14%

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2073	203.1	516.8574023	154.48%	250.4617834	23.32%	205.5271919	1.20%
2074	202.5	526.7715794	160.13%	251.0695889	23.98%	204.938757	1.20%
2075	201.8	536.8759267	166.04%	251.6579432	24.71%	204.3502219	1.26%
2076	201.2	547.1740921	171.96%	252.2273757	25.36%	203.6634711	1.22%
2077	200.4	557.6697933	178.28%	252.7784077	26.14%	203.0747191	1.33%
2078	199.7	568.3668193	184.61%	253.3115519	26.85%	202.2895606	1.30%
2079	198.9	579.2690319	191.24%	253.8273127	27.62%	201.602401	1.36%
2080	198.1	590.3803669	198.02%	254.3261852	28.38%	200.8169087	1.37%
2081	197.2	601.7048357	205.12%	254.8086556	29.21%	200.0312385	1.44%
2082	196.3	613.2465264	212.4%	255.2752003	30.04%	199.1471467	1.45%
2083	195.4	625.0096058	219.86%	255.7262867	30.87%	198.2628297	1.47%
2084	194.4	636.9983204	227.67%	256.1623722	31.77%	197.3782873	1.53%
2085	193.4	649.2169984	235.69%	256.5839045	32.67%	196.3951982	1.55%
2086	192.4	661.6700507	243.9%	256.9913215	33.57%	195.411831	1.57%
2087	191.4	674.3619731	252.33%	257.3850513	34.47%	194.4281856	1.58%
2088	190.3	687.2973476	261.17%	257.765512	35.45%	193.4442621	1.65%
2089	189.2	700.4808438	270.23%	258.1331119	36.43%	192.361625	1.67%
2090	188.1	713.9172212	279.54%	258.4882494	37.42%	191.2786513	1.69%
2091	187	727.6113305	289.09%	258.8313132	38.41%	190.1953411	1.71%
2092	185.86	741.5681153	298.99%	259.1626821	39.44%	189.1116943	1.75%
2093	184.69	755.7926143	309.2%	259.4827253	40.49%	187.9882871	1.79%
2094	183.5	770.2899627	319.78%	259.7918023	41.58%	186.8349407	1.82%
2095	182.3	785.0653941	330.64%	260.0902632	42.67%	185.6614883	1.84%
2096	181.1	800.1242426	341.81%	260.3784489	43.78%	184.4777762	1.87%
2097	179.95	815.4719447	353.17%	260.6566906	44.85%	183.2936636	1.86%
2098	178.76	831.114041	364.93%	260.9253108	45.96%	182.1585131	1.90%
2099	177.56	847.0561785	377.05%	261.1846228	47.097%	180.9834919	1.93%
2100	176.3	863.3041125	389.68%	261.434931	48.29%	179.7981977	1.98%
Mean Absolute Percentage Error			105.92%		16.70%		0.84%

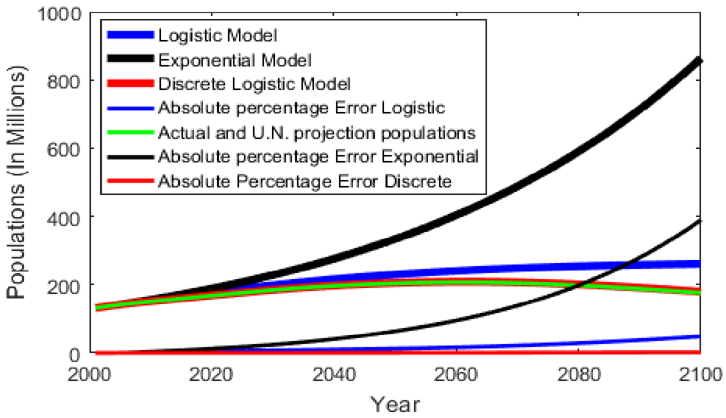

Figure 6: Forecasting Population of Bangladesh using Exponential, Logistic, and Discrete Logistic Model with corresponding Absolute Percentage Error

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5. RESULTS AND DISCUSSIONS

The predicted population of Bangladesh using three models is presented in Table 2 from 2001 to 2022 with actual data. We have calculated the predicted population of Bangladesh from equations (16), (19), and (20). In Table 2 we get that the predicted populations of Bangladesh are expected to be 196.13 million (approximately), 181.92 million (approximately), and 171.73 million in 2022 by using the Exponential model, the Logistic model, and the Discrete Logistic model respectively. Let us now present the numerical simulation results of the actual and projected populations of Bangladesh from the year 2001 to 2022 using the Exponential model (16), Logistic model (19), and Discrete Logistic model in Figures 2, 3, and 4 respectively. Now, a combined graph of the Exponential, Logistic and Discrete Logistic models showing a comparative population size in Bangladesh from 2001 to 2022 is shown in Figure 5.

Again, we have calculated the predicted population of Bangladesh from 2001 to 2100 using the equations (16), (19), and (20). As shown in Table 3, we get that the predicted populations of Bangladesh are expected to be 863.30 million (approximately), 261.43 million (approximately), and 179.798 million in 2100 by using the Exponential model, the Logistic model, and Discrete Logistic model respectively. In this situation, population growth is a pressing problem in Bangladesh like in every developing country. With our limited resources, it will too difficult to cope with this overpopulation but Bangladesh tries to reduce the population growth rate, increase the Literacy rate and create mass awareness to overcome this problem. Now we present the numerical simulation results of the actual and projected populations of Bangladesh from the year 2001 to 2100 using the Exponential model (16), Logistic model (19), and Discrete Logistic model (20) in Figures 7, 8, and 9 respectively.

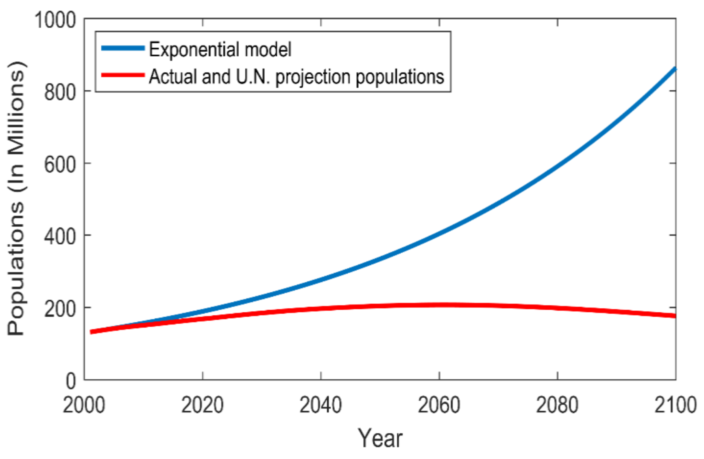

Figure 7: The actual (red) and predicted (blue) populations of Bangladesh using the Exponential model (16) from 2001 to 2100

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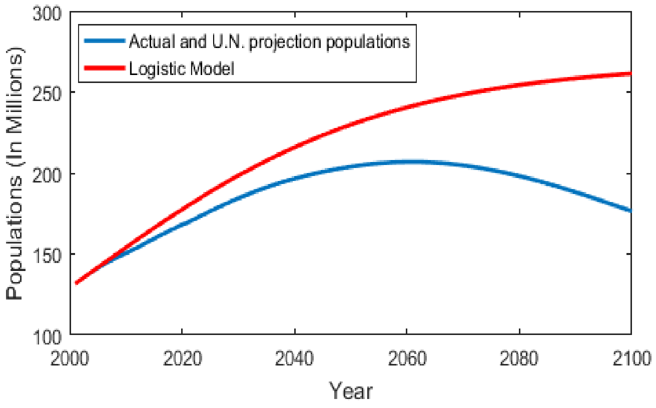

Figure 8: The actual (blue) and predicted (red) populations of Bangladesh using the Logistic model (19) from 2001 to 2100

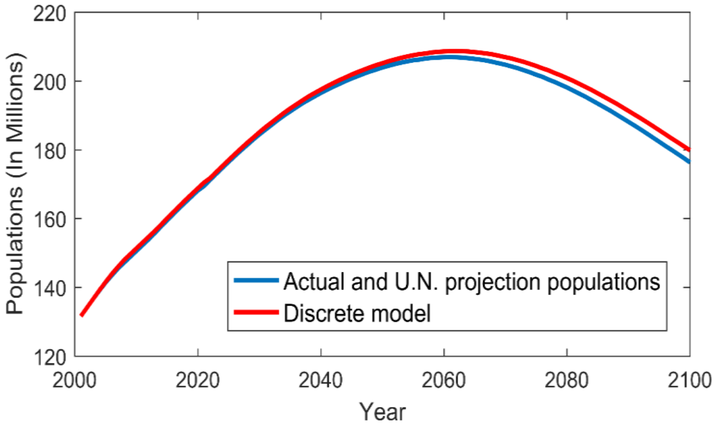

Figure 9: The actual (blue) and predicted (red) populations of Bangladesh using the Discrete Logistic model (20) from 2001 to 2100

6. CONCLUSION

Finally, we find that according to the exponential model the predicted growth rate is approximately 2%, and the predicted population of Bangladesh is 863.30 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 105.92%. According to the logistic model, the population growth rate of Bangladesh is approximately 4% and the carrying capacity for the population of Bangladesh is calculated as 268.2 million. By the Logistic model, the population of Bangladesh is calculated to be 261.43 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 16.70%. On the other hand, assuming the carrying capacity for the population of Bangladesh is 268.2 million and the population growth rate of Bangladesh is approximately 4% according to the Discrete Logistic model. By the Discrete Logistic model, the population of Bangladesh is calculated to be 179.798 million in 2100 with a Mean Absolute Percentage Error (MAPE) of 0.84%. It is also shown that Mean Absolute Percentage Error (MAPE) is very

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low in the Discrete Logistic model. From this point of view, we can finally conclude that the Discrete Logistic model gives a good forecasting result as compared to the Exponential and Logistic model for long-term prediction.

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