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THE ACTION OF SHOCK WAVES ON CYLINDRICAL PANELS PLACED IN A BOUNDLESS IDEAL FLUID

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Abstract

In this scientific article, the problems associated with the interaction of a plane shock pressure wave on an elastic circular cylindrical panel are considered. A cylindrical panel fixed in a cylindrical screen is placed in a boundless ideal fluid. The transverse oscillations of the panel are described by the well-known finite deflection equations according to the theory of thin shallow shells. The problem of non-linear motion of an elastic panel under the action of a weak shock wave is a difficult task. To simplify the problem, the pressure of reflected and radiated waves is determined approximately without taking into account diffraction from boundary edges. Based on these simplifications, the basic formulas for a smooth cylindrical shell are derived. Nonlinear differential equations of motion of a cylindrical panel placed in an infinite ideal fluid are solved numerically using the Maple-17 program. The results of the change in the amplitude of the deflection and displacement of the middle surface of the panel of the cylindrical shell from time to time at different angle-β and coefficient λ are obtained. The graphs obtained show that the oscillations of the panel in an ideal liquid are close to aperiodic. This is due to its large damping.

Keywords: task, plane, impact, wave, pressure, elastic, circular, cylindrical, panel, limitless, ideal, liquid, transverse. fluctuations. equations, non-linear, motion, diffraction, edges, differential, fluid, method, program, amplitude, deflection, displacement, surface, damping.

INTRODUCTION

The problems associated with the interaction of a plane pressure wave on an elastic circular cylindrical panel are considered [1-3].

A cylindrical panel, fixed in a cylindrical screen, is placed in a boundless ideal fluid.

The transverse oscillations of the panel are described by the well-known finite deflection equations of the theory of thin shallow shells.

The problem of non-linear motion of an elastic panel under the action of a weak shock wave is a difficult task.

To simplify the problem, the pressure of reflected and radiated waves is determined approximately without taking into account diffraction from boundary ribs based on formulas derived for a smooth cylindrical shell.

Statement of the problem and differential equations of nonlinear oscillations of a cylindrical panel. Let us investigate the case of non-linear vibrations of a cylindrical panel with length a and width b (Figure 1)

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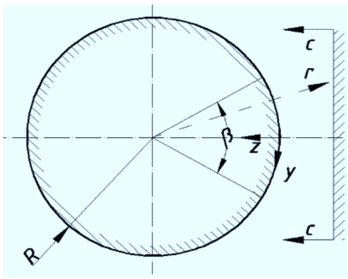

Figure 1: **Elastically circular cylindrical shallow shell, placed compressible liquid**

When solving the problem, we will proceed from the nonlinear equations of the theory of shallow shells with respect to displacement w(x, y, t) and stress functions Φ(x, y, t).

$$\frac{D}{h}\nabla^4 w + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial x \partial y} + \frac{1}{R}\frac{\partial^2 \Phi}{\partial x^2} + \frac{q}{h},$$

$$\frac{1}{E}\partial^4 \Phi + \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} \tag{1.1}$$

Here: x, y are the coordinates along the generatrix and along the arc, t is the time, c is the speed of sound in the liquid, q (x, y, t) is the intensity of the transverse load[4-7].

R-radius, h-thickness, $D = \frac{Eh^3}{12(1-\mu^2)}$, μ - Poisson's ratio, E - modulus of elasticity of the material of the cylindrical shell.

Equation (1.1) takes into account only the transverse inertia of the shell. Since taking into account the inertia in the middle surface of the shell has little effect on the nature of the motion.

Boundary conditions of the problem:

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \ \text{ for } x = \pm\frac{a}{2}\ ; \ w = \frac{\partial^2 w}{\partial x^2} = 0$$

$$\text{for } x = \pm\frac{a}{2}\ ; \ w = \frac{\partial^2 w}{\partial x^2} = 0\ ; \tag{1.2}$$

Imagine the external load on the shell as follows:

$$q = p - \rho_o h\frac{\partial^2 w}{\partial t^2} - \rho_o h\varepsilon\frac{\partial w}{\partial t} \tag{1.3}$$

Here: p- is the hydrodynamic pressure, ρ_o -is the density of the shell material, ε is the attenuation coefficient.

The deflection of a gently sloping hinged cylindrical shell can be represented as:

$$w(x, y, t) = f(t)\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}\ , \tag{1.4}$$

where: f(t) - is the time-dependent deflection amplitude. For this case, the solution of the second equation (1.1) with respect to the stress function Φ will be:

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$$\frac{1}{E}\Phi = \frac{f^2}{32}\left(-\left(\frac{a}{b}\right)^2 cos\frac{2\pi x}{a} - \left(\frac{a}{b}\right)^2 cos\frac{2\pi y}{b}\right) + \frac{1}{R}\frac{f}{(\pi a)^2(a^{-2}+b^{-2})^2} cos\frac{\pi x}{a}\, cos\frac{\pi y}{b} - \frac{\sigma_1 y^2+\sigma_2 x}{2E} \quad (1.5)$$

Here: σ_1, σ_2 - chain stresses resulting from the interaction of the shell panel with reinforcing ribs. The mean shear stresses are assumed to be zero.

Considering that when the averaged edge approaches are equal to zero, we have the following condition to determine σ_1, σ_2:

$$-\int_{-\frac{b}{2}}^{\frac{b}{2}} dy \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{\partial u}{\partial x} dx = -\int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\partial v}{\partial y} dy = 0 \quad (1.6)$$

where: u, v - displacement of the points of the middle surface in the direction of the x, y axes.

Taking into account the ratios for the relative elongations of the middle surface $\varepsilon_1, \varepsilon_2$, we have:

$$\varepsilon_1 = \frac{1}{E}\left(\frac{\partial^2\Phi}{\partial y^2} - \mu\frac{\partial^2\Phi}{\partial x^2}\right), \varepsilon_2 = \frac{1}{E}\left(\frac{\partial^2\Phi}{\partial x^2} - \mu\frac{\partial^2\Phi}{\partial y^2}\right), \quad (1.7)$$

$$\frac{\partial u}{\partial x} = \varepsilon_1 - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \frac{\partial v}{\partial y} = \varepsilon_2 - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{w}{R} \quad (1.8)$$

Taking into account (1.4) – (1.8), we obtain expressions for chain voltages:

$$\sigma_1 = \frac{E}{1-\mu^2}\left[-\frac{(\pi f)^2}{8a^2}(1+\mu\lambda^2) + \frac{4f}{\pi^2 R}\frac{\mu(1+\lambda^2)^2-\lambda^2(1-\mu^2)}{(1+\lambda^2)^2}\right],$$

$$\sigma_2 = \frac{E}{1-\mu^2}\left[-\frac{(\pi f)^2}{8a^2}(\mu+\lambda^2) + \frac{4f}{\pi^2 R}\frac{\mu^2+2\lambda^2+\lambda^4}{(1+\lambda^2)^2}\right] \quad (1.9)$$

Hydrodynamic pressure on the panel surface

The pressure $p(x, y, t)$ acting on the shell can be represented as: $p = p_1 + p_2 + p_3$; here: p_1 is the pressure of the incident wave, p_2 is the pressure in the wave reflected from the stationary non-deformable shell, p_3 is the pressure of the radiated waves due to panel oscillation.

The screen is considered to be stationary. The moment of contact of the incident wave front with the shell is taken as the initial moment of time $t = 0$.

The wave front is parallel to the generatrix of the cylinder, and its propagated direction coincides with the z axis (Fig. 1). At the initial moment of time, the panel is considered to be stationary: $w(x, y, 0) = \acute{w}(x, y, \acute{0}) = 0$

Let us consider the effect of a potential pressure wave on a cylindrical shell.

Since a flat panel is considered ($\beta = 30°$, Fig. 1), we can assume that the pressure in the reflected wave $p_r(r, \theta, \tau)$ does not depend on the angle θ, and the nature of its change in time is the same as at $\theta = 0$. Then the specific dynamic transverse pressure acting on the surface of

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the shell has the form:

$$q=(p_1+p_2)+p_3-\rho_o h\frac{\partial^2 w}{\partial t^2}-\rho_o h\varepsilon\frac{\partial w}{\partial t}=\left(2e^{-\delta\tau}-\frac{1}{1-2\delta}e^{-\delta\tau}+\frac{1}{1-2\delta}e^{-\frac{1}{2}\delta\tau}\right)\rho_0 H(\dots)-\frac{\rho c^2}{R}\left[\acute{w}-\frac{w}{2}+\frac{1}{4}\int_o^\tau we^{-\frac{1}{2}(\tau-\tau_1)}d\tau_1\right]-\frac{\rho_0 hc^2}{R^2}\ddot{w}-\frac{2\rho_0 h\varepsilon c}{R}\acute{w} \quad (1.10)$$

where δ is a constant that determines the rate of pressure drop behind the wave front.

Methods for solving the equation of motion of a cylindrical panel placed in an infinite ideal fluid

To solve the problem, we apply the Bubnov-Galerkin method to the first equation (1.1) taking into account (1.4) and (1.5). Then we obtain the following equation for without dimensional deflection of the center of the panel $\xi=\frac{f}{h}$:

$$\gamma_0\ddot{\xi}+\frac{\gamma_1+\varepsilon_0}{k}\dot{\xi}+\frac{\pi^4k^2}{16\beta^4}\left(1+\frac{1}{\lambda^4}\right)\xi^3-\left[\frac{32}{3(1+\lambda^2)^2}+\frac{2}{3}\right]\frac{k\xi^2}{\beta^2}+\left[\frac{1}{(1+\lambda^2)^2}-\frac{\pi^2}{\beta^2}\left(\frac{\sigma_1^*}{\lambda^2}+\sigma_2^*\right)+\frac{\pi^4k^2}{12(1-\mu^2)\beta^4}\left(1+\frac{1}{\lambda^2}\right)^2-\frac{\gamma_1}{2k}\right]\xi+\frac{16}{k\pi^2}\sigma_2^*-\frac{16}{k^2\pi^2}p_0^*Q(\tau)+\frac{\gamma_1}{4k}e^{-0.5\tau}\int_0^\tau\xi(\tau_1)e^{0.5\tau_1}d\tau_1=0 \quad (1.11)$$

Here: $\gamma_0=\frac{\rho_0c^2}{E}$, $\gamma_1=\frac{\rho c^2}{E}$, $\varepsilon_0=\frac{\rho_0 h\varepsilon c}{E}$, $k=\frac{h}{R}$, $\sigma_1^*=\frac{\sigma_1}{E}$, $\sigma_2^*=\frac{\sigma_2}{E}$, $p^*=\frac{p_0}{E}$;

$$\text{a) } Q(\tau)=\left(2e^{-\delta\tau}-\frac{1}{1-2\delta}e^{-\delta\tau}+\frac{1}{1-2\delta}e^{-\frac{1}{2}\delta\tau}\right)\sin\left[\frac{\pi}{\beta}\arccos(1-\tau)\right] \quad \text{at } 0\le\tau\le 1-\cos\frac{\beta}{2}$$

$$\text{б) } Q(\tau)=\left(2e^{-\delta\tau}-\frac{1}{1-2\delta}e^{-\delta\tau}+\frac{1}{1-2\delta}e^{-\frac{1}{2}\delta\tau}\right) \quad \text{at } \tau\ge 1-\cos\frac{\beta}{2} \quad (1.12)$$

Initial conditions: $\xi(0)=0$, $\dot{\xi}(0)=0$ (1.13)

Nonlinear differential equations (1.11) are solved numerically using the Maple-17 program [8].

Initial data: shell material duralumin.

$\lambda=1;\frac{1}{2};\frac{1}{3};\frac{1}{5};\frac{1}{7};\frac{1}{9}$; $R=0.125$м; $h=0.002$; $\mu=0.3$;, $k=0.008$, $\beta=\frac{\pi}{2};\frac{\pi}{3};\frac{\pi}{6};\frac{\pi}{8}$; $\gamma_0=78.2*10^{-2}$; $\gamma_1=3.01*10^{-2}$; $\varepsilon_0=1$; $p_0^*=\frac{p_0}{E}=0.01*10^{-3}$; $\varepsilon=0.35\frac{1}{c}$; $\rho_{dra}=2800\frac{кг}{м^3}$; $\rho_{вод}=102\frac{кг}{м^3}$; $c=1500\frac{м}{c}$;

$$E=7300\frac{Н}{м^2};\ P_0=10атм;$$

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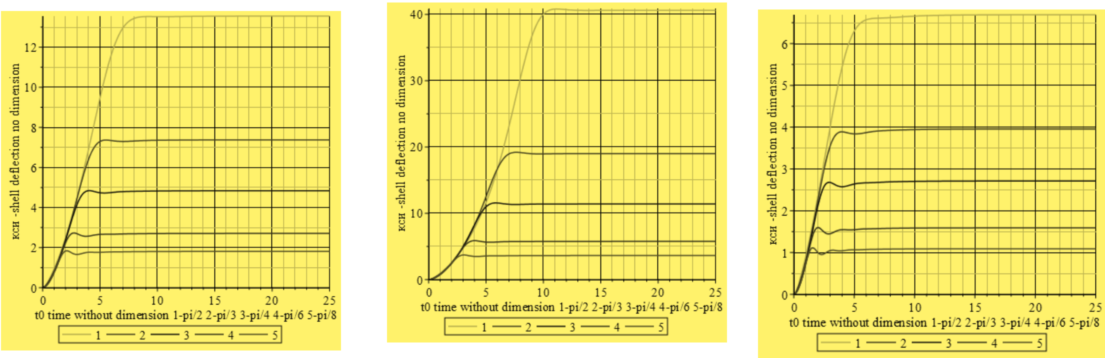
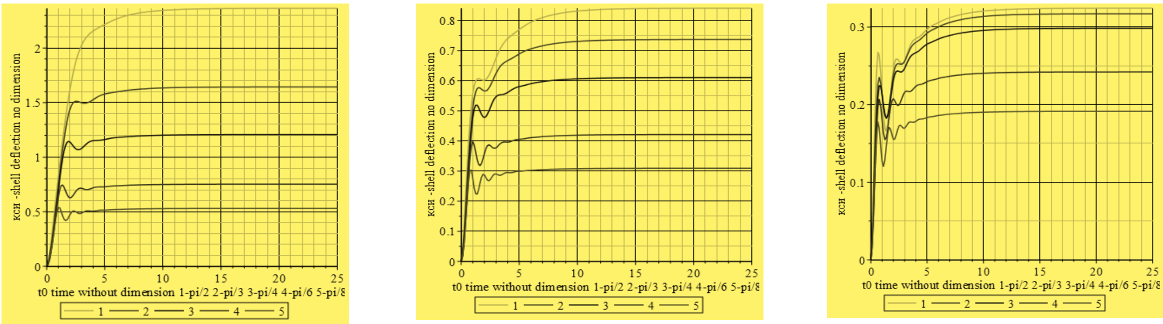
ANALYSIS OF RESULTS.



Lambda=1 Lambda=1/2 Lambda=1/3

Figure 2: Change ξ(τ)- the amplitude of the deflection of the panel of the cylindrical shell from time to time at different angle- β and coefficient λ



Lambda=1/5 Lambda=1/7 Lambda=1/9

Figure 3: Change ξ(τ)- the amplitude of the deflection of the panel of the cylindrical shell from time to time at different angle- β and coefficient λ.

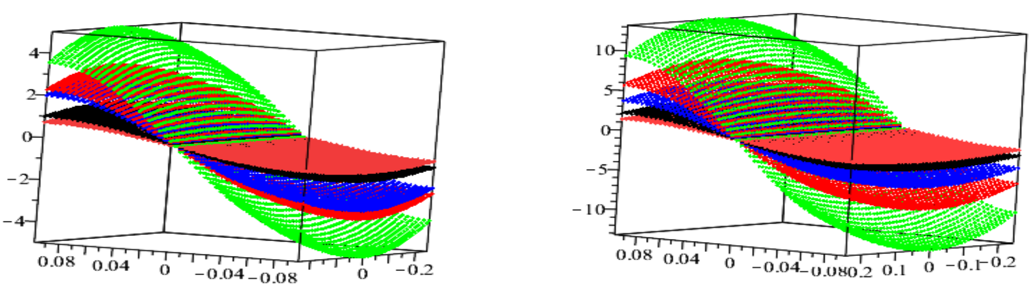


Figure 4: Change in the normal displacement of the middle surface of the panel of the cylindrical shell - $w(x,y,t)$ from the coordinate - x, y at different coefficients λ

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а) б)

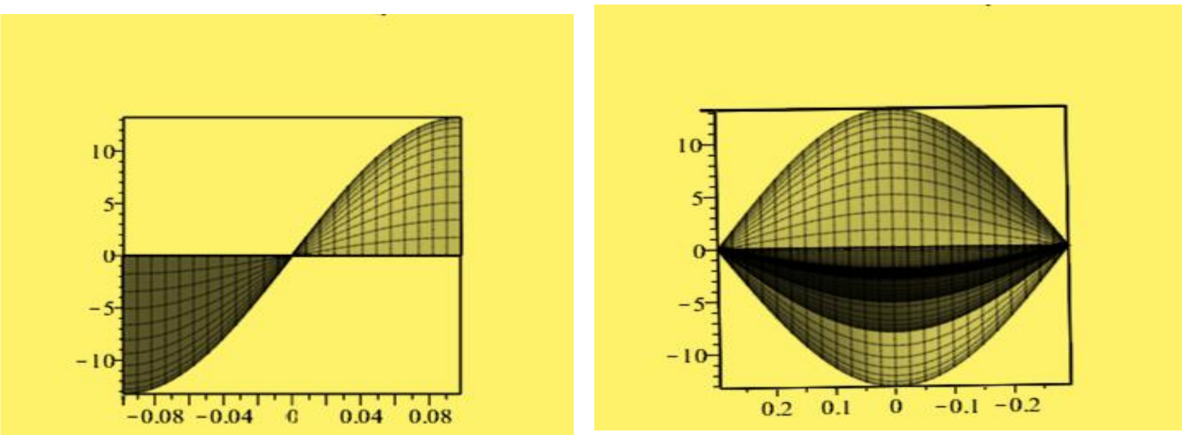

Figure: 5 a,b. Change in the normal displacement of the middle surface of the cylindrical shell panel - $w(x, y, t)$ from the coordinates - x (5a-Figure) and y (5a-Figure) at different coefficients λ.

The graphs in Figures 2 and 3 show that with an increase in the coefficient λ, the amplitude of the deflection of the panel of the cylindrical shell increases. The same character of the change in the deflection is observed with an increase in the angle-β.

Under the action of the pressure wave, the amplitude of the deflection of the panel of the cylindrical shell increases exponentially to the maximum values. Once the pressure waves cover the surface of the panel, the deflection values become constant.

The change in the normal displacement of the middle surface of the panel of the cylindrical shell - $w(x, y, t)$ from the coordinates - x, y at different coefficients λ is shown in the 4.5-figure. The oscillations of the panel in an ideal fluid are close to aperiodic, which is due to its large damping.

Static formulation of the problem

Let the cylindrical panel of the shell be statically loaded with uniform pressure - $q_0^* = \frac{q_0}{E}$ acting from the convex side. That is, the load is distributed over the entire surfaces and applied to the panel statically. Then from (1.11) we obtain the following equation with respect to without dimensional deflection of the panel - ξ:

$$\frac{\pi^4 k^2}{16\beta^4}\left(1+\frac{1}{\lambda^4}\right)\xi^3 - \left[\frac{32}{3(1+\lambda^2)^2}+\frac{2}{3}\right]\frac{k\xi^2}{\beta^2} + \left[\frac{1}{(1+\lambda^2)^2} - \frac{\pi^2}{\beta^2}\left(\frac{\sigma_1^*}{\lambda^2}+\sigma_2^*\right) + \frac{\pi^4 k^2}{12(1-\mu^2)\beta^4}\left(1+\frac{1}{\lambda^2}\right)^2 - \frac{\gamma_1}{2k}\right]\xi + \frac{16}{k\pi^2}\sigma_2^* - \frac{16}{k^2\pi^2}q_0^* = 0$$

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$$\frac{16}{k^2\pi^2}q_0^* = \frac{\pi^4k^2}{16\beta^4}\left(1+\frac{1}{\lambda^4}\right)\xi^3 - \left[\frac{32}{3(1+\lambda^2)^2}+\frac{2}{3}\right]\frac{k\xi^2}{\beta^2} +$$

$$\left[\frac{1}{(1+\lambda^2)^2} - \frac{\pi^2}{\beta^2}\left(\frac{\sigma_1^*}{\lambda^2}+\sigma_2^*\right) + \frac{\pi^4k^2}{12(1-\mu^2)\beta^4}\left(1+\frac{1}{\lambda^2}\right)^2 - \frac{\gamma_1}{2k}\right]\xi + \frac{16}{k\pi^2}\sigma_2^* \qquad (1.14)$$

Obtained numerical values for q_0^* at various coefficients a given in Table-1. $\beta = \frac{\pi}{6}$

Table 1: Change in pressure value depending on the coefficient λ.

λ	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{9}$
q_0^*	0.554	0.07	0.014	0.002	0.0008

From the table it can be observed that with a decrease in the coefficient λ, the pressure values - q_0^* decrease.

CONCLUSIONS

1. A mathematical model has been compiled for the equation of motion of a cylindrical panel placed in a boundless ideal fluid.
2. Nonlinear differential equations of motion of a cylindrical panel placed in an infinite ideal fluid are solved numerically using the Maple-17 program.
3. The results of the change $\xi(\tau)$- - of the amplitude of the deflection of the panel of the cylindrical shell from time to time at different angle - β and coefficient λ are obtained.
4. 4. The results of the change in the normal displacement of the middle surface of the cylindrical shell panel - $w(x, y, t)$ from the coordinate - x at different coefficients λ are obtained.
5. It can be seen from the graphs that the oscillations of the panel in an ideal liquid are close to aperiodic. This is due to its great vibration damping.

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