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SUBDIVISION NUMBER OF SPLIT POINT SET DOMINATION OF A GRAPH

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Abstract

Let G= (V,E) be a connected, nontrivial, simple, finite graph. In this paper a new parameter called subdivision split point set domination is introduced and is defined by a set D of vertices in a graph G is a subdivision point set domination, if (i) The graph obtained from a graph G by subdividing each edge of G exactly once (ii) For every set S⊆ V-D such that v∈ D, that is <S ∪ {v}> is connected. (iii) The induced sub graph <V-D> is disconnected. The minimum cardinality of subdivision split point set dominating set is denoted by γ_{sp} (S (G)). Besides some bounds, exact values of γ_{sp}(S (G)) are determined. Some theorems based on split point Set Domination are also discussed

Keywords: Domination Number, Split domination number, Point set domination number, Subdivision number of split point set domination number

1. INTRODUCTION

Let G=(V,E) be a simple, undirected, finite nontrivial graph with vertex set V and edge set E. The maximum, minimum degree among the vertices of G is denoted by δ(G),Δ(G) respectively. If deg v=0 then v is called an isolated vertex of G. If deg v=1 then v is called a pendant-vertex of G. And K_n,C_n,P_n, W_n and $K_{1,n-1}$ denote the complete graph, the cycle, the Path, the Star and the Wheel on n vertices respectively. The floor function is the inverse function of a ceiling function. It gives the largest nearest integer of specific value.

A non-empty set D ⊆V of vertices in a graph G is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ(G) of G is the minimum cardinality of a dominating set. A dominating set D of a graph G=(V,E) is a split set dominating set if there exists a non-empty set R⊆ S such that <R∪T> is connected for every set T⊆V-D and the induced subgraph <V-D> is not connected. The split domination number γ_s (G) of a graph G is the minimum cardinality of a split dominating set. A set D ⊆ V in a graph G = (V, E) is called a point-set dominating set of G if for every non-empty subset S ⊆ V - D there exists a vertex v ∈ D such that the induced subgraph <S ∪ {v} > is connected and the point-set domination number of a graph G is the minimum cardinality of the Point-set domination number of a graph G and is denoted by γ_p(G).A subdivision of an edge e = uv of a graph G is the replacement of the edge e by a path (u, v, w). The graph obtained from a graph G by subdividing graph of G and is denoted by S (G).

The purpose of this paper is to obtain some bounds for γ_{sp}(S(G)) and find its exact values for many classes of graphs including trees. Some theorems based on split point Set Domination are also discussed.

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Henceforth, by a graph G we mean a connected graph, and V denotes its vertex set and $|V|$=n.

Definition: 1.1

A set D of vertices in a graph G is a subdivision split point set domination, if (i)The graph obtained from a graph G by subdividing each edge of G exactly once (ii) For every set S⊆ V-D such that v∈ D, that is <S ∪ {v}> is connected. (iii)The induced subgraph <V-D> is disconnected.

The minimum carnality of subdivision point set dominating set is denoted by γ_{sp} (S(G)) and subdivision point set dominating set is denoted by γ_{sp}-set.

Example: 1.2

Consider the following graph G:

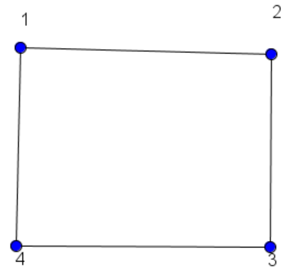

Figure (1)

Here the dominating set D = {1, 3}. Therefore, the domination number γ(G) = 2. And D is also point set domination. Therefore, γ_p(G) = 2. When we subdivide each edge of a graph G, it produces a new graph H as follows:

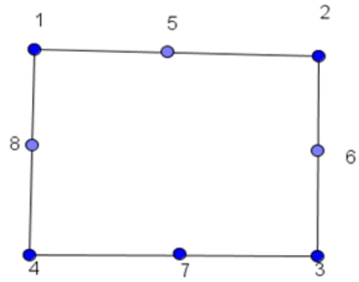

Figure (2)

Now the dominating set D = {1, 2, 3, 4}. Therefore the domination number

γ(G) = 2. But D is not a point set dominating set, since for S= {5,7}⊆V-D there does not exist v∈D such that <S∪{v}> is connected. Therefore the subdivision number of split point set dominating set D = {1, 2, 3, 4, 5, 6}. Clearly γ_p (S(G)) = 6.

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2. CHARACTERIZATION OF SUBDIVISION OF SPLIT POINT SET DOMINATING SET

Observation: 2.1

(i) For any graph G, $\gamma(G) \leq \gamma_p(G) \leq \gamma_{sp}(G) \leq \gamma_{sp}(S(G))$.

(ii) For any graph G,γ_{sp} = n-Δ where n is the number of vertices in S(G) and Δ is the maximum degree of S(G).

3. BOUNDS OF SUBDIVISION NUMBER OF SPLIT POINT SET DOMINATION:

Theorem: 3.1

Let G be a complete graph K_n with n vertices then γ_{sp} (S(G)) = $\lfloor 3n/4 \rfloor$ where n ≥ 3.

Proof:

Let G be a graph K_n on n-vertices. Then the split point set dominating set is

D = V – {v_i,v_j}, where v_i and v_j are non-adjacent vertices. Also <V-D> is disconnected.

Thus γ_{sp} (S(G)) = n-2 for all n≥ 3.

Remark: 3.2

In the above theorem, γ_{sp} (S(G)) = 1 where n = 2.

Theorem: 3.3

Let G be a path P_n on n-vertices namely $v_1,v_2,\ldots,v_n$, then γ_{sp} (S(G)) = n-2 where n ≥ 3.

Proof:

Let G be a graph P_n on n-vertices namely $v_1,v_2,\ldots,v_n$. Then, the split point – set dominating set D= V – {v_i,v_j}, where v_i and v_j are non-adjacent vertices. Also <V-D> is disconnected. Thus γ_{sp} (S(G)) = n-2 for all n≥ 3.

Theorem: 3.4

Let G be a Cycle C_n on n-vertices namely $v_1,v_2,\ldots,v_n$, then γ_{sp}(S(G)) = n-2 where n ≥ 6.

Proof:

Let G be a cycle C_n on n-vertices namely $v_1,v_2,\ldots,v_n$. Then, the split point set dominating set D= V – {v_i,v_j}, where v_i and v_j are non-adjacent vertices. Also

<V-D> is disconnected.

Thus γ_{sp} (S(G)) = n-2 for all n≥ 6.

Theorem: 3.5

For a star graph, γ_{sp} (S(G)) = $\lfloor n/2 \rfloor + 1$ where n ≥ 5.

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Proof:

Let u be a vertex of degree n-1 and $v_1,v_2,\ldots,v_n$ be the pendant vertices of G. Subdivide each edge and an vertex say $u_1,u_2,\ldots,u_n$ each of degree 2. Then the split point set dominating set D = $\{u,v_1,v_2,\ldots,v_n\}$, which is also the point set dominating set.Also <V-D> is disconnected.

Hence γ_{sp} (S(G)) = $\lfloor n/2 \rfloor + 1$ where n≥ 5.

Theorem: 3.6

Let G be a bistar graph with order n vertices, then γ_{sp} (S(G)) = $\lfloor 3n/2 \rfloor - 1$ where n ≥ 3.

Proof:

Let G be a bistar graph that joining two star graph by a path P_2.

Let $s_1, s_2,\ldots, s_n$ and $u_1,u_2,\ldots,u_n$ be the pendent vertices of G.

Let u_i be a new vertex by subdivide the path P_2. Subdivide the incident edges of v and u namely $s_1, s_2,\ldots, s_n$ and $t_1,t_2,\ldots,t_n$ respectively.

The split point set domination set D = V- $\{u_i, s_1, s_2,\ldots, s_n\}$ (or) V – $\{u_i, t_1,t_2,\ldots,t_n\}$.Also <V-D> is disconnected.

Therefore γ_{sp} (S(G)) = $\lfloor 3n/2 \rfloor - 1$.

Theorem: 3.7

For a comb graph G, γ_{sp} (S(G)) = 2n – 4 if n≥ 6.

Proof:

Let G be a comb graph, having a path vertices namely $v_1,v_2,\ldots,v_n$ and pendant vertices $u_1,u_2,\ldots,u_n$. Then in G, point set domination set of D = V- $\{u_i ,v_j, v_k\}$.

Then in S (G), selecting u_i in V-D, then u_i is adjacent to v_j and u_k in D and also v_i is adjacent to v_j and u_k. .Also <V-D> is disconnected.

Therefore, γ_{sp} (S(G)) =2n – 4 if n≥ 6.

Remark: 3.8

For a comb graph G, γ_{sp} (S(G)) =2n – 3 if n= 4.

4. MORE THEOREMS BASED ON SUBDIVISION NUMBER OF SPLIT POINT SET DOMINATION

Theorem: 4.1

Let u be a cut vertex of a graph G. Then u is in every γ_{sp} -set D of G if either

(i) G-u has at least three components or

(ii) G-u has exactly two components G_1 and G_2 and neither $<G_1\cup\{u\}>$ nor $<G_2\cup\{u\}>$ is a path.

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Proof:

Let D be a γ_{sp}-set. Suppose u $\notin$D. Condider vertices v and w in different components of G-u that are not in D. Since u is on every v-w path in G and u$\notin$D, there is no set S $\subset$ D such that the subgraph <S$\cup${v,w}> is connected. Which is a contradiction. Hence there are two cases arises

Case 1

G-u has at least three components say G_1,G_2 and G_3. And since all vertices in all except possibly one component of G-u belong to D say G_1 belong to D. Let u_1 and u_2 are in D. Then the set D_1=(D-{u_1,u_2})$\cup$(v) is also a γ_{sp}-set of G with $|D_1|=|D|$-1, which is a contradiction to D is a γ_{sp}-set. This proves u$\in$D.

Case 2

G-u has exactly two components say G_1 and G_2 such that neither H_1=<$G_1\cup${u}> nor H_2=<$G_2\cup${u}> is a path.

If u$\notin$D then all vertices in either G_1 or G_2 say G_1 are in D. Let T be a spanning tree of H_1 such that T has at least two end vertices u_1 and u_2 other than u. Clearly the set

D_1=(D-{u_1,u_2})$\cup$(v) is γ_{sp}-set.with $|D_1|=|D|$-1, which is a contradiction to D is a γ_{sp}-set. This proves u$\in$D.

Theorem: 4.2

For any connected graph G, γ_{sp} (S(G)) $\geq$ n.

Proof:

Let D be any split point set dominating set of S(G), let D_1 = D ∩ V(G) and

D_2 = D ∩ (V(S(G))\ V(G)). Since D is a split point set dominating set of S(G),

$D_1 \neq \emptyset$ and $D_2 \neq \emptyset$ and each element of D_2 dominates at most one vertex of V(G). Hence it follows $|D_2| \geq n - |D_1|$. Then $|D| = |D_1|+|D_2| \geq n$ so that γ_{sp} (S(G)) $\geq$ n.

Theorem: 4.3

Let G be a tree graph with n vertices then γ_{sp} (S(G)) =n-s_1-1

Proof:

Let G be a tree graph with n vertices and let B_1 and B_2 be the set of maximum and minimum support vertices containing in G with $|B_1| = s_1$ and $|B_2| = s_2$ respectively . Now let u be the vertex which is adjacent to the support s_1 in G. Therefore the subdivision split point set D=V-B_1-{u}.Therfore γ_{sp} (S(G)) =n-s_1-1

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