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INVESTIGATION OF BENDING VIBRATIONS OF ROTATING SHAFTS WITH DISTRIBUTED INERTIAL CHARACTERISTICS

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Abstract

One of the main problems of ensuring elastic stability and long-term strength of the system under cyclically changing external influences of rotating shafts is investigated. **Method:** A review and comparative study of a number of analytical and numerical methods for calculating bending vibrations of rotating shafts is carried out. The construction of a direct analytical solution was carried out using the continuation method by parameter. **The main results**: This model allows us to take into account the distribution of elastic, inertial and eccentric properties of the system, and is also applicable for any operating frequency range. Free and forced shaft oscillations caused by the presence of eccentricity are investigated. **Practical significance:** The sensitivity of the installation to an additional external force factor of a given order is investigated. The frequency characteristics of the system under consideration are constructed and compared with the results of field tests. Methods of damping shaft vibrations using controlled (active) magnetic bearings are proposed.

Keywords: Unsteady Bending Vibrations, Shaft, Center Of Mass, Inertia Forces, Shaft Deformations, Elastic Shaft Forces, Distributed Characteristics, Strength, Disk Eccentricity, Analytical Methods, Numerical Methods, Experiment.

INTRODUCTION

Consider a circular shaft rotating with angular velocity ω. In the middle of the shaft there is an eccentrically mounted disk (Fig.1). Denote by *e* the eccentricity of the disk–the distance from its center of mass to the undeformed axis of the shaft. The mass of the shaft in comparison with the mass of the disk m will be neglected. When rotating, the inertia force $\vec{J}$ (centrifugal force) acts on the shaft from the disk side, under the action of which the shaft acquires a transverse deformation (Fig.2). Let r be the transverse deformation of the shaft at the place of the disk attachment. Then the centrifugal force modulus J=m (l+r) $\cdot\omega^2$. This force is balanced by the elastic force of the shaft, its modulus [1].

The stiffness coefficient c of the shaft is determined during the solution of the following problem of the resistance of materials: we have a beam of length 퇼, a force $\vec{F}$ is applied to the middle of the beam. Find the deflection of the beam at the point of application of this force (Fig.3).

Obviously, $R_A = R_B = \frac{F}{2}$. The differential equation of the curved axis of the beam has the form

$$E \cdot J \cdot y^{//} = M \,,$$

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where E is the modulus of elasticity; J is the moment of inertia of the shaft section, equal to $J = \frac{\pi R^4}{4}$, if the shaft is round.The bending moment in the cross section $M = R_A x = \frac{F}{2}x$, so [2]

$y^{//} = \frac{F}{2EJ}x.$

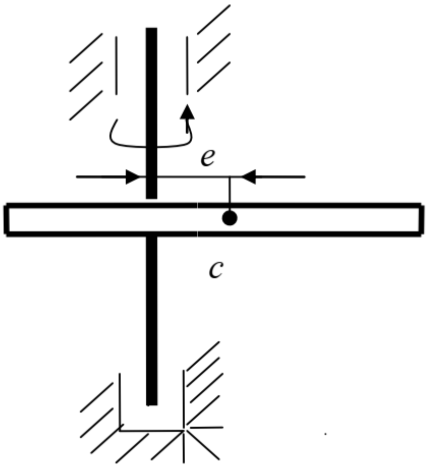

Fig 1

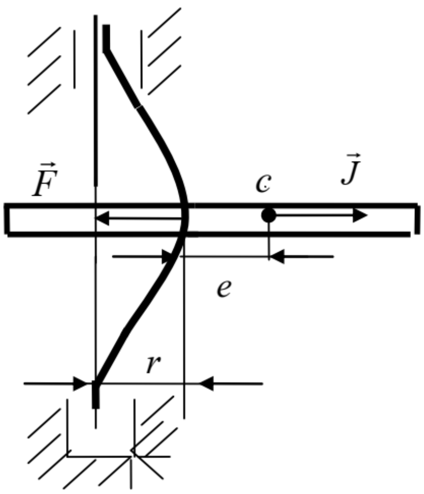

Fig 2

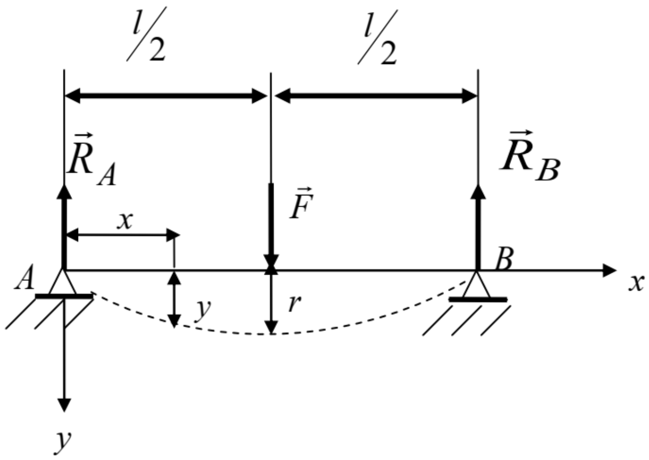

Fig 3

Integrating this expression for the first time:

$$y^{'} = \frac{F}{4\,EJ}x^2 + C.$$

The constant C_1 is sought from the condition that the section in the middle of the shaft does not rotate: $y^{'}/_{x=\frac{l}{2}} = 0.$

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Then $C_1 = -\frac{F}{4\,EJ} \cdot \frac{l^2}{4}$ and therefore,

$$y' = \frac{F}{4\,EJ}\left(x^2 - \frac{l^2}{4}\right)$$

Integrating the second time:

$$y = \frac{F}{4\,EJ}\left(\frac{x^3}{3} - \frac{l^2}{4}x\right) + C_2$$

There is no deflection at point A, hence$y/_{x=0} = 0$. [3] Therefore,$C_2 = 0$, we obtain the following equation of the elastic shaft line:

$$y = \frac{F}{4\,EJ}\left(\frac{x^3}{3} - \frac{l^2}{4}x\right)$$

To find the value of r, you need to substitute in (1) $x = \frac{l}{2}$.

Then we get $r = \frac{Fl^3}{48EJ}$. Hence, the shaft stiffness coefficient

$$c = \frac{F}{r} = 48 \cdot \frac{EJ}{l^3}$$

From the condition that the inertia force $\vec{J}$ (centrifugal force) is balanced by the elastic force $\vec{F}$ of the shaft, i.e., J=F, we obtain:

$m(e + r)\omega^2 = cr$ where $r = \frac{me\omega^2}{c - m\omega^2}$.

Denote $k = \sqrt{\frac{c}{m}}$ the frequency of natural oscillations of the disk on a non-rotating shaft, then

$$r = \frac{e}{\frac{k^2}{\omega^2} - 1}$$

If ω→k, then r→∞. The frequency at which the transverse deflections of the shaft increase indefinitely is called the critical rotation frequency [4]. In this problem, the critical rotation frequency coincides with the natural frequency of transverse vibrations of the disk on a non-rotating shaft: $\omega_{\text{кр}} = k$.

In general, the critical frequency does not coincide with its own. Let the disk be mounted asymmetrically relative to the supports, then with transverse vibrations it will rotate. At the same time, due to the influence of Coriolis inertia forces, a gyroscopic moment arises, and the critical frequency increases with respect to its own [5].

We investigate the case when the shaft section has different main moments of inertia, and hence different bending stiffness. Let's also assume that there is no initial eccentricity (e=0). Consider

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the perturbed position of the disk shown in Fig.4, the *x*, *y* coordinate axes are rigidly connected to the disk and selected parallel to the main axes of inertia of the shaft section (1→1 and 2→2). In this case, the coordinate system of the Ohu rotates with an angular velocity ω of the shaft 6.

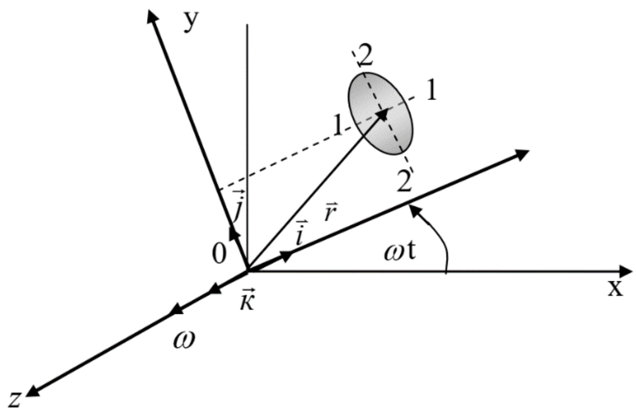

Fig 4

The equation of transverse vibrations of a disk in a rotating coordinate system has the form:

$$m\vec{w}_r = \vec{F} + \vec{J}_e + \vec{J}_c$$

Where w ⃗_r is the relative acceleration of the center of mass of the shaft; F ⃗ is the elastic force; J ⃗_e and J ⃗ _c are, respectively, the portable and Coriolis forces of inertia [7].

Let the center of mass of the shaft in the moving coordinate system be given by the radius vector R ⃗=xi ⃗+yj ⃗ (through the coordinates of the center of mass). Given that the vectors of the angular velocity of the shaft and the relative velocity of the center of mass of the shaft are determined by the ratios ω ⃗=ωk ⃗ and v ⃗ _r=x i ⃗+y j ⃗, we obtain expressions for the forces included in (2):

A ⃗=-c_1 xi ⃗-c_2 but ⃗,

Where c_1 and c_2 are the bending stiffness of the shaft in directions 1-1 and 2-2;

J ⃗_e=m·ω^2·R ⃗;

J ⃗_c=-2m·ω ⃗·v ⃗_r=-2m·ω·k ⃗·(x ̇ı ⃗+y ̇ȷ ⃗) =-2mω(x ̇ı ⃗+y ̇ȷ ⃗)

Since w ⃗_r=x ̈ı ⃗+y ̈ȷ ⃗, then writing (2) in projections on the axis of the movable coordinate system, we get

$$\begin{cases} m\ddot{x} = -c_1 x + m\omega^2 x + 2m\omega\dot{y}; \\ m\ddot{y} = -c_2 y + m\omega^2 y + 2m\omega\dot{x}; \end{cases}$$

Or by reducing by mass m,

$$\begin{cases} \ddot{x} + (x_1^2 - \omega^2)x - 2\omega\dot{y} = 0; \\ \ddot{y} + (x_2^2 - \omega^2)y - 2\omega\dot{x} = 0; \end{cases} \quad (3)$$

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$$\text{где } k_1 = \sqrt{\frac{c_1}{m}},\ k_2 = \sqrt{\frac{c_2}{m}}.$$

We are looking for the solution of system (3) in the form

$$x = A_1 \cdot e^{rt},\ y = A_2 \cdot e^{rt} \qquad (4)$$

Substituting (4) into (3), we obtain a system of two homogeneous equations with respect to A1 and A2

$$\begin{cases}(r^2 + k_1^2 - \omega^2)A_1 - 2\omega r A_2 = 0;\\ 2\omega r A_1 + (r^2 + k_2^2 - \omega^2)A_2 = 0;\end{cases}$$

To find a non-zero solution, it is necessary to equate the determinant of the system to zero:

$$\begin{vmatrix} r^2 + k_1^2 - \omega^2 - 2\omega r \\ 2\omega r (r^2 + k_1^2 - \omega^2) \end{vmatrix} = 0$$

or

$$r^4 + r^2(r_1^2 + k_2^2 - 2\omega^2) - (r_1^2 - \omega^2)(r_2^2 - \omega^2) = 0$$

Solving this biquadrate equation, we get two real roots:

$$r_{1,2}^2 = \frac{1}{2}\left[-(r_1^2 + k_2^2 - 2\omega^2) \pm \sqrt{(r_1^2 - \omega^2) + 8\omega^2(r_1^2 + \omega^2)}\right] \qquad (4)$$

If $r_{1,2}^2 < 0$, then the characteristic indicators in expressions (4) will be imaginary, which corresponds to a stable oscillation of the shaft near a stationary position. If r_1^2 or r_2^2 is greater than zero, then the characteristic indicators will be positive and the deviations of the shaft x and y will increase a periodically with time, which corresponds to the case of instability [8]. Thus, it follows from (5) that the condition corresponds to the critical condition of the shaft

$$\sqrt{(r_1^2 - \omega^2) + 8\omega^2(r_1^2 + \omega^2)} \geq k_1^2 + k_2^2 + 2\omega^2.$$

Squaring both parts of the last equality and bringing similar terms, we get

$$(\omega^2 - k_1^2)(\omega^2 - k_2^2) \leq 0$$

This inequality is satisfied if ω lies within

$$k_1 \leq \omega \leq k_2 \qquad (6)$$

This proves the instability of the shaft movement in a certain range of angular speeds of rotation.

In the case of a round shaft, its bending stiffness is the same in any direction $c_1 = c_2 = c$, hence k_$k_1 = k_2 = \sqrt{\frac{c}{m}}$ and the region (6) degenerates into a point

$$\omega = \omega_{cr} = \sqrt{\frac{c}{m}}.$$

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Thus, the round shaft loses the stability of the stationary position when the angular velocity reaches a critical value:

$$\omega_{cr} = \sqrt{\frac{c}{m}}$$

Suppose that the shaft axis (Fig.5) horizontal. When composing the equations of motion in this case, it is also necessary to take into account the force of weight, the projections of which are respectively equal to *-mq sinωt,-mq cosωt*, then we get the system

$$\begin{cases} \ddot{x} + (x_1^2 - \omega^2)x - 2\omega\dot{y} = 0; \\ \ddot{y} + (x_2^2 - \omega^2)y - 2\omega\dot{x} = 0; \end{cases} \qquad (7)$$

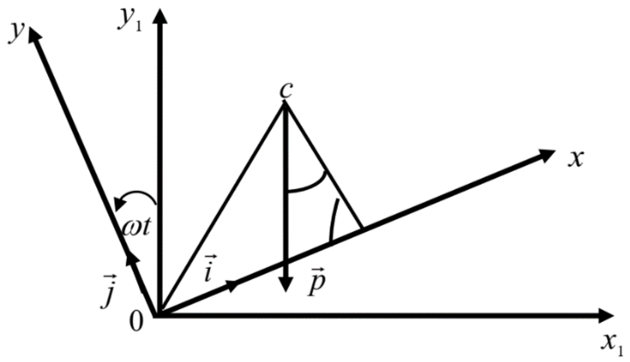

Fig 5

which differs from system (3) by having right-hand sides. The general integral of systems (7) consists of the sum of the solution of the homogeneous problem (3) and the partial integral of equations (7). Consider the partial integral of the system (7) to find out another possibility of a critical state. We are looking for purely forced fluctuations in the form of

$$x = A_1 \sin wt\, , y = \cos wt. \qquad (8)$$

Substituting (8) into equations (7), we arrive at an inhomogeneous system of algebraic equations for amplitudes

$$\begin{cases} (x_1^2 - 2\omega^2)A_1 + 2\omega^2 A_2 = -q \\ 2\omega^2 A_1 + (x_1^2 - 2\omega^2)A_2 = -q \end{cases}$$

Having solved the system, we get

$$A_1 = -q\frac{k_2^2 - 4\omega^2}{k_1^2 k_2^2 - 2\omega^2(k_1^2 + k_2^2)}$$

$$A_2 = -q\frac{k_1^2 - 4\omega^2}{k_1^2 k_2^2 - 2\omega^2(k_1^2 + k_2^2)}$$

A critical state occurs if the denominator of the expressions obtained tends to zero; this is possible under the condition

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$$\omega^2 = \frac{k_1^2 k_2^2}{2(x_1^2+k_2^2)}, \quad (9)$$

Assuming in (9) $k_1 \approx k_2$ we get.

$$\omega \approx \frac{k_1}{2} \approx \frac{k_2}{2}, \quad (10)$$

Thus, for a round shaft, a critical state is possible at an angular velocity equal to half of the usual critical velocity.

The experiments used a device similar to the one shown in photo 3, but manufactured in Austria (see photo 5). The total weight is 24 kg. The fluctuating mass is 20 kg. Debalance weight 0.2 kg. The radius of rotation of the center of mass of the imbalance is 0.02 m. The gear ratio of the gearbox is 1.6666.

The device was installed horizontally on the stand. The working link (asterisk) was fixed with a short rod, the end of which was attached to the stand. A strain gauge 3 was attached to the rod, connected to the measuring equipment [9].

Depending on the applied moment, the sensor showed the amount of electrical voltage proportional to the deformation of the short rod.

At the same time, the following initial data were obtained. The control moment of force at the end of the rod 2 is M = 42.75 Nm. The reference voltage of sensor 3 is U = 3.8 volts. The conversion factor, voltage/moment, is k_M= 0.088 v/Nm. The moment M corresponds to the work $E=M_\varphi=42.75\cdot2\pi\cdot10.71/360=7.991$J. The conversion factor, voltage/operation, is equal to k_E = 0.475 v/J. The oscillation power of the asterisk is equal to

$$P_{expe}=U\cdot f/k_E \text{ ,} \quad (11)$$

where U is the voltage at the sensor output (v), f is the frequency of torsional vibrations of the sprocket (hz) [10].

Measurements were made for five frequency values. The results are summarized in Table 1, graphs in Fig.6.

Table 1

№	U	I	P_{vx}	f	M	M_E	P	P_{expe}
1	4,9	6	29,4	15,38	7,47	1,31	36	4,0
2	8	8	64	22,16	15,5	93,06	108	386
3	8,5	10	85	25,19	20,0	93,06	159	438
4	9,7	12	116,4	25,79	21,0	94,19	170	454
5	10	14	140	26,04	21,4	198	175	965

U – power supply voltage, V, I – amperage, A, Rix – power consumption, f – vibration frequency of the switch, hz, Mε - moment measured at the output, Nm. P – vibration power at the output of the device, theory. Calculation, W. REKSP – the power measured on the asterisk, W.

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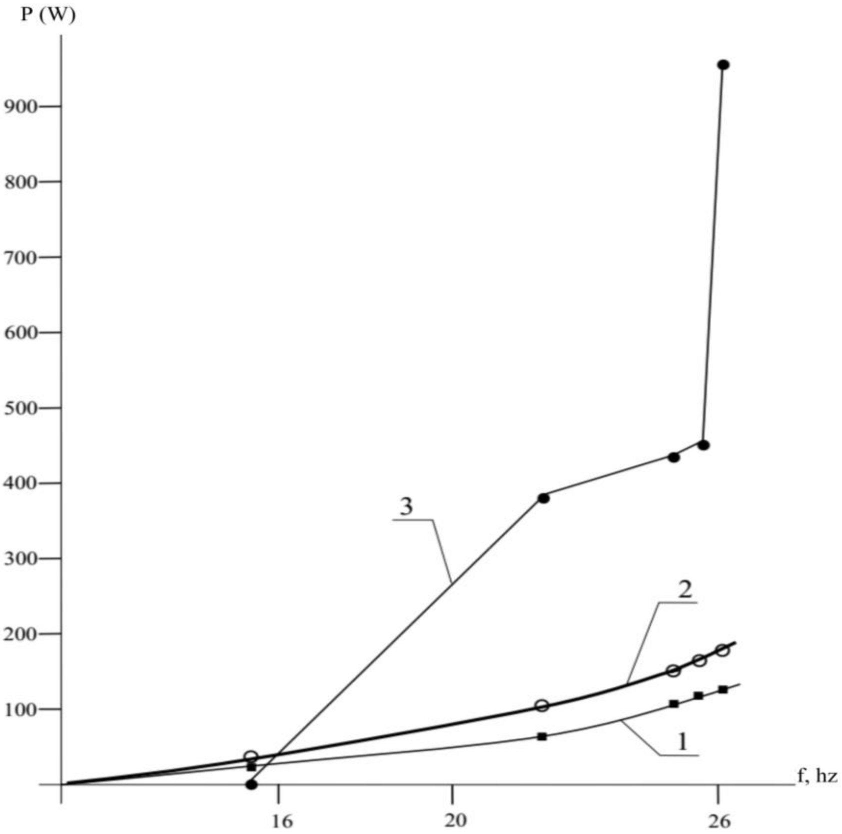

Fig 6: Graph to Table 2, dependence of power P on the frequency of oscillations f: 1- P_{vx}.; 2-P, theoretical calculation 3- P_{expe}.

Table 2

№	U	I	P_{vx}	f	M	M_E	P	P_{expe}
1	3,15	5	15,8	6,6	1,38	6,8	11,4	11,5
2	6,6	8	52	17,86	10,1	35,9	56,5	165
3	16,7	10	167	57,47	104,3	345	1883	5108
4	18	12	216	59,52	111,9	389,7	2092	5965
5	18,6	14	260	60,24	114,6	382,9	2169	5932
6	18,7	16	299	58,82	109,3	369,3	2019	5587
7	19,1	18	344	58,14	106,8	410	1950	6131
8	19,3	20	386	58,14	106,6	406,8	1950	6080
9	19,6	22	431	57,47	104,3	416,8	1883	6160
10	17,3	24	415	47,17	70,3	372,7	1041	4522
11	18,2	26	473	64,94	133,2	267,7	2717	4471
12	27	19	513	92,6	270,8	267,7	7878	6375
13	28	18,25	511	96,15	292	227	8820	5614

U – power supply voltage, V, I – amperage, A, P_{ix} – power consumption, f – vibration frequency of the switch, hz, M_ε - moment measured at the output, Nm. P – vibration power at

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the output of the device, theory. Calculation, W. P_{expe} – the power measured on the asterisk, W.

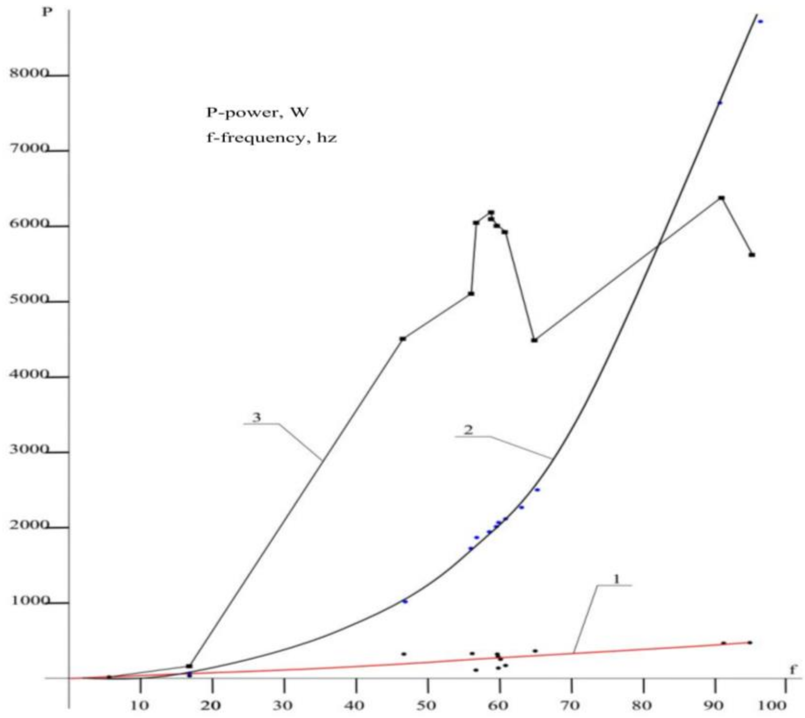

Fig 7: Graph to Table 3, the dependence of power P on the frequency of oscillations f: 1- P_{vx}.; 2-P, theoretical calculation 3- P_{expe}.

CONCLUSIONS

The use of shaft sensors allows you to solve a number of tasks automatically, while in the absence of shaft sensors, these tasks require special measurements and at the same time do not provide the necessary accuracy of the results of defect predictions or the required value of correction parameters (alignment, gaps, thickness of pads, etc.).

To fully assess the reliability of the system, it is necessary to consider the variable shaft stresses in the field of static and transient centrifugal and temperature fields and the corresponding stresses. This approach will make it possible to control machines of any class that have structural, operational, technological and any other accidental shaft defects, the appearance or development of which will immediately affect the change in shaft trends.

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