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CONTROLLING OF CHAOTIC RUCKLIDGE SYSTEM WITH DYNAMICAL BEHAVIORS

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Abstract

In this article, we study the basic concept of the Rucklidge system, a generalized Lorenz-like system. First, we have tried to explain which systems are generalized Lorenz-like systems. Then, we have concentrated on some dynamical characteristics, including nonlinearity, stability, and instability; sensitivity to numerical inaccuracy, sensitivity to initial conditions; vector field analysis; time series analysis, strange attractor, and bifurcation of the Rucklidge System. Mainly, controlling the chaos of this system is our main focus. Moreover, we have also displayed the two-scroll chaotic attractor of this system. Finally, we have controlled the Rucklidge system through trajectories for different parameter values. We have found a boundary of parameter values where the system has minimum oscillation, is non-periodic, or may be chaotic.

Keywords: Chaos; Strange Attractor; Sensitivity; Bifurcation; Time Series; Chaotification, Three dimensional, Lorenz-Like System

1. INTRODUCTION

Chaos is a fascinating research topic in mathematics, science, and engineering. It is a rigorous field in recent studies all over the world. Recently, chaos has been very useful and has great potential in many technological disciplines like biomedical sciences, computer engineering and information technology, encryption, communications, fluid dynamics, and so on. [Chen &Dong, 1998; Chen, 1999; Lu¨et al., 2002e; Chen & Lu¨, 2003, and references therein].

Therefore, academic research on chaotic dynamics has evolved from the traditional trend of analyzing and understanding chaos [Hao, 1984] to the new direction of controlling and utilizing it [Ott et al., 1990; Chen, 1993; Chen & Dong, 1998; Chen, 1999; Wang & Chen, 2000; Lu¨ et al., 2002e; Chen & Lu, 2003]. In extension, chaos control can be divided into two types:

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(i) to suppress the chaotic dynamical behavior when it is harmful [Ott et al., 1990; Chen, 1993], and (ii) to create or enhance chaos when it is desirable known as chaotification or anticontrol of chaos [Chen & Lai, 1998; Chen & Dong, 1998; Wang & Chen, 1999; B´enard problem [Festa et al., 2002]. Recently, there has been increasing attraction to using chaotic dynamics in engineering applications. Some of this emphasis has been on successfully generating chaos using fundamental physical systems in electronic circuits. [Tang et al., 2001; Wang & Chen, 2000] and switching piecewise-linear controllers [Lu¨ et al., 2002f; Lu¨ et al., 2003]. Chaotification is a fascinating theoretical topic that is quite challenging technically, and it involves a few complicated but well-decorated dynamical behaviors, which usually include bifurcations, fractals, etc. In the past few years, a lot of work has been invested in achieving this goal through computer simulations and the creation of rigorous mathematical theories. In the endeavor of chaotification, purposefully creating discrete chaos has gained great success [Chen & Lai, 1998; Wang & Chen, 1999]. However, chaos is also rapidly growing in continuous time systems simultaneously. For example, the Lorenz system [Lorenz 1963] may be evolved from a nonchaotic state to a chaotic one using a straightforward liner partial state-feedback controller.

The main objective of this article is to represent and further study simple, interesting chaotic behaviors and a three-dimensional Rucklidge autonomous chaotic system, which can display two 1-scroll attractors simultaneously or two complex 2-scroll chaotic attractors simultaneously. This new chaotic system is introduced Rucklidge system. This generalized Lorenz-like system explains how to find this new chaotic system and investigates the dynamical behaviors of this chaotic system by employing some tools used in [Ueta & Chen, 2000; Lu¨ et al., 2002b]. First, the compound structure of chaotic attractors [Elwakil & Kennedy, 2001; Lu¨ et al., 2002c, 2002g] is analyzed for the two 2-scroll attractors of this new system. Then explore the relationship and connecting function for the 2-scroll attractor.

In fluid dynamics, two-dimensional convection in a horizontal layer of Boussinesq fluid with lateral constraints was considered by Rucklidge [Yu & Xia, 2001]. However, when the convection happens in a fluid layer rotating uniformly about a vertical axis and in the limit of high thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modeled by a new three of ordinary differential equations, which produces chaotic solutions like the Lorenz model [Zhang, 1991]. The 3-D model describes the double convection Rucklidge chaotic system.

$$\begin{cases} \dot{x} = -ax + by - yz \\ \dot{y} = x \\ \dot{z} = -z + y^2 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where a, b are unfolding real parameters of the system and where $x, y, z \in \boldsymbol{R}^3$ are the state variables. The system described by equation (1) was derived in the work **[**Zhong & Tang, 2002**]** from partial differential equations with the help of Galerkin method. This system displays a 2-scroll chaotic attractor when $a = 2$, $b = 6.7$ [Sparrow, 1982**].**

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2. METHODOLOGY

There are some rigorous tools to investigate the chaotic dynamics of different chaotic models. In our research, we endeavour to represent the dynamical behaviours three-dimensional Rucklidge system in the several chaotic properties in dynamical systems. For this purpose, we have used mathematical software like MATHEMATICA and MATLAB to analyze this model numerically and graphically.

3. DYNAMICAL PROPERTIES OF RUCKLIDGE SYSTEM

3.1.1 Nonlinearity: The Rucklidge system has three first order ordinary differential equation with two nonlinear terms yz and y^2.

3.1.2 Symmetry: The Lorenz System is invariant under the symmetry

$(x,\ y,\ z)\to(-x,\ -y,\ z)$. Since $-\dot{x}=-a(-x)+b(-y)-(-y)z \Rightarrow \dot{x}=-ax+by-yz$

$$-\dot{y}=(-x)\Rightarrow \dot{y}=x \text{ and } \dot{z}=-z+(-y)^2=-z+y^2$$

3.1.3 Equilibrium points of Rucklidge system

If $X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $F=\begin{pmatrix} -ax+by-yz \\ x \\ -z+y^2 \end{pmatrix}$, in vector form the system becomes $\dot{X}=F$. The equilibrium points are given by $F=0$, solving these system reveals that the equilibrium points $C_0=(0,0,0), C^{\pm}=(0,\pm\sqrt{b},b)$. Here, $C_0=(0,0,0)$ is an equilibrium point for all values of the parameters. For $b\geq 0$ the pair of fixed points are $C^{\pm}=\left(0,\ \pm\sqrt{b},\ b\right)$. In order to study the stability of $C^{\pm}$ it is sufficient only to study the stability of C^{+} due to the symmetry $(x,\ y,\ z)\to(-x,\ -y,\ z)$ presented by system (1). In general, to decide the stability of a non-hyperbolic equilibrium point of a system in $\boldsymbol{R}^3$ is very difficult even for quadratic systems. As far as we know, the stabilities of C_0 and $C^{\pm}$ were analyzed in [JINHU LU, *2001*]. For different values of parameters, there are different equilibrium points. In this system, a variety of equilibrium points depend only on the parameter value of *b*.

3.1.4 Eigenvalues

We calculate the real eigenvalues of system (1) for the parameters $a=2,\ b=6.7$ are $\lambda_1=1.7749,\ \lambda_2=-3.7749,\ \lambda_3=-1$. Here $\lambda_1,\ \lambda_2,\ \lambda_3\in \boldsymbol{R}^3$ such that the absolute values eigenvalues satisfy this $-\lambda_2>\lambda_1>-\lambda_3>0$. Thus, by definition, system (1) is Generalized Lorenz-like system.

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3.1.5 Dissipative

The Rucklidge chaotic system is a dissipative system that tends to dissipate energy over time. It is seen in the third question in the Rucklidge model, where the derivative of the third equation has a negative sign. This means that the third variable of the negative sign, interpreted as the system's energy tends to decrease with time. Furthermore, the dissipative nature of the system can also be seen in the dynamic behaviors of this model. The system exhibits a high sensitivity to initial conditions, which means small changes in the initial configuration of the system can generate vastly different trajectories of the system. This scenario also expresses this model's dissipative behaviors, which express chaotic behaviors due to the energy-dissipating nature.

For system (1), one has $$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = a - 1.$$

Hence, with $a < 1$, system (1) is dissipative, with an exponential contraction rate:

$$\frac{dV}{dt} = e^{(a-1)}.$$

That is, a volume element V_0 is contracted by the flow into a volume element $V_0 e^{(a-1)t}$ in time t. This means that each volume containing the system trajectory shrinks to zero as $t \to \infty$ at an exponential rate $a-1$. Therefore, all system orbits are ultimately confined into a specific subset of zero volume, and this asymptotic motion settles onto an attractor [DAVIES, *1988*]**.**

4. DYNAMICAL BEHAVIORS OF RUCKLIDGE SYSTEM

4.1. Sensitivity to Numerical Inaccuracies

The numerical solution of the Rucklidge model may cause trouble because the model is chaotic. The basic property of such models is the extreme sensitivity of the solution to numerical inaccuracies and to the initial conditions. To demonstrate the effect of numerical inaccuracies, we first calculate one solution of the system with $a = 2.6,\ b = 7.7$ and ask for the value of the solution at $t = 100$ with initial condition $\gamma = (x_0, y_0, z_0) = (1.5, 0.5, 0.8)$, we have a solution $s1 = (\text{-0.162194, -4.5546, 10.2347})$.

Then we calculate the solution with high-precision numbers by asking that the calculations are done to 15 decimals with same parameter values and initial condition at t=100, we have wholly different solution which is

$s2 = (\text{-0.1687115049454505, -4.557908462070945963, 10.2343999190812319})$.

Thus, the Rucklidge system has sensitivity to numerical inaccuracy.

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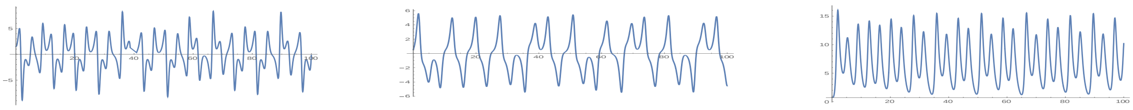
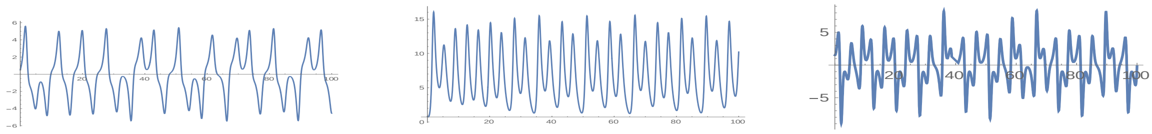
(a) (b)

Figure 4.1: $a = 2.6,\ b = 7.7$, $\gamma = (1.5, 0.5, 0.8)$, t=$\left[0 \quad 100\right]$ **(a) Trajectories without working precision and Accuracy, (b) Trajectories with working precision 20 and Accuracy 15 decimal.**



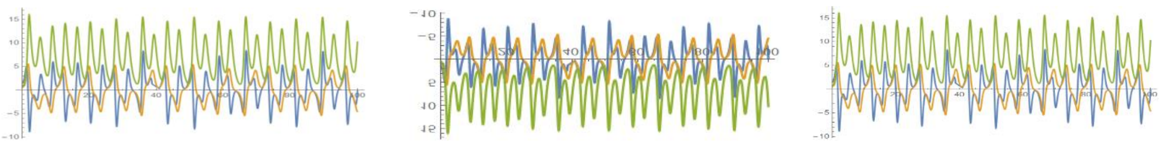
(a) (b) (c)

Figure 4.2: $a = 2.6,\ b = 7.7$, **Initial condition** $\gamma = (1.5, 0.5, 0.8)$ **(a) Trajectories for** x **-axis, (b) Trajectories for** y **-axis, (c) Trajectories for** z **-axis, [Without precision and Accuracy].**



(a) (b) (c)

Figure 4.3: $a = 2.6,\ b = 7.7$, **Initial condition** $\gamma = (1.5, 0.5, 0.8)$ **(a) Trajectories for** x **-axis, (b) Trajectories for** y **-axis, (c) Trajectories for** z **-axis, [With precision 20 and Accuracy 15 decimal].**



(a) (b) (c)

Figure 4.4: The solution trajectories of Rucklidge system for $a = 2.6,\ b = 7.7$, **Initial condition** $\gamma = (1.5, 0.5, 0.8)$ **(a) Trajectory solution s1, (b) Trajectory solution s2, (c) Trajectory solution s3.**

The figures show that from approximately on, the solutions differ greatly. We can trust the solution $s2$ more, but, actually, how accurate it is? We can solve the equations once more and

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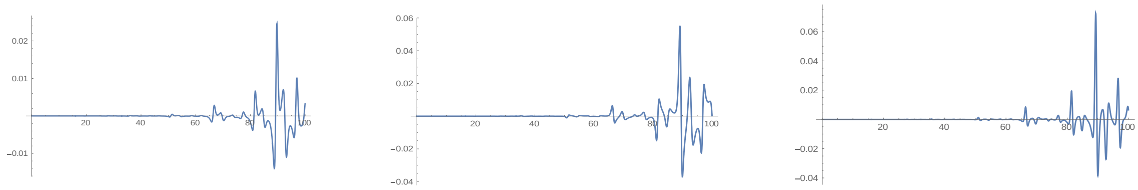
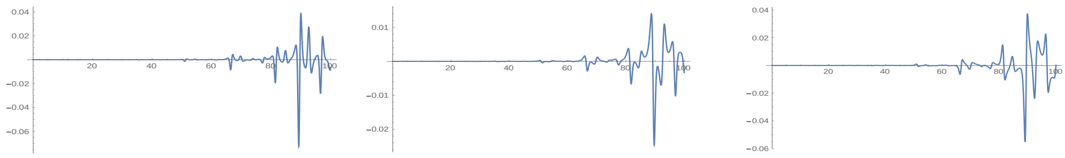
use even tighter precision and accuracy condition, then we will get again a difference with $s2$. Thus, the Rucklidge system has sensitivity to numerical inaccuracy.



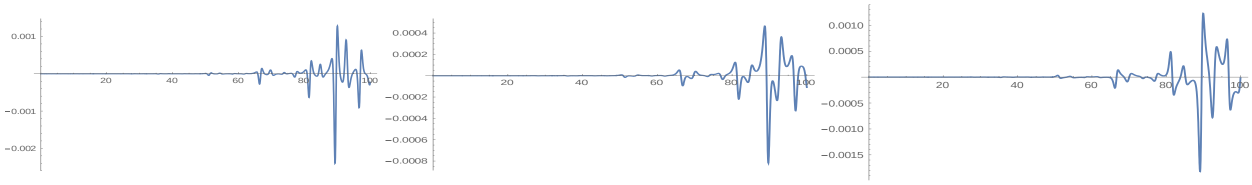
(a)



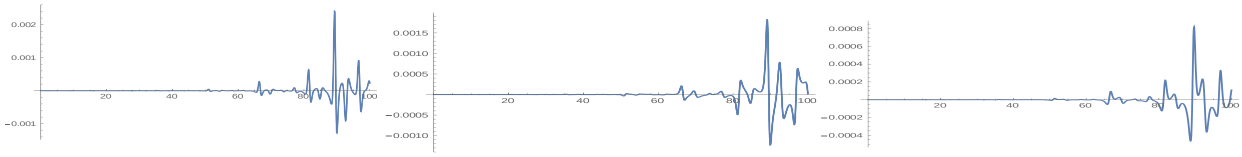
(b)

Figure 4.5: The solution trajectories of Rucklidge system for $a = 2.6,\ b = 7.7$, Initial condition $\gamma = (1.5, 0.5, 0.8)$ (a) Trajectory solution for s1-s2, (b) Trajectory solution for s2-s1

The figures show that from approximately on, the solutions differ greatly. Thus, the Rucklidge system has sensitivity to numerical inaccuracy.



(a)



(b)

Figure 4.6: The solution trajectories of Rucklidge system for $a = 2.6,\ b = 7.7$, Initial condition $\gamma = (1.5, 0.5, 0.8)$ (a) Trajectory solution for s2-s3, (b) Trajectory solution for s3-s2

The figures show that from approximately on, the solutions differ greatly. Thus, the Rucklidge system has sensitivity to numerical inaccuracy.

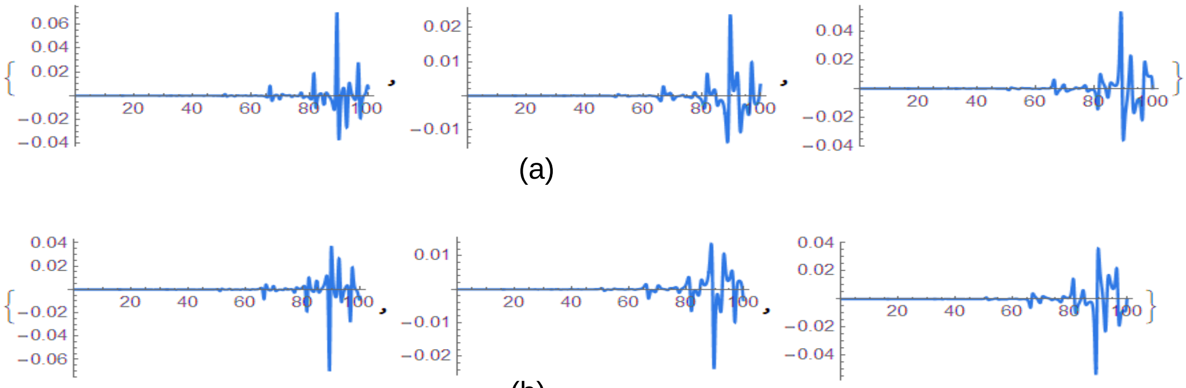


Figure 4.7: The solution trajectories of Rucklidge system for $a = 2.6,\ b = 7.7$, Initial condition $\gamma = (1.5, 0.5, 0.8)$ (a) Trajectory solution for s1-s3, (b) Trajectory solution for s3-s1

The figures show that from approximately on, the solutions differ greatly. Thus, the Rucklidge system has sensitivity to numerical inaccuracy [Ruskeepää, H, 2009].

5. SENSITIVITY TO INITIAL CONDITION

We have already known that the Rucklidge system is a generalized Lorenz-like system by definition-1 & definition-2 of generalized Lorenz-like system. Since the Lorenz system has sensitivity to the initial condition, the Rucklidge system has sensitivity to the initial condition. But how can we clear it? To demonstrate sensitivity of the Rucklidge to the initial conditions we solve the equations with parameter value $a = 2,\ b = 6.7$ and two initial conditions $x_1 = x_2 = 1.2,\ y_1 = y_2 = 0.8,\ z_1 = 1.4,\ z_2 = 1.4002$ which are very close to each other. Then we compare these two solutions by plotting $x(t)$. Figure **5.7** illustrates the sensitivity to initial condition of Rucklidge system.

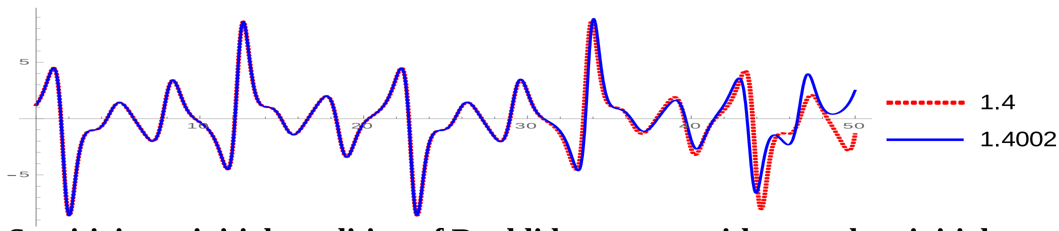


Fig 5.1: Sensitivity to initial condition of Rucklidge system with very close initial conditions $(1.2,\ 0.8,\ 1.4)$ & $(1.2,\ 0.8,\ 1.4002)$. Here only z value have slight changed

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Two trajectories begin with very close initial conditions. For the first 36-time units, the two trajectories seem identical, and then from $t = 42$ on the solutions are wholly different. However, beyond 42 time units, they seem completely unrelated to each other. This sensitivity makes the prediction of a chaotic system impossible for a long time period because the initial conditions are hardly known exactly. Due to the presents of the behavior of sensitivity to initial condition, we can confirm that the Rucklidge system is a chaotic system [Ruskeepää, H, 2009].

6. TIME SERIES ANALYSIS OF RUCKLIDGE SYSTEM

The Rucklidge model is a three-dimensional system of ordinary differential equations that exhibits chaotic behavior for specific parameter values. However, one of the vital tools is the time series analysis to investigate the chaotic behaviors of the Rucklidge model. We have drawn the three-time series graphs separately, where vertical axes are considered variables and horizontal axes are time. In this case, two sets of very closely nearby initial values are taken for the same iteration number. We see those small changes in the initial values can lead to drastically different over time, and it is visualized in the figures, which is called sensitivity to initial conditions [Ruskeepää, H, 2009].

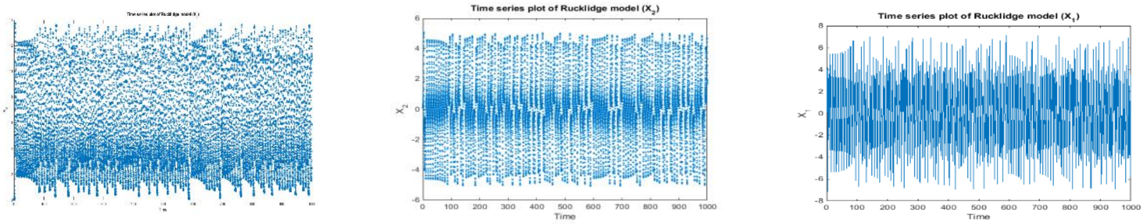

Fig 6.1: The Time Series Analysis of Rucklidge system for a=2.6, b=6.7, n=1000 and the initial values are $x_0 = 0.1, y_0 = 0.1, z_0 = 0.1$

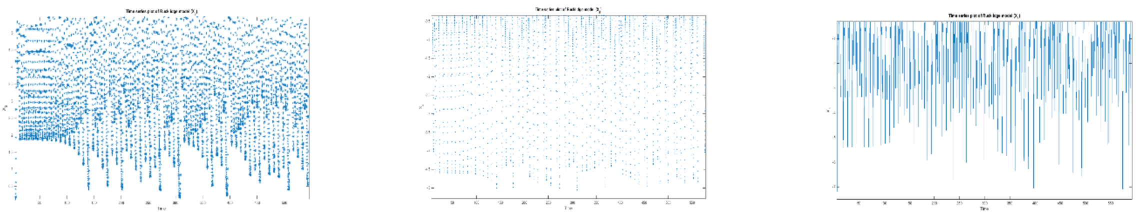

Fig 6.2: The Time Series Analysis (zoom out) of Rucklidge system for a=2.6, b=6.7, n=1000 and the initial values are $x_0 = 0.0001, y_0 = 0.0001, z_0 = 0.0001$

7. VECTOR FIELD ANALYSIS OF RUCKLIDGE SYSTEM

It will better understand the dynamics of the Rucklidge system, and we can plot the vector field, which is a plot of the direction field of the system at each point in the three-dimensional phase space. In other words, at each point in the phase space, we plot an arrow showing the direction in which the system is evolving. To plot the vector field of the Rucklidge system, Mathematica Software can be used to generate the three-dimensional vector plot, which is

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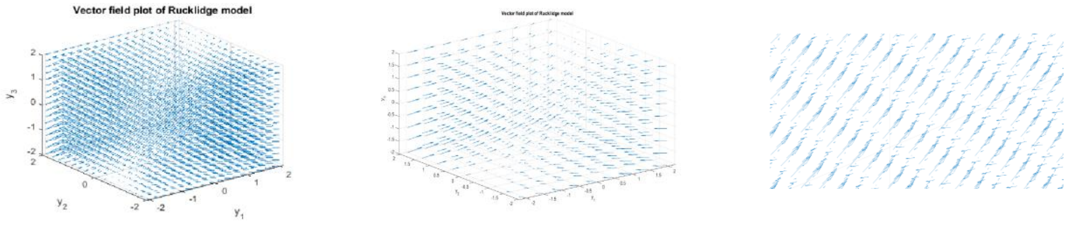
visualized by scattering arrows in different directions. To see clear visualization parameters are a significant factor. We have to choose appropriate parameters for a better graph of the vector field. The direction of the arrows indicates the direction in which the system is evolving at each point. Overall, the vector field of the Rucklidge system provides us with a visual representation of the system's dynamics and helps us understand how the system evolves in different regions of the phase space that exhibits the chaotic regime. For example, the following figures are the vector field for some parameter values [Ruskeepää, H, 2009].



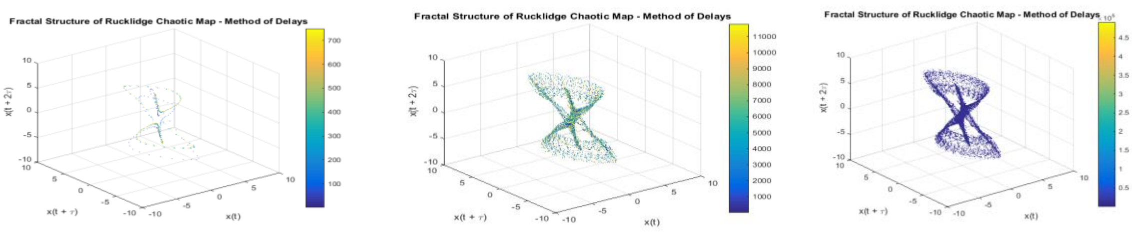
(a) (b) (c)

Fig 7.1: Vector Field diagram for parameters values a=2.6, b=6.7 and with different step size=0.5, 0.25 and 0.1 of Rucklidge System

8. FRACTAL STRUCTURE OF RUCKLIDGE SYSTEM

We know that the Rucklidge system exhibits chaotic behavior in a dynamical system. But we investigate this system using MATLAB code to discuss different chaotic tools that are used to show the graphically chaotic of this model. Analyzing this model, we have found fractal nature in this chaotic regime. To change the parameter values, initial values, and the number of iterations, we can generate the fractal in chaos, which is shown in this model.

For parameter values, iterations and initial values $a = 2,6, b = 6.7, n = 100,1000,40000$ respectively we have found a shape in figure 3.6.1used for setting. Similarly, for parameters values, iterations and initial values $a = 2,6, b = 12.7, n = 500,20000,40000$ respectively, we also have got a picture which look like a bird in the figure 3.6.2 [Ruskeepää, H, 2009].



(a) (b) (c)

Fig 8.1: Fractal of Rucklidge model for parameter values (a) A=2.6, b=6.7 n=100, (b) A=2.6, b=6.7 n=1000, (c) A=2.6, b=6.7 n=40000

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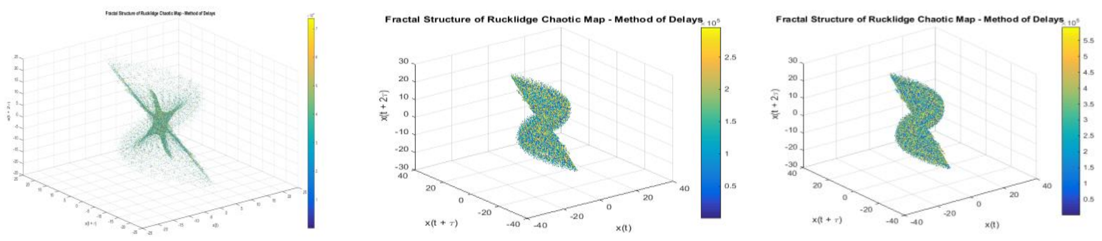
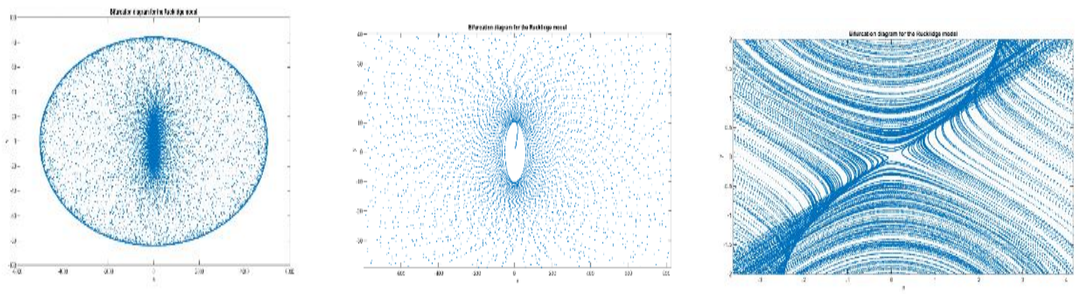


(a) (b) (c)

Fig 8.2: Fractal of Rucklidge model for parameter values (a) a=2.6, b=12.7 n=5000, (b) a=2.6, b=12.7 n=20000, (c) a=2.6, b=12.7 n=40000

9. BIFURCATION OF RUCKLIDGE SYSTEM

The Rucklidge chaotic system is a nonlinear first-order differential equation that exhibits a versatile variety of changes in dynamical behaviors. Bifurcation is one of the vital tools to show whether a map or system is chaotic. A bifurcation happens when a slight change in a system parameter causes a qualitative change in the system's behavior. In the Rucklidge system, the parameters a and b are very important that affect the dynamical behaviours. For different parameter values, the model expresses the different bifurcations. For example, we investigate that if the parameter b=0 and a stable equilibrium point split into one stable and another unstable, the model gives Pitchfork bifurcation. In the Rucklidge model, a pitchfork bifurcation can occur when b = 0. In changing the parameter values, we can generate Hopf bifurcation, Period-doubling bifurcation, and so on. In the following, I have created some bifurcations changing parameter values which express the high sensitivity and spreading chaotically that is chaos [Ruskeepää, H, 2009].



(a) (b) (c)

Fig 9.1: The Bifurcation Diagram of Rucklidge system for parameter (a) a=0.19, b=1, n=200000, (b) a=0.19, b=10, n=200000, and (c) a=2, b=10, n=200000. In all cases the initial values are $x_0 = 0.1, y_0 = 0.1, z_0 = 0.1$

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10. STRANGE ATTRACTOR OF RUCKLIDGE SYSTEM

The Rucklidge system is a three-dimensional chaotic system that exhibits a strange attractor. The strange attractor has a complex geometrical structure, and its behaviors are highly sensitive to initial conditions. The applications of strange attractors are widely studied and have many important in various fields such as mathematics, physics, and economics. This system attractor is formed due to the system's nonlinear dynamics. The system parameters and initial values are very important for such types of chaotic behaviours characterized by extreme sensitivity to initial conditions. The strange attractor is a set of states that the system repeatedly visits over time. These states form a geometrical structure closely self-similar at different scales, that is, the fractal dimension. The strange attractor's geometry is highly irregular and appears to be random, but it is, in fact, deterministic [Ruskeepää, H, 2009].

(a) (b) (c)

Fig 10.1: The Strange attractor of Rucklidge system for parameter (a) a=1.6, b=2.7, initials=1.0005, n=200, (b) a=2.6, b=7.7, initials=1.5, n=500, and (c) a=2.6, b=12.7, initials=1, n=1000

The Rucklidge attractor has several notable features, including its butterfly-shaped structure and the existence of periodic windows, where the attractor periodically returns to a specific state before continuing its chaotic behaviours. The attractor is also highly sensitive to initial conditions, which means that even slight variations in the initial conditions can lead to vastly different trajectories over time. For other parameters and initial values, some attractors, such as butterflies, are generated here that look like fractals.

11. CONTROLLING CHAOS

11.1. Trajectories of Phase Portrait

To control the Chaos in the Rucklidge system, we have changed one of the system parameters. Here, we have taken a fixed which is $a = 1.6$ and changed the parameter value of b. After long time drawing phase portrait and trajectories of system (1) for different values of b, we can able to write a control statement of Rucklidge system. If we have observed that if the parameter value b ranging between 3.2 to 36.8 *i.e.* $3.2 < b < 36.8$ the oscillations are chaotic

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and out of these interval we have totally controlled Rucklidge system. To illustrate this more clearly figure **11.1, 11.2, 11.3** are generated below:

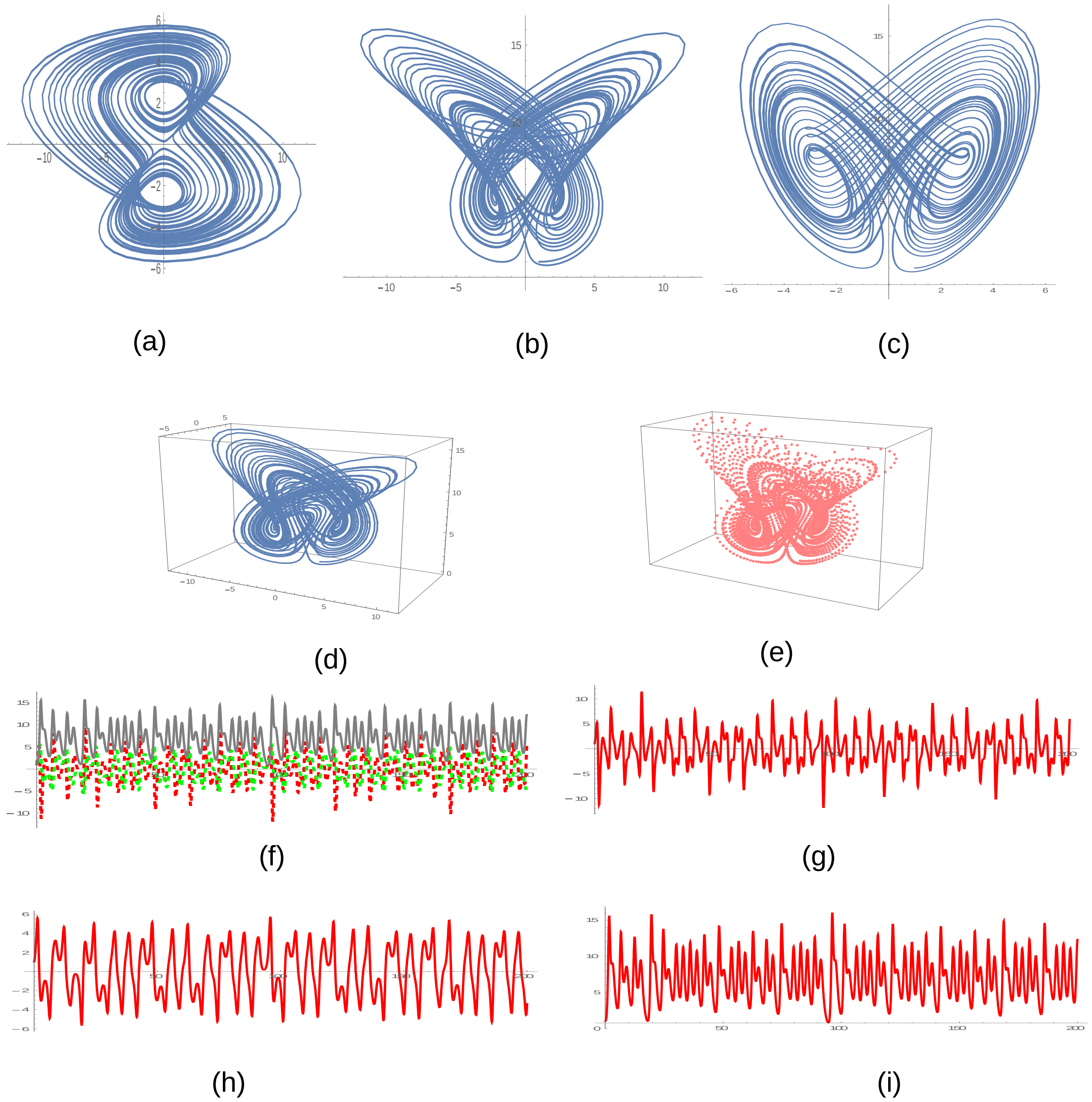

Fig 11.1: (a) 2D-Phase Portrait of $x-y$ Plane, (b) 2D-Phase Portrait of $y-z$ Plane (c) 2D-Phase Portrait of $z-x$ Plane, (d) & (e) 3D-Phase Portrait, (f) Trajectories of x, y, z values in one axis, (g),(h) & (i) Trajectories of x, y, z values in different axis. (a=1.6, b=6.7; Initial condition (1, 1, 1); T = [0 200]

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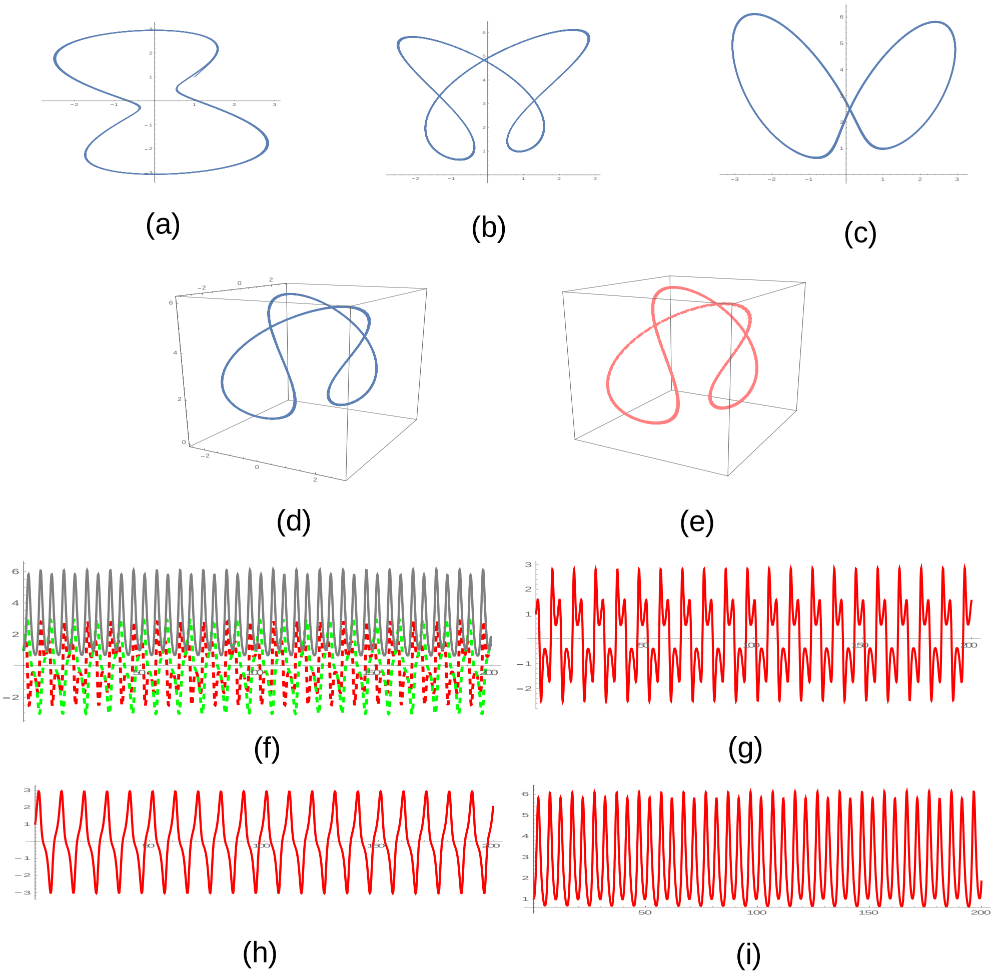
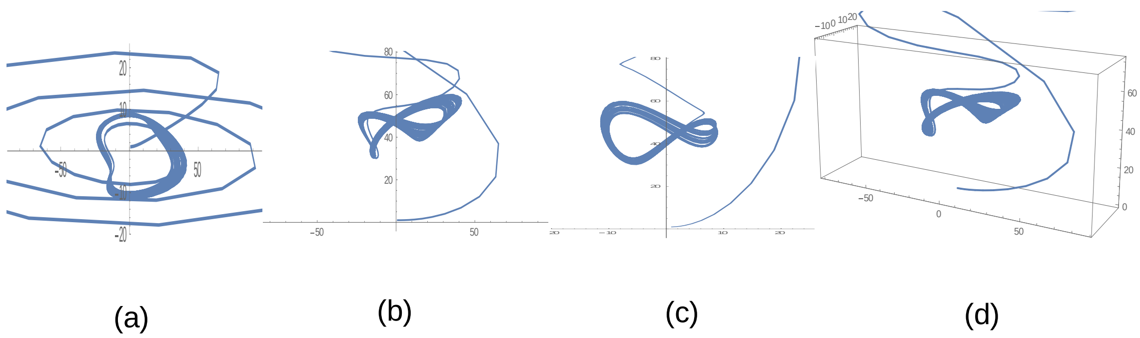


Fig 11.2: (a) 2D-Phase Portrait of $x-y$ Plane, (b) 2D-Phase Portrait of $y-z$ Plane (c) 2D-Phase Portrait of $z-x$ Plane, (d) & (e) 3D-Phase Portrait, (f) Trajectories of x, y, z values in one axis, (g), (h) & (i) Trajectories of x, y, z values in different axis. (a=1.6, b=3.2; Initial condition (1, 1, 1); *T*= [0 200]



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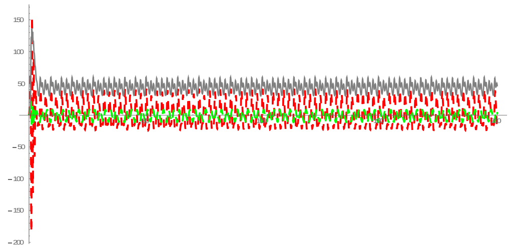
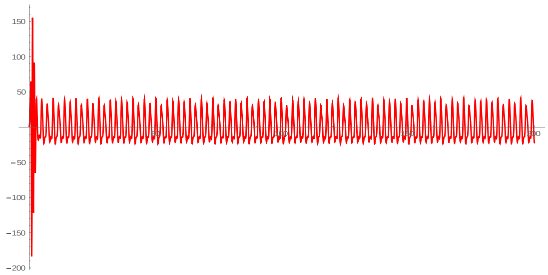
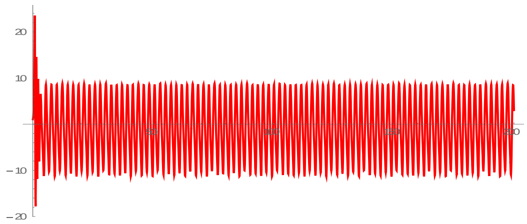


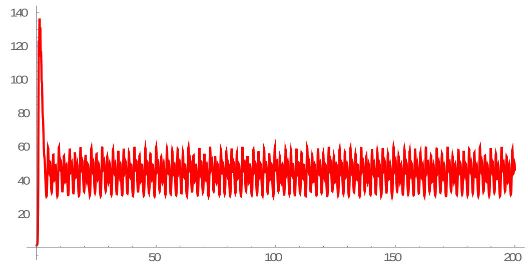
(e)



(f)



(g)



(h)

Fig 11.3: (a) 2D-Phase Portrait of $x-y$ Plane, (b) 2D-Phase Portrait of $y-z$ Plane (c) 2D-Phase Portrait of $z-x$ Plane, (d) 3D-Phase Portrait, (e) Trajectories of x, y, z values in one axis, (f), (g) & (h) Trajectories of x, y, z values in different axis. (a=1.6, b=36.8; Initial condition $(1,\ 1,\ 1)$; $T=[0\ \ 200]$

12. RESULT AND DISCUSSION

Figure 11.1 illustrates that system (1) is chaotic at b=6.7 since all trajectories of this oscillate irregularly, which is not periodic, and there is no possibility of being periodic. Also, from the phase portrait, we see that at a=1.6, b=6.7, this system has a 2-scroll chaotic attractor. But Figure 11.2 & 11.3 illustrates at b=3.2 & b=36.8, the system (1) is controlled since all trajectories corresponding to these parameter values are periodic. Even for any values of b in the interval (3.2 36.8), this system has a minimum oscillation, i.e., all trajectories between them are non-periodic. Experimentally, system (1) is controlled out of this interval.

To be more precise we have drawn the **Figures 11.4 & 11.5** which are given and discussed below:

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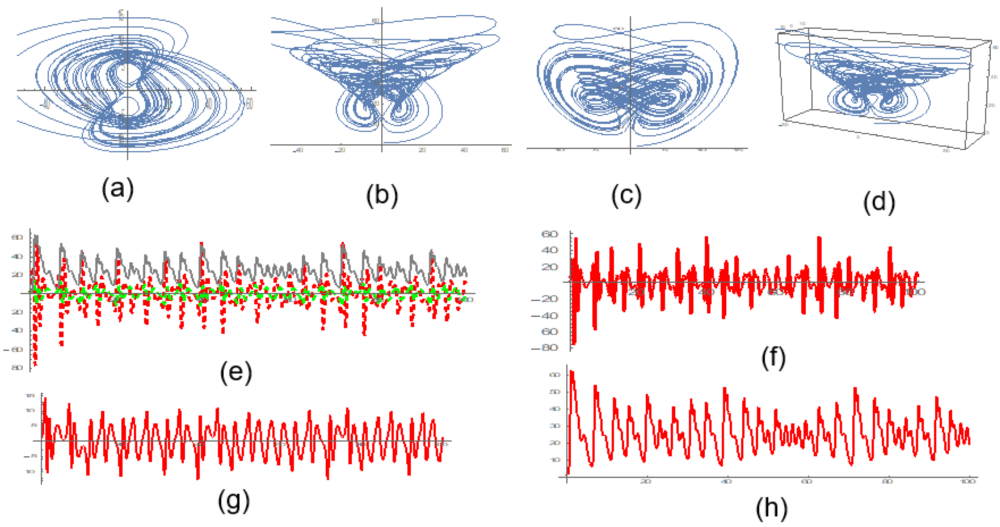

Fig 11.4: (a) 2D-Phase Portrait of x-y Plane, (b) 2D-Phase Portrait of y-z Plane (c) 2D-Phase Portrait of z-x Plane, (d) 3D-Phase Portrait, (e) Trajectories of x, y, z values in one axis, (f), (g) & (h) Trajectories of x, y, z values in different axis. (a=1.6, b=20.7; Initial condition (1, 1, 1); *T* = [0 200]

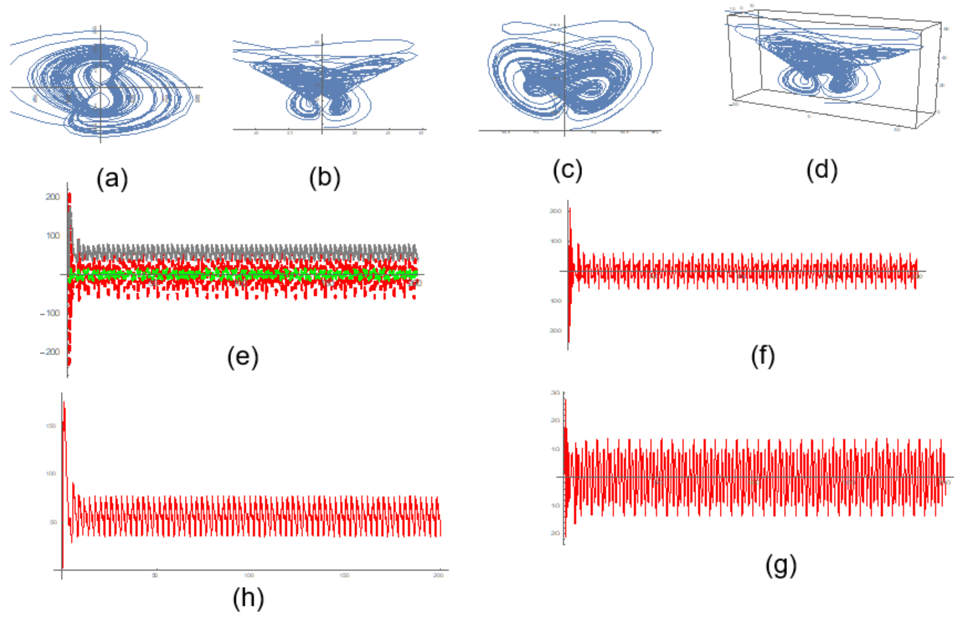

Fig 11.5: (a) 2D-Phase Portrait of x-y Plane, (b) 2D-Phase Portrait of y-z Plane (c) 2D-Phase Portrait of z-x Plane, (d) 3D-Phase Portrait, (e) Trajectories of x, y, z values in one axis, (f), (g) & (h) Trajectories of x, y, z values in different axis. (a=1.6, b=45); Initial condition (1, 1, 1); *T*= [0 200]

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Figure 11.4 illustrates, at b=20.7 [i.e., b ∈ (3.2 36.8)], the trajectories of the system (1) are not periodic, i.e., not controlled. But Figure 11.5 illustrates, b=45 [i.e., b ∉ (3.2 36.8)] the trajectories of the system (1) are periodic, i.e., controlled. Thus, this chaotic system can be controlled.

13. CONCLUSION

Dynamical Behaviors of Generalized Lorenz-like Chaotic Systems, that is Rucklidge System, introduce interesting and exciting parts of chaotic dynamics. We have shown some dynamical characteristics, including nonlinearity, stability, and instability; sensitivity to numerical inaccuracy, sensitivity to initial conditions; vector field analysis; time series analysis, strange attractor, and bifurcation of the Rucklidge System. Mainly, controlling the chaos of this system is our main focus. We have also displayed the two-scroll chaotic attractor of this system. Moreover, the graphical representations show these systems are chaotic in different senses. Finally, we have controlled the Rucklidge system through trajectories for different parameter values. We have found a boundary of parameter values where the system has minimum oscillation, is non-periodic, or may be chaotic. In the future, we will endeavor to investigate the mathematical theorem that has brought revolutionary changes in chaotic dynamical systems. The new researchers will benefit from this research.

References

1) Chen, G. [1993] "Controlling Chua's global unfolding circuit family," *IEEE Trans. Circuits Syst.-I* 40, 829–832.

2) Chen, G. & Dong, X. [1998] *From Chaos to Order: Methodologies, Perspectives and Applications* (World Scientific, Singapore).

3) Chen, G. & Lai, D. [1998] "Anti control of chaos via feed-back," *Int. J. Bifurcation and Chaos* 8, 1585–1590.

4) Chen, G. [1999] *Controlling Chaos and Bifurcations inEngineering Systems* (CRC Press, Boca Raton, FL).

5) Chen, G. & Ueta, T. [1999] "Yet another chaotic attractor," *Int. J. Bifurcation and Chaos* 9, 1465–1466.

6) Chen, G. & Lu¨, J. [2003] Dynamical Analysis, Control and Synchronization of the Generalized Lorenz Systems Family (in Chinese) (Science Press, Beijing).

7) El wakil, A. S. & Kennedy, M. P. [2001] "Construction of classes of circuit independent chaotic oscillators using passive-only nonlinear devices," IEEE Trans. Circuits Syst.-I 48, 289–307.

8) Festa, R., Mazzino, A. & Vincenzi, D. [2002] "Lorenz-like systems and classical dynamical equations with memory forcing: An alternate point of view for singling out the origin of chaos," Phys. Rev. E65, 046205.

9) Hao, B. L. [1984] Chaos (World Scientific, Singapore).

10) Liu, W. B. & Chen, G. [2003] "A new chaotic system and its generation," Int. J. Bifurcation and Chaos 12, 261–267.

11) Lorenz, E. N. [1963] "Deterministic no periodic flow," J. Atmos. Sci. 20, 130–141.

ISSN 1533-9211

DOI 10.17605/OSF.IO/AJUBY

12) Lu¨, J. & Chen, G. [2002] "A new chaotic attractor coined," Int. J. Bifurcation and Chaos 12, 659–661.

13) Lu¨, J., Chen, G. & Zhang, S. [2002b] "Dynamical analysis of a new chaotic attractor," Int. J. Bifurcation and Chaos 12, 1001–1015.

14) Lu¨, J., Chen, G. & Zhang, S. [2002c] "The compound structure of a new chaotic attractor," Chaos Soliton. Fractal. 14, 669–672.

15) Lu¨, J., Zhou, T., Chen, G. & Yang, X. [2002f] "Generating chaos with a switching piecewise-linear controller," Chaos 12, 344–349.

16) Lu¨, J., Zhou, T., Chen, G. & Zhang, S. [2002g] "The compound structure of Chen's attractor," Int. J. Bifurcation and Chaos 12, 855–858.

17) Ott, E., Grebogi, C. & Yorke, J. A. [1990] "Controlling chaos," Phys. Rev. Lett. 64, 1196–1199.

18) Tang, K. S., Man, K. F., Zhong, G. Q. & Chen, G. [2001] "Generating chaos via x x ," IEEE Trans. Circuits Syst.-I 48, 636–641. | |

19) Ueta, T. & Chen, G. [2000] "Bifurcation analysis of Chen's attractor," Int. J. Bifurcation and Chaos 10, 1917–1931.

20) Wang, X. & Chen, G. [1999] "On feedback anticontrol of discrete chaos," Int. J. Bifurcation and Chaos 9, 1435–1441.

21) Wang, X. & Chen, G. [2000] "Chaotification via arbitrarily small feedback controls: Theory, method, and applications," Int. J. Bifurcation and Chaos 10, 549–570.

22) Yu, X. & Xia, Y. [2001] "Detecting unstable periodic orbits in Chen's chaotic attractor," Int. J. Bifurcation and Chaos 10, 1987-1991.

23) Zhang, S. [1991] Mathematical Theory and Numerical Method of Modern Oscillatory Reaction (Henan Scientific and Technic Press, Henan, China).

24) Zhong, G.-Q. & Tang, K. S. [2002] "Circuitry implementation and synchronization of Chen's attractor," Int. J. Bifurcation and Chaos 12, 1423-1427.

25) C. Sparrow. The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors. Applied Mathematical Sciences. Springer New York, 1982.

26) JINHU LU, "Local Bifurcation of the Chen System" Int. J. Bifurcation and Chaos (2001)

27) Jinhu Lu, Bridge the Gap between the Lorenz system and the Chen system", Int. J. Bifurcation and Chaos (2002) 2917-2926

28) DAVIES, Paul. "The Cosmic Blueprint." Orion Productions, 1988. Simon and Schuster, New York.

29) Ruskeepää, H, Mathematica® Navigator, Mathematics, Statistics, and Graphics, Third Edition, 2009, Department of Mathematics University of Turku, Finland.