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RELAXED SKOLEM MEAN LABELING OF 6 - STAR GRAPHS WITH PARTITION (5, 1)

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Abstract

Existence Relaxed skolem mean labeling for a 6 – star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\alpha_4} \cup K_{1,\alpha_5} \cup K_{1,\beta}$ with partition 5, 1 with a certain condition is the core topic of the following article. Trial and error method is used to find the existence of the relaxed skolem mean labeling of 6 – star graph with partition 5, 1 holding a specific condition.

Keywords: Star Graph, Union of Star Graphs, Labeling, Skolem Mean Labeling, Relaxed Skolem Mean Graph.

1) INTRODUCTION

Labeling of Graphs is a branch of graph theory which is widely used in the area of networking and routing. There are various types of labeling functions introduced by different mathematicians. In this research article we discuss about one of these types of labeling namely Relaxed Skolem Mean Labeling which is derived from Skolem Mean Labeling of Graphs introduced by V. Balaji et.al. [5] In the year 2010. Basic properties of Relaxed Skolem Mean Labeling were already discussed by V. Balaji et.al. [5].

2) PRELIMINARIES

Definition:

A graph G = (V, E) with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function $f : V \rightarrow \{1,2,3,..., p+1 = |V| + 1\}$ such that the induced edge map

$f^* : E \rightarrow \{2,3,\ldots..,p+1= |V| +1\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } \left(f(u)+f(v)\right) \text{is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } \left(f(u)+f(v)+1\right) \text{is even} \end{cases}$$

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The resulting distinct edge labels are from the set $\{2,3,\ldots,p+1=|V|+1\}$

Note:

There are p vertices and available vertex labels are p + 1 and hence one number from the set$\{1, 2, 3, \ldots, p+1=|V|+1\}$is not used and we call that number as the relaxed label. When the relaxed label is p + 1, the relaxed skolem mean labeling becomes a skolem mean labeling.

Result:

The three star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolem mean labeling if $a+b \le c \le a+b+c$.

3) MAIN RESULT

Theorem:

The six star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\alpha_4} \cup K_{1,\alpha_5} \cup K_{1,\beta}$with partition (5, 1); $\alpha_1 \le \alpha_2 \le \alpha_3 \le \alpha_4 \le \alpha_5$is a relaxed skolem mean graph if$\beta - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 = 5$.

Proof:

Let $\sigma_k = \sum_{j=1}^{k} \alpha_j; 1 \le k \le 5$. Hence we have, $\sigma_1 = \alpha_1;\ \sigma_2 = \alpha_1 + \alpha_2;\ \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3;$ $\sigma_4 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$ and $\sigma_5 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$.

Consider the six star graph$G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\alpha_4} \cup K_{1,\alpha_5} \cup K_{1,\beta}$. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6$ be the vertex set of G where $V_k = \left\{v_{k,i} : 0 \le i \le \alpha_k\right\};$ for $1 \le k \le 5$ and $V_6 = \left\{v_{6,i} : 0 \le i \le \beta\right\}$.Let $E = \bigcup_{k=1}^{5} \{v_{k,0} v_{k,i} : 1 \le i \le \alpha_k\}$ $\cup \{v_{6,0} v_{6,i} : 1 \le i \le \beta\}$be the edge set of G.

Case:

Let $\beta = \sigma_5 + 5$

G has $\sigma_5 + \beta + 6 = 2\sigma_5 + 11$vertices and $\sigma_5 + \beta = 2\sigma_5 + 5$ edges.

The rsv function $f : V \to \{1, 2, \ldots, p+1 = \sigma_5 + \beta + 6 + 1 = 2\sigma_5 + 12\}$is defined as follows:

$f(v_{1,0}) = 1;\ f(v_{2,0}) = 2;\ f(v_{3,0}) = 3;\ f(v_{4,0}) = 5;\ \ f(v_{5,0}) = 7;$

$f(v_{6,0}) = \sigma_5 + \beta + 5 = 2\sigma_5 + 11$

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$f(v_{1,\kappa}) = 2\kappa + 7 \quad 1 \le \kappa \le \alpha_1$

$f(v_{2,\kappa}) = 2\sigma_1 + 2\kappa + 7 \quad 1 \le \kappa \le \alpha_2$

$f(v_{3,\kappa}) = 2\sigma_2 + 2\kappa + 7 \quad 1 \le \kappa \le \alpha_3$

$f(v_{4,\kappa}) = 2\sigma_3 + 2\kappa + 7 \quad 1 \le \kappa \le \alpha_4$

$f(v_{5,\kappa}) = 2\sigma_4 + 2\kappa + 7 \quad 1 \le \kappa \le \alpha_5$

$f(v_{6,\kappa}) = 2\kappa + 2 \quad 1 \le \kappa \le \beta = \sigma_5 + 5$

Here $2\sigma_5 + 9$ is the relaxed label.

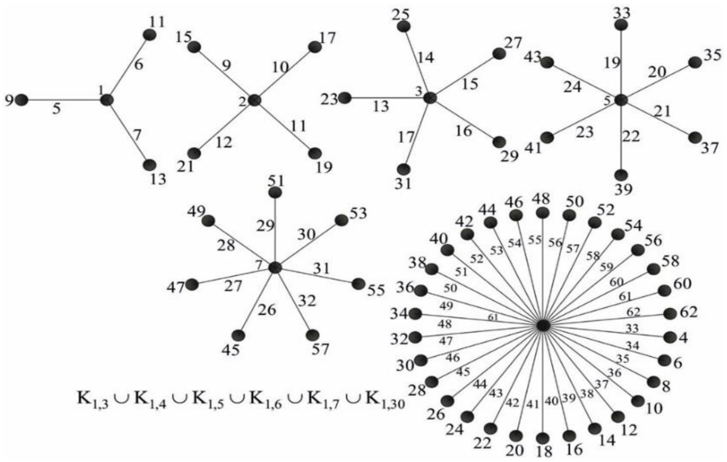
The corresponding edge labels are as follows:

The edge label of $V_{1,0}V_{1,\kappa}$ is $4+\kappa$ for $1 \le i \le \alpha_1$ $(5, 6, \ldots, \alpha_1 + 4 = \sigma_1 + 4)$, $V_{2,0}V_{2,\kappa}$ is $\sigma_1 + 5 + \kappa$ for $1 \le \kappa \le \alpha_2$ $(\sigma_1 + 6, \sigma_1 + 7, \ldots, \sigma_1 + \alpha_2 + 5 = \sigma_2 + 5)$, $V_{3,0}V_{3,\kappa}$ is $\sigma_2 + 5 + \kappa$ for $1 \le \kappa \le \alpha_3$ $(\sigma_2 + 6, \sigma_2 + 7, \ldots, , \sigma_2 + \alpha_3 + 5 = \sigma_3 + 5)$, $v_{4,0}v_{4,\kappa}$ is $\sigma_3 + 6 + \kappa$ for $1 \le \kappa \le \alpha_4$ $(\sigma_3 + 7, \sigma_3 + 8, \ldots, \sigma_3 + \alpha_4 + 6 = \sigma_4 + 6)$, $v_{5,0}v_{5,\kappa}$ is $\sigma_4 + 7 + \kappa$ for $1 \le \kappa \le \alpha_5$ $(\sigma_4 + 8, \sigma_4 + 9, \ldots, \sigma_4 + \alpha_5 + 7 = \sigma_5 + 7)$, $v_{6,0}v_{6,i}$ is $\sigma_5 + 7 + \kappa$ for $1 \le i \le \beta = \sigma_5 + 5$ $(\sigma_5 + 8, \sigma_5 + 9, \ldots, 2\sigma_5 + 12)$.

The edge labels hence are $5, 6, \ldots, \sigma_1 + 4, ,$ $\sigma_1 + 6, \sigma_1 + 7, \ldots, \sigma_2 + 5,$ $\sigma_2 + 6, \sigma_2 + 7, \ldots, \sigma_3 + 5,$ $\sigma_3 + 7, \sigma_3 + 8, \ldots, \sigma_4 + 6,$ $\sigma_4 + 8, \sigma_4 + 9, \ldots, \sigma_5 + 7,$ $\sigma_5 + 8, \sigma_5 + 9, \ldots, 2\sigma_5 + 12$.

Hence the induced edge labels of G are distinct and the graph G is a relaxed skolem mean graph.

Example:



$K_{1,3} \cup K_{1,4} \cup K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,30}$

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4) CONCLUSION

In this research article we concentrated mainly on the existence of relaxed skolem mean labeling of a 6 – star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\alpha_4} \cup K_{1,\alpha_5} \cup K_{1,\beta}$ with the condition $\beta - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5 = 5$. Trial and error method is used to find the existence of the labeling function.

Reference

1) M. Apostal, "Introduction to Analytic Number Theory", Narosa Publishing House, Second edition, 1991.

2) J. A. Bondy and U. S. R. Murty, "Graph Theory with Applications", Macmillan press, London, 1976.

3) J. C. Bermond," Graceful Graphs, Radio Antennae and French Wind Mills", Graph Theory and Combinatories, Pitman, London, 1979, 13 – 37.

4) V. Balaji, D. S. T. Ramesh and A. Subramanian, "Skolem Mean Labeling", Bulletin of Pure and Applied Sciences, vol. 26E No. 2, 2007, 245 – 248.

5) V. Balaji, D. S. T. Ramesh and A. Subramanian, "Relaxed Skolem Mean Labeling", Advances and Applications in Discrete Mathematics, vol. 5(1), January 2010, 11 – 22.

6) V. Balaji, D. S. T. Ramesh and A. Subramanian, "Some Results On Relaxed Skolem Mean Graphs", Bulletin of Kerala Mathematics Association, vol. 5(2), December 2009, 33 – 44.

7) J. A. Gallian, "A Dynamic Survey of Graph Labeling", The Electronic Journal of combinatorics 14(2007).