The Seybold Report
ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

USING A GREY CENTRALIZED NETWORK DATA ENVELOPMENT ANALYSIS METHOD FOR PERFORMANCE EVALUATION

TABASI MARYAM[1], NAVABAKHSH MEHRZAD[2]*, HAFEZALKOTOB ASHKAN[3] and TAVAKKOLI-MOGHADDAM REZA[4]

[1, 2, 3]Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.
[4]Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.
*Corresponding Author Email: m_navabakhsh@azad.ac.ir

Abstract

The operation of any dynamic organization needs to be evaluated to help the organization to improve. One of the most practical methods of evaluating is Network Data Envelopment Analysis (NDEA). NDEA compares the decision making units via a linear or non-linear model. NDEA considers all inputs and outputs during the network process, while standard DEA just considers the inputs of first stage and output of last stage. Intermediate data are not considered by standard DEA. NDEA models can be solved via different approaches; one of them is Game Theory approach. The efficiency of NDEA model has been calculated by centralized game. Also NDEA model has been developed by considering Grey numbers to manage the uncertainty of the real world. Grey Centralized NDEA model has evaluated the performance of decision making units (DMUs). The proposed model has been used in Iran Khodro Company which is one of the most important companies at automobile industry in Iran.

Keywords: Network Data Envelopment Analysis, Grey System Theory, Game Theory, Centralized game and Performance Evaluation

1. INTRODUCTION

Performance evaluation helps to survey current situation and discover improvement opportunities. DEA is one of the best models of evaluating. As it has the ability to consider many inputs and outputs. NDEA considers the details of the process. Most of the processes have two or more stages. In this paper first stage has one output which is the input of the second stage and we call it intermediate data. Paying attention to intermediate data, we will have more actual performance evaluation.

There are conflicting objectives and uncertainty in real organizational decision making. Grey numbers theory has been very successful to manage the uncertain performance of systems.

Charnes, Cooper and Rhodes [1] were the first researchers who introduced DEA to evaluate efficiency in 1978. DEA can estimate efficiency in absence of a priori assumptions, so DEA has been used for efficient frontier estimation.

When we use Standard DEA, we consider a black box which has inputs and outputs. But Network DEA (NDEA) considers interrelated processes for example in [2], [3], [4], [5], [6], [7]. The two-stage system is one of multi-stage series system, which has a simple network structure and is extensively studied.

Because of the uncertainty at the real world, overall efficiency can have different decompositions so the efficiency can be declared by an interval. Game theory approach has

ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

been used in some researches to calculate the efficiency. For example Nash bargaining was used by Zhou et al. to achieve single efficiency decomposition in two-stage systems [8]. Section 2 reviews literature about NDEA, game theory and Grey number system. Section 3 demonstrates the Grey model. Section 4 applies the model in Iran Khodro Company and conclusions are in last Section.

2. LITERATURE REVIEW

Seiford and Zhu [9] searched about the profitability and marketability of US banks by a two-stage network. At their research, labor and assets are inputs of the first stage, and the profits and revenues are outputs of that stage. The revenue and the profits are inputs in the second stage, and returns and market value and earnings per share are outputs. In first stage Profitability is measured and marketability is measured in second stage.

Farzipoor Saen and Mohammad Izadikhah [10] use a two-stage DEA method which is stochastic. Undesirable data was used in their model. Some linear models were applied to calculate upper and lower bounds of efficiencies of stages. Also, the overall efficiency of DMUs was obtained via a linear model.

Shafiei et al. [11] suggest a two stage model to calculate performance of supply chains. They adjusted NDEA model to assurance region to obtain more real results. Then they compered the results with the results of conventional DEA. Then they use Kendall's-Tau correlation tests for validity of results.

A Stackelberg game is proposed by Liang et al. [12]. It is difficult to extend these leader-follower games. The Stackelberg game is a cooperative game and can be used to the multistage systems.

Wade et al. [13] use a centralized game to obtain the NDEA efficiency. They do not consider uncertainty.

Zhongshan Yang and Xiaoxue Wei [14] use a competitive game DEA to measure energy efficiency. They analyze efficiency of 26 cities in Chinese from 2005 to 2015. Then, a comparative and research is done and the urban energy efficiency measured.

Emrouznejad et al. [15] combined a fuzzy game with DEA model for performance evaluation of 288 hospitals in Iran. First a fuzzy C-means Technique was use to cluster the DMUs. Then DEA combined with the game theory was applied. Also for each cluster, Core and Shapley Value approaches were applied.

Vaezi et al. [16] considered a three-stage network with optimal desirable and undesirable inputs and outputs. They use Stackelberg game model and Goal programming. For performance evaluation four models are investigated. And finally the nonlinear models convert into linear by a heuristic method.

Hatami-Marbini and Saati [17] use a two stage fuzzy DEA with series structure to generate common weights. And overall efficiency and the component efficiencies are calculated.

The Seybold Report

ISSN 1533-9211

DOI 10.17605/OSF.IO/4Y5PN

Khalili-Damghani and Shahmir [18] use an interval NDEA method to calculate the efficiency of electric power distribution and production systems. Also undesirable outputs are considered in their research.

Khalili-Damghani and Tavana [19] apply a two-stage fuzzy DEA method. They use the Stackelberg game theory approach to evaluate the efficiency for a DMU and its sub-DMUs. The efficiencies were ranked with Monte Carlo simulation. Banking industry is their case study.

Tavana et al. [20] define a fuzzy NDEA model to calculate the dynamic performance. The model was multi-objective and multi-period. They also consider undesirable outputs. They define the efficiency levels by a standard fuzzy operator. The model was applied to optimize the performance of refineries. Also Malmquist framework is used in their research.

Tavana et al. [21] combined game theory with a fuzzy two-stage DEA model. They used bargaining game. DMUs are serially connected. Because of uncertainty of outputs, intermediate measures, and inputs, linguistic terms via fuzzy sets were used. For each DMU and sub-DMU, interval efficiency is obtained. Sixty branches of the Saman bank in Iran were investigated.

Cooper et al. [22–25] investigate imprecise data such as bounded data in DEA. They present an imprecise DEA (IDEA) model and a linear programming problem which is obtained from a nonlinear programming problem.

Lee et al. [25] use an additive IDEA model. They claim that their model makes rapid increase in computation but Cooper et al.'s model is complicated.

Smirlis and Despotis [26] suggest a model for imprecise DEA. The approach transforms a nonlinear model to a linear one. They only transform the variables. The resulting efficiency scores are intervals.

Hatami-Marbini and et al. [27] present an Interval DEA model to evaluate performance which is Returns-to-Scale (RTS). They described six models of DEA consisting decreasing, increasing, constant, variable, non-decreasing and non-increasing approaches. Then a case study and two numerical examples are investigated.

Aineth Torres-Ruiz and Ravi Ravindran [28] use an IDEA model for economic performance with an index. This index helps suppliers to be pre-qualified and allocated orders are used multiple criteria approaches for single and multiple sourcing. Finally the Malmquist productivity index is used to assess the index change.

3. PRESENTING THE APPROACH

In this section we describe the approach of performance evaluation which is according to a developed model of NDEA.

3.1. Grey number system

Grey number system provides a mathematical way to evaluate more exactly. We use grey numbers with upper and lower limits, which are called Interval Grey numbers.

ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

So the developed model will be presented at following sections.

3.1.1. Definitions of Grey Linear programming

First some useful definitions are presented, and then grey linear programming will be proposed [29].

Definition1. Suppose x is a real closed and bounded set. Grey number $x^{\mp}$ is an interval with an uncertain distribution [30].

$$x^{\mp} = [x^{-}, x^{+}] = [t \in x \mid x^{-} \leq t \leq x^{+}], \tag{1}$$

Where x^{+}, x^{-} are the upper and lower bounds of grey number $x^{\mp}$. If $x^{-} = x^{+}$ so $x^{\mp}$ is a diterministic number and $x^{\mp} = x^{-} = x^{+}$.

Definition2. The whitened number of $x^{\mp}$ is a certain one between x^{-} *and* x^{+}.

$$x^{-} \leq x_v^{\mp} \leq x^{+}, \tag{2}$$

Where $x_v^{\mp}$ is the whitened number of $x^{\mp}$.

Definition3. A grey system contains information of grey numbers.

Definition4. We have the following relations for grey numbers $x^{\mp}$ *and* $y^{\mp}$[31], [32].

$$x^{\mp} + y^{\mp} = [x^{-} + y^{-}, x^{+} + y^{+}], \tag{3}$$

$$x^{\mp} - y^{\mp} = [x^{-} - y^{+}, x^{+} - y^{-}] \tag{4}$$

$$x^{\mp} \times y^{\mp} = [min\{x \times y\}, max\,\{x \times y\}], x^{-} \leq x \leq x^{+}\ ,\ y^{-} \leq y \leq y^{+}. \tag{5}$$

$$x^{\mp} \div y^{\mp} = [min\{x \div y\}, max\,\{x \div y\}], x^{-} \leq x \leq x^{+}\ ,\ y^{-} \leq y \leq y^{+}. \tag{6}$$

Definition5. We have the following relations for the grey number $x^{\mp}$[30].

$$x^{\mp} \geq 0 \quad if \quad x^{+} \geq 0,\ x^{-} \geq 0 \tag{7}$$

$$x^{\mp} \leq 0 \quad if \quad x^{+} \leq 0, x^{-} \leq 0 \tag{8}$$

Definition6. For $x^{\mp} = [x^{-}, x^{+}]$ *and* $y^{\mp} = [y^{-}, y^{+}]$, we have

$$x^{\mp} \leq y^{\mp} \quad if \quad x^{-} \leq y^{-}\ ,\ x^{+} \leq y^{+} \tag{9}$$

$$x^{\mp} < y^{\mp} \quad if \quad x^{-} < y^{-}\ ,\quad x^{+} < y^{+} \tag{10}$$

Definition7. Suppose $x^{\mp}$ is an interval grey number. Sign function related to $x^{\mp}$ is as follows.

$$Sign(x^{\mp}) = \begin{cases} 1, & x^{\mp} \geq 0 \\ -1, & x^{\mp} < 0 \end{cases} \tag{11}$$

The Seybold Report
ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

Definition8. The absolute of $x^{\mp}$ is as relation below.

$$|x|^{\mp} = \begin{cases} x^{\mp}, & x^{\mp} \geq 0 \\ -x^{\mp}, & x^{\mp} < 0 \end{cases} \quad (12)$$

So we have:

$$|x|^{-} = \begin{cases} x^{-}, & x^{\mp} \geq 0 \\ -x^{+}, & x^{\mp} < 0 \end{cases} \quad (13)$$

$$|x|^{+} = \begin{cases} x^{+}, & x^{\mp} \geq 0 \\ -x^{-}, & x^{\mp} < 0 \end{cases} \quad (14)$$

Definition9. According to the above definition, the linear programming model

$$max\, f^{\mp} = C^{\mp}X^{\mp} \quad (15)$$

s.t. $C^{\mp}X^{\mp} \leq B^{\mp}$

$x_j^{\mp} = grey\ integer\ value, \quad x_j^{\mp} \in X^{\mp}, \ j = 1,2, \dots, n$

$x_j^{\mp} \geq 0, \ j = 1,2, \dots, n$

And $A^{\mp} \in \{R^{\mp}\}^{m\times n}$, $B^{\mp} \in \{R^{\mp}\}^{m\times 1}$, $C^{\mp} \in \{R^{\mp}\}^{1\times n}$

Also $R^{\mp}$is a set of grey numbers.

As the uncertainty existence, the optimal solution and the objective value are grey numbers.

The solution is as follows.

$$x_{opt}^{\mp} = \{x_j^{\mp}{}_{opt} \mid j = 1,2, \dots, n\} \quad (16)$$

$$x_j^{\mp}{}_{opt} = [x_j^{-}{}_{opt}, x_j^{+}{}_{opt}] \ , \ x_j^{+}{}_{opt} \geq x_j^{-}{}_{opt} \quad \forall j \quad (17)$$

$$= [f_{opt}^{-}, f_{opt}^{+}] \ , \ f_{opt}^{+} \geq f^{-}{}_{opt} f^{\mp}{}_{opt} \quad (18)$$

3.1.2. The solving method

3.1.2.1. The relation between parameters and model variables

When cost coefficients are grey, we have the above relations for lower and upper bounds.

For grey coefficients of model (15) , $c_j^{\mp}$, where $j = 1,2, \dots, n$, if k_1 of them are positive namely $c_j^{\mp} \geq 0$ where $j = 1,2, \dots, n$, and if k_2 of them are negative namely $c_j^{\mp} \leq 0$ where $j = k_1 + 1, k_1 + 2, \dots, n$ such that $k_1 + k_2 = n$ (the model does not contain the situation that upper bound and lower bound have different signs), the above description are true for upper bound and lower bound.

$$f^{+} = \sum_{j=1}^{k_1} c_j^{+} x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+} x_j^{-} \quad (19)$$

ISSN 1533-9211

DOI 10.17605/OSF.IO/4Y5PN

$$f^{-} = \sum_{j=1}^{k_1} c_j^{-}\, x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-}\, x_j^{+} \tag{20}$$

The constraints related to f^{+} in objective function, are developed as:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{+} \le b_i^{\mp} \quad \forall i, \tag{21}$$

Similarly the constraints related to f^{-} are developed as:

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{-} \le b_i^{\mp} \quad \forall i \tag{22}$$

For the maximizing function, sub-model of upper bound of objective function regarding to constraints is first solving (21).

Vice versa if the function is minimizing, we first solve sub-model of lower bound regarding to constraints (22).

So $x_j^{\mp}{}_{opt}$=[$x_j^{-}{}_{opt}, x_j^{+}{}_{opt}$] and $f_{opt}^{\mp}$=[f_{opt}^{-}, f_{opt}^{+}] which are obtained of upper and lower bounds of objective function, are optimal.

.There are some relations about right hand side of constraints or $b_i^{\mp} = [b_i^{-}, b_i^{+}]$

If $b_i^{-} = b_i^{+}$ then $b_i^{\mp}$ is a definite number and $b_i^{\mp} = b_i^{-} = b_i^{+} = b_i$. So the constraints (21) & (22) will be as following:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} \le b_i \quad \forall i, \tag{23}$$

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} \le b_i \quad \forall i, \tag{24}$$

If $b_i^{-} < b_i^{+}$ so $b_i^{\mp}$ is a grey number and there are two situations:

1- When $b_i^{\mp}$ does not include zero which means $b_i^{\mp} > 0$ or $b_i^{\mp} < 0$, then we have $b_i^{\mp}$ as following:

1-1- If $b_i^{\mp} > 0$ the constraints (21) and (22) will be as following:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+}/b_i^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-}/b_i^{-} \le 1 \ \forall i, \tag{25}$$

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-}/b_i^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+}/b_i^{+} \le 1 \ \forall i, \tag{26}$$

1-2- If $b_i^{\mp} < 0$ the constraints (21) and (22) will be as following:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+}/b_i^{+\prime} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-}/b_i^{-\prime} \le -1 \ \forall i, \tag{27}$$

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-}/b_i^{-\prime} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+}/b_i^{+\prime} \le -1 \ \forall i, \tag{28}$$

Where $b_i^{\mp\prime} = -b_i^{\mp}$.

ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

2- When $b_i > 0$ *and* interval related to $b_i^{\mp}$ contains zero, we have $b_i^{\mp} = [0, b_i]$ or $b_i^{\mp} = [-b_i, 0]$. (The model does not contain the situation which the upper bound and lower bound of $b_i^{\mp}$ has different signs.)

2-1- When $b_i^{\mp} = [0, b_i]$ and $b_i > 0$, the constraint of upper bound of function will be as following:

$$\sum_{j=1}^{k_1} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} \le b_i \quad \forall i, \tag{29}$$

The above relation is the same as relation (24), and when $b_i^{\mp} = [0, b_i]$ and $b_i > 0$, the constraint related to lower bound of objective function will be as following:

$$\sum_{j=1}^{k_1} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} \le 0 \quad \forall i, \tag{30}$$

2-2- When $b_i^{\mp} = [-b_i, 0]$ and $b_i > 0$, the constraint related to upper bound of objective function is as following:

$$\sum_{j=1}^{n} |a_{ij}|^{-} Sign(a_{ij}^{-}) x_j^{+} \le -b_i \quad \forall i, \tag{31}$$

And when $b_i^{\mp} = [-b_i, 0]$ and $b_i > 0$, the constraint related to lower bound of objective function is as following:

$$\sum_{j=1}^{n} |a_{ij}|^{+} Sign(a_{ij}^{+}) x_j^{-} \le 0 \quad \forall i, \tag{32}$$

3.2. NDEA Approach

In the standard DEA, the units are like black-boxes so that there is no internal relation inside DMUs. So NDEA is suggested. Two-stage structures are processes where outputs from the first stage act like inputs to the second stage. These are called intermediate measures. The standard DEA does not manage conflicts between the two stages. For example, in order to obtain efficiency, the second stage will reduce its inputs (intermediate measures). Such an action would reduce the first stage outputs, so the efficiency of that stage will reduce. A centralized game model which is used by Liang et al. [33] solved such conflict via a number of DEA models.

3.3. Game theory

Game theory helps NDEA to obtain the efficiencies of DMUs. There are different cooperative games; one of them is centralized game.

3.3.1. Centralized model

Centralized game is a cooperative game, in which the two stages can be treated as one. So in order to maximize their efficiency scores, the stages jointly define optimal weights on the intermediate factors [33].For example the retailer and the manufacturer jointly define prices to achieve maximum benefit [34]. So the centralized approach lets both stages be optimized simultaneously. We have $e_j^1 . e_j^2 = \frac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} \gamma_i x_{ij}}$ because we assumed that $\omega_d^1 = \omega_d^2$. So we do not

ISSN 1533-9211

DOI 10.17605/OSF.IO/4Y5PN

maximize the average of e_0^1, e_0^2 , instead we have

$$e_0^{centralized} = Max\, e_0^1 . e_0^2 = Max\, \frac{\sum_{r=1}^{s} \mu_r\ y_{r0}}{\sum_{i=1}^{m} \gamma_i x_{i0}}$$

$$s.t. e_j^1 \leq 1\,, e_j^2 \leq 1\,, \omega_d^1 = \omega_d^2 \qquad (33)$$

We transform Model (1) into linear program:

$$e_0^{centralized} = Max \sum_{r=1}^{s} \mu_r y_{r0}$$

$$s.t. \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{d=1}^{D} \omega_d z_{dj} \leq 0 \quad j = 1, \dots, n$$

$$\sum_{d=1}^{D} \omega_d z_{dj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0 \quad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$$

$$\omega_d \geq 0, d = 1, \dots, D; \quad \gamma_i \geq 0,\ i = 1, \dots, m; \quad \mu_r \geq 0,\ r = 1, \dots, s \qquad (34)$$

Model (34) is the centralized model and is available in [33] and the Hwang and Kao [35] model.

The above model will obtain the unique overall efficiency. As mentioned before k_1 of the variables are positive and the others are negative. For example k_1 numbers of the variable ω_d are positive and others from $k_1 + 1$ to D are negative numbers. So we have D numbers of the variableω_d.

As described before the lower bound of the function is as following.

3.3.3. Standard DEA model

The CRS DEA model is as follows:

$$e_0^{standard\ DEA} = Max \sum_{r=1}^{s} \mu_r y_{r0}$$

$$s.t. \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \gamma_i x_{ij} \leq 0 \quad j = 1, \dots, n$$

$$\sum_{i=1}^{m} \gamma_i x_{i0} = 1$$

$$\gamma_i \geq 0,\ i = 1, \dots, m; \quad \mu_r \geq 0,\ r = 1, \dots, s \qquad (37)$$

3.3.4. Grey Standard DEA Model

Upper bound of grey standard model

According to Grey DEA model at Ying research, the upper and lower bound of standard DEA methods are as following [36].

The Seybold REPORT

ISSN 1533-9211

DOI 10.17605/OSF.IO/4Y5PN

$$e_0^{U-standard\ DEA} = Max \sum_{r=1}^{s} \mu_r\, y_{r0}^U$$
$$s.t. \sum_{r=1}^{s} \mu_r\, y_{r0}^U - \sum_{i=1}^{m} \gamma_i x_{ij}^L \le 0 \quad j = 1, \dots, n$$
$$\sum_{i=1}^{m} \gamma_i x_{ij}^L = 1$$
$$\gamma_i \ge 0,\ \ i = 1, \dots, m; \qquad \mu_r \ge 0,\ \ r = 1, \dots, s \tag{38}$$

Lower bound of grey standard model

$$e_0^{L-standard\ DEA} = Max \sum_{r=1}^{s} \mu_r\, y_{r0}^L$$
$$s.t. \sum_{r=1}^{s} \mu_r\, y_{r0}^U - \sum_{i=1}^{m} \gamma_i x_{ij}^L \le 0 \quad j = 1, \dots, n$$
$$\sum_{i=1}^{m} \gamma_i x_{ij}^U = 1$$
$$\gamma_i \ge 0,\ \ i = 1, \dots, m; \qquad \mu_r \ge 0,\ \ r = 1, \dots, s \tag{39}$$

3.4. The efficiency of Grey Model

Calculating efficiency score is consisting of following steps:

Step1. Convert linguistic or fuzzy variables to grey numbers according table 1. Fuzzy variables are related to grey values in table 1.

Table 1: Linguistic variables related to interval numbers

Linguistic variable for positive criterion	normal interval number
Very bad	[0,10]
bad	[10,30]
partly bad	[30,40]
mediate	[40,60]
partly good	[60,70]
good	[70,90]
Very good	[90,100]

Step2. Use lower and upper limits of network structure with two stages in the developed model.

Step3. Use lower and upper limits of the structure which has just one stage in the developed model.

Step4. Compare the results of two previous steps.

4. CASE STUDY

At this research the delivery department of Iran Khodro Company will be evaluated. This company delivers 21 types of cars to their owners. In fact every car delivery is a DMU which is compared to other DMUs. Car delivery is a two stage process. At the first stage which is

ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

called PDI (Pre Delivery Inspection), visual defects are checked. If a car has no defect, it will enter second stage which is sending. Inputs and outputs of the two stages are denoted at figure 1. The number of cars entered to PDI, the number of cars exited from PDI and the number of PDI's personals are grey numbers. Online delivery score and customer satisfaction score are linguistic variables.

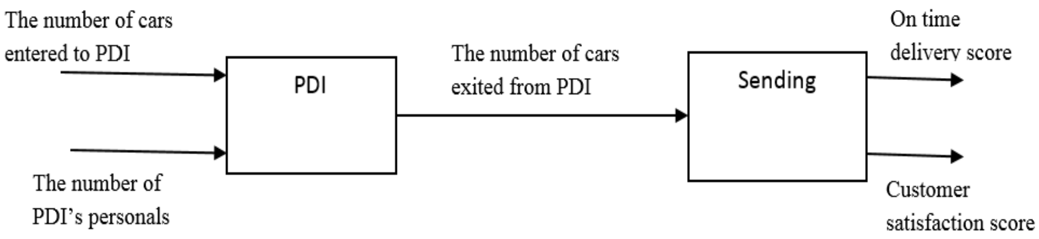

Figure 1: Inputs and outputs of the two stages process of car delivery at Iran Khodro Company

Table 2: The data of Iran Khodro Car Delivery

	Cars	The number of entered cars	The number of personals	The number of exited cars	On time delivery score	Customer satisfaction score
1	Automatic Tondar 90+	[16841,16860]	[10,12]	[16807,16830]	mediate	very bad
2	Tondar 90+	[1443,1463]	[10,12]	[1438,1460]	Partly bad	bad
3	H30 CROSS AT	[31907,31930]	[14,18]	[31522,31540]	Partly good	mediate
4	Automatic Peugeot 2008-EP6	[2540,2560]	[10,12]	[2426,2440]	Partly bad	bad
5	Peugeot 206-1600 cc	[38404,38430]	[18,20]	[38285,38300]	very good	Partly good
6	Peugeot 206 SD-1600 cc	[38659,38700]	[18,20]	[38533,38570]	good	good
7	Peugeot 206	[59677,59700]	[20,22]	[59499,59520]	Partly bad	good
8	Automatic Peugeot 207i	[17407,17600]	[10,12]	[17177,17200]	very good	Partly bad
9	Peugeot207i	[26071,26100]	[10,12]	[25881,25900]	good	very bad
10	Peugeot pars TU5	[34975,35000]	[12,14]	[34953,34970]	mediate	mediate
11	Peugeot pars hybrid	[9358,9370]	[8,10]	[9355,9370]	good	mediate
12	Tondar 90	[26047,26200]	[16,18]	[26024,26040]	very good	Partly good
13	Automatic Tondar 90	[2539,2570]	[10,12]	[2519,2530]	very good	very good
14	Tondar pick up	[4203,4230]	[10,12]	[4196,4200]	very good	Partly bad
15	Dena	[27269,27300]	[14,18]	[26986,27000]	good	very bad
16	Tourbocharged Dena+	[833,850]	[10,12]	[759,770]	good	bad
17	Dena+	[14018,14040]	[14,18]	[13744,13800]	good	mediate
18	Runna	[6900,7000]	[12,14]	[6849,6860]	very good	Partly good
19	Samand SE	[145,160]	[8,10]	[143,160]	very good	very good
20	Tourbocharged Soren EF7-TC	[1752,1770]	[10,12]	[1751,1760]	very good	good
21	Soren P2	[210,230]	[8,10]	[209,220]	Partly good	very good

ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

Table 3: The grey outputs of Iran Khodro Car Delivery

	cars	Grey on time delivery score	Grey customer satisfaction score
1	Automatic Tondar 90+	[40,60]	[0,10]
2	Tondar 90+	[30,40]	[10,30]
3	H30 CROSS AT	[60,70]	[40,60]
4	Automatic Peugeot 2008-EP6	[30,40]	[10,30]
5	Peugeot 206-1600 cc	[90,100]	[60,70]
6	Peugeot 206 SD-1600 cc	[70,90]	[70,90]
7	Peugeot 206	[30,40]	[70,90]
8	Automatic Peugeot 207i	[90,100]	[30,40]
9	Peugeot207i	[70,90]	[0,10]
10	Peugeot pars TU5	[40,60]	[40,60]
11	Peugeot pars hybrid	[70,90]	[40,60]
12	Tondar 90	[90,100]	[60,70]
13	Automatic Tondar 90	[90,100]	[90,100]
14	Tondar pick up	[90,100]	[30,40]
15	Dena	[70,90]	[0,10]
16	Tourbocharged Dena+	[70,90]	[10,30]
17	Dena+	[70,90]	[40,60]
18	Runna	[90,100]	[60,70]
19	Samand SE	[0,10]	[90,100]
20	Tourbocharged Soren EF7-TC	[70,90]	[70,90]
21	Soren P2	[60,70]	[90,100]

Table 4: The results of Grey Centralized game model

	cars	e_0^{-c}	e_0^{+c}
1	Automatic Tondar 90+	0.007460	0.012416
2	Tondar 90+	0.064447	0.096589
3	H30 CROSS AT	0.005910	0.007647
4	Automatic Peugeot 2008-EP6	0.036830	0.054873
5	Peugeot 206-1600 cc	0.007365	0.009075
6	Peugeot 206 SD-1600 cc	0.005689	0.008114
7	Peugeot 206	0.002268	0.003143
8	Automatic Peugeot 207i	0.016079	0.020021
9	Peugeot207i	0.008436	0.012032
10	Peugeot pars TU5	0.003595	0.005979
11	Peugeot pars hybrid	0.023479	0.033511
12	Tondar 90	0.010801	0.013379
13	Automatic Tondar 90	0.110061	0.137237
14	Tondar pick up	0.066869	0.082904
15	Dena	0.008063	0.011503
16	Tourbocharged Dena+	0.258824	0.376471
17	Dena+	0.015670	0.022371
18	Runna	0.040408	0.050499
19	Samand SE	0.900000	1.000000
20	Tourbocharged Soren EF7-TC	0.124294	0.178996
21	Soren P2	0.847826	1.000000

The Seybold Report
ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

Table 5: The results of Grey CRS models for standard DEA

	cars	$e_0^{L-standard\ DEA}$	$e_0^{U-standard\ DEA}$
1	Automatic Tondar 90+	0.296296	0.533333
2	Tondar 90+	0.269533	0.425613
3	H30 CROSS AT	0.296296	0.476190
4	Automatic Peugeot 2008-EP6	0.255696	0.399995
5	Peugeot 206-1600 cc	0.400000	0.500000
6	Peugeot 206 SD-1600 cc	0.350000	0.500000
7	Peugeot 206	0.254545	0.360000
8	Automatic Peugeot 207i	0.666667	0.888889
9	Peugeot207i	0.518519	0.800000
10	Peugeot pars TU5	0.285714	0.500000
11	Peugeot pars hybrid	0.640303	1.000000
12	Tondar 90	0.444444	0.562500
13	Automatic Tondar 90	0.766728	1.000000
14	Tondar pick up	0.740041	0.977793
15	Dena	0.345679	0.571429
16	Tourbocharged Dena+	0.648523	0.993021
17	Dena+	0.362836	0.583351
18	Runna	0.621966	0.798346
19	Samand SE	0.815625	1.000000
20	Tourbocharged Soren EF7-TC	0.619528	0.940648
21	Soren P2	0.807609	1.000000

Comparing table 4 and 5, as it is obvious in most of the DMUs the efficiency scores of standard DEA are greater than centralized game. In standard DEA we ignore intermediate data which are an important part of a network structure, so the unreal efficiency has been occurred.

5. CONCLUSION

In this research centralized game which is a cooperative game, is used to obtain efficiency score of DMUs. To achieve a better adjustment to real world, NDEA models were used. Also we made the NDEA model grey. The efficiencies of grey NDEA model are more precise than grey DEA model as the details related to stages called intermediate data have been considered.

References

1) A. Charnes, W.W. Cooper, E. Rhodes. (1978)."Measuring efficiency of decision-making units", European Journal of Operational Research, 2 , 429-444.

2) Färe, R. and Grosskopf, S. (2000)."Network DEA", Socio- Economic Planning Sciences, 34,35-49.

3) Färe, R., Grosskopf, S. and Whittaker, G. (2007)."Network DEA. In J. Zhu and W.D. Cook (eds.),"Modelling Data Irregularities and Structural Complexities in Data Envelopment Analysis, 209-240.

4) Fukuyama, H. and Weber, W.L., (2010). "A slacks-based inefficiency measure for a two-stage system with bad outputs", Omega, 38,398-409.

The Seybold REPORT
ISSN 1533-9211
DOI 10.17605/OSF.IO/4Y5PN

5) Lozano, S. (2011)."Scale and cost efficiency analysis of networks of process", Expert Systems with Applications, 38, 6612-6617.

6) Tone, K. and Tsutsui, M. (2009)."Network DEA: A slacks based measure approach", European Journal of Operational Research, 197, 243-252.

7) Wei, Q.L. and Chang, T.S. (2011)."Optimal system design series-network DEA models", Journal of the Operational Research Society, 42, 1109-1119.

8) Zhou, Z., Sun., L., Yang, W., Liu, W. and Ma, C. (2013). "A bargaining game model for efficiency decomposition in the centralized model of two-stage systems", Computers and Industrial Engineering, 64, 103–108.

9) Seiford LM, Zhu J. (1999)."Profitability and marketability of the top 55 US commercial banks", Management Science; 45, 1270–1288.

10) Izadikhah M., Farzipoor Saen R. (2018). "Assessing sustainability of supply chains by chance-constrained two-stage DEA model in the presence of undesirable factors", Computers & Operations Research 100, 343-367.

11) Shafiei Kaleibari, S., Gharizadeh Beiragh, R., Alizadeh, R., (2016). " A framework for performance evaluation of energy supply chain by a compatible Network Data Envelopment Analysis model", Scientiairanica, 23(4), 1904-1917.

12) Wu, Jie., Liang Liang, Chen Yao, (2016). "DEA game cross-efficiency approach to Olympic rankings", Omega 37, 909-918.

13) Wade D.Cook, Liang Liang, JoeZhu. (2010). "Measuring performance of two-stage network structures by DEA: A review and future perspective", Omega 38, 423–430.

14) Yang Z., Wei X., (2019)."The measurement and influences of China's urban total factor energy efficiency under environmental pollution: Based on the game cross-efficiency DEA", Journal of Cleaner Production, 209, 439-450.

15) Omrani, H., Shafaat, Kh., Emrouznejad, A.,. (2018)."An Integrated fuzzy clustering cooperative game data envelopment analysis model with application in hospital efficiency", Expert Systems with Applications 114, 615–628.

16) Vaezi, E., Najafi, S.E., Hajimolana, S.M., Hosseinzadeh Lotfi, F., Ahadzadeh Namin, M.,(2019). "Efficiency Evaluation of a Three-Stage Leader-Follower Model by the Data Envelopment Analysis with Double-Frontier Viewpoint", Scientiairanica, Articles in Press, Available Online from 21 May 2019.

17) Hatami-Marbini, A., Saati, S., (2018)."Efficiency evaluation in two-stage data envelopment analysis under a fuzzy environment: A common-weights approach", Applied Soft Computing, 72, 156-165.

18) Khalili-Damghani, K., Shahmir, Z., (2015). "Uncertain network data envelopment analysis with undesirable outputs to evaluate the efficiency of electricity power production and distribution processes", Computers & Industrial Engineering, 88, 131-150.

19) Tavana, M., Khalili-Damghani, K., (2014)."A new two-stage Stackelberg fuzzy data envelopment analysis model", Measurement, 53, 277-296.

20) Tavana, M., Khalili-Damghani, K., J.Santos Arteaga, F., Hosseini, A., (2019). "A fuzzy multi-objective multi-period network DEA model for efficiency measurement in oil refineries", Computers & Industrial Engineering, 135, 143-155.

21) Tavana, M., Khalili-Damghani, K., J.Santos Arteaga, F., Mahmoudi, R., Hafezalkotob, A., (2018). "Efficiency decomposition and measurement in two-stage fuzzy DEA models using a bargaining game approach", Computers & Industrial Engineering, 118, 394-408.

22) W.W. Cooper, K.S. Park, G. Yu," (1999). IDEA and AR-IDEA: models for dealing with imprecise data in DEA", Management Sci.45, 597–607.

23) W.W. Cooper, K.S. Park, G. Yu, (2001)."An illustrative application of IDEA (imprecise data envelopment analysis) to a Korean mobile telecommunication company", Operations Research, 49, 807–820.

24) W.W. Cooper, K.S. Park, G. Yu, (2001)." IDEA (imprecise data envelopment analysis) with CMDs (column maximum decision making units"), J. Operations Research. Soc. 52, 176–181

25) Y.K. Lee, K.S. Park, S.H. Kim, (2002). "Identification of inefficiencies in an additive model based IDEA (imprecise data envelopment analysis)", Computers and Operations Research. 29, 1661–1676.

26) D.K. Despotis, Y.G. Smirlis, (2002)."Data envelopment analysis with imprecise data", European J. Operations Research 140, 24–36.

27) Hatami-Marbini, A., Ghelej, Z., Jens, B., Hougaard, L.,Gholami, K., (2018)."Measurement of returns-to-scale using grey data envelopment analysis models", Computers & Industrial Engineering, 117(3), 94-107.

28) Aineth Torres-Ruiz and Ravi Ravindran, A., (2019)."Use of grey data envelopment analysis, goal programming and dynamic eco-efficiency assessment for sustainable supplier management", Computers & Industrial Engineering, 131, 211-226.

29) Huang, Guo. Baetz, Brian, W., Patry, Gilles, G., (1995)."Grey integer programming: An application to waste management planning under uncertainty", European Journal of Operational Research. 83, 594-620.

30) Huang, G., Baetz, B.W., and Patry, G.G., (1992)."A grey linear programming approach for municipal solid waste management planning under uncertainty", Civil Engineering Systems. 9, 319-335.

31) Huang, G., Baetz, B.W., and Patry, G.G., (1994b)."Grey dynamic programming (GDP) for solid waste management planning under uncertainty", ASCE Journal of Urban Planning and Development. 120, 132-156.

32) Huang, G., Baetz, B.W., and Patry, G.GA., (1993)."Grey fuzzy linear programming approach for municipal solid waste management planning under uncertainty", Civil Engineering Systems. 10, 123-146.

33) Liang, L., Cook, W.D., Zhu, J., (2008)."DEA models for two-stage processes: Game approach and efficiency decomposition", Naval Research Logistics, 55, 643–653.

34) Huang ZM, Li SX. Co-Op., (2001)."Advertising Models in a Manufacturing-Retailing Supply Chain: A Game Theory Approach", European Journal of Operational Research; 135, 27–44.

35) Kao, C., Hwang, S.N., (2008). " Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan", European Journal of Operational Research, 185 , 418–429.

36) Ying-Ming Wang, (2005). "Grey efficiency assessment using data envelopment analysis" Fuzzy Sets and Systems, 153, 347–370.