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FORECASTING FUTURE STOCK RETURN IN INFORMATIONALLY INEFFICIENT MARKETS

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Abstract

As a characteristic of informationally inefficient economies, asymmetric information has frequently led to distorted financial markets. By using stochastic optimization techniques, we constructed a model that addresses the problem of mispricing financial instruments in markets that do not conform to the traditional Markowitz portfolio optimization. The model does not suffer from adverse observations and is hence insensitive to sudden changes in the model parameters. We propose forecasting future stock market returns using the model constructed under informationally inefficient markets. Using data from BofA Merrill Lynch, we estimated an optimal forecasting model for this work. The sample period is from December 1998 to November 2017, and the forecast period starts 14 years after the beginning of the sample. We conclude that there is substantial predictability in stock market returns, as with our constructed model, an investor could have timed the market and gained up to 6.11% over the course of five years.

Keywords: Inefficient Market; Log Return; Return Predictability; Information Asymmetry.

1. INTRODUCTION

The selection of portfolios and asset prices are currently the key topics in financial markets. If the market is efficient, it is anticipated that asset prices will disclose already-known information and that all investors will have an equal amount of knowledge to choose their portfolios (Bekele et al. (2018)). But most of the financial markets that exist in today's real-world markets are inefficient. An inefficient market is one that does not succeed in incorporating all available information into a true reflection of an asset's fair price. Market inefficiencies exist due to information asymmetry, transaction costs, market psychology, and human emotions, among other reasons. In each investment, investors seek larger returns relative to their initial capital invested, but occasionally they encounter difficulties in determining how to create the finest trading methods to satiate their desire to profit from a specific invested asset.

In the financial market, returns are usually accompanied by risk (the adage is that the greater the risk, the greater the return). The ability to manage risk in the financial market is crucial for achieving higher future profits (Komunte et al. (2021); Yang et al. (2022); Islam et al. (2016);

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Andersen et al. (2007)). When predicting stock market returns, there is a substantial body of literature. Ferreira & Santa-Clara (2011) used the same data as Welch & Goyal (2008) for the 1927–2005 period and applied the sum-of-the-parts forecasting approach. Their method's performance clearly outperforms both the historical mean and conventional predictive regressions.

Economists have suggested a whole range of predictive variables that investors could or should use to predict the future stock market return. Some of those variables (with the author(s) who applied the same in brackets) are: the consumption, wealth, and income ratio (Lettau & Ludvigson (2001)); the book-to-market ratio (Kothari & Shanken (1997) and Pontiff & Schall (1998)); the dividend price ratio and the dividend yield (Menzly et al. (2004), Lewellen & Shanken (2002), Campbell & Yogo (2006), Campbell et al. (2002), and Hodrick (1992)); the short-term interest rate (Ang & Bekaert (2007), Campbell (1987), and Hodrick (1992)); the earnings price ratio and dividend earnings (payout) ratio (Lamont (1998)); inflation (Fama (1981), Campbell & Vuolteenaho (2004), and Vuolteenaho et al. (2004)); the corporate issuing activity (Boudoukh et al. (2008) and Baker & Wurgler (2000)); the stock market volatility (Guo (2006) and Ghysels et al. (2005)).

All of these investigations discover proof of return predictability in the sample. These results, however, have drawn criticism from a number of authors who contend that the forecasting variables' persistence and the link between their innovations and returns may bias the regression coefficients and have an impact on t-statistics (Torous et al. (2004), Lewellen (2004), Stambaugh (1999), Cavanagh et al. (1995), and Indrayono (2019)).

2. METHODOLOGY

2.1. The Model

We consider a maximization equation as given by Zhang (2007), and we assume there is a zero consumption throughout the investment period and we incorporate the skew Brownian motion. The maximization problem is then given by

$$\max_{\pi \in \mathcal{A}_0(x)} E\left[u\left(X^{\pi}(T)\right)\right]$$

s.t

$$dX^{\pi}(t) = X^{\pi}(t)\left[R(t)dt + \pi^{T}(t)\sigma(t)\left(\phi(t)dt + \mu\left(Z(t)\right)dt + dW(t) + (2p-1)dL_t^0(Z)\right)\right]$$

$$X^{\pi}(0) = x, \qquad (1)$$

With

$$\mathcal{A}_0(x) = \{(\pi, 0) \in \mathcal{A}(x) : E\left[u\left(X^{\pi}(T)\right)\right] < \infty\}$$

And

$$u(x) = \frac{x^{1+\gamma}}{1+\gamma}, \quad \gamma \neq -1.$$

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Where,

π represents the investment, $X^{\pi}(t) \geq 0$ is the wealth process, $x > 0$ is the initial capital of an investor, $\phi(t)$is the market price of risk, $R(t)$ is the nominal interest rate, $W(t)$ is the Brownian motion, $\sigma(t)$ is the volatility, $u(\cdot)$ represents the investor's utility function, $\mathcal{A}(x)$ represents the class of admissible pairs, $L_t^0(Z)$ is the symmetric local time of $Z(t)$, and $Z(t)$ is the skew Brownian motion with a skew parameter p.

Remark 2.1.

The terms skew Brownian motion $Z(t)$, symmetric local time $L_t^0(Z)$, and skew parameter p have been explained in detail by Gairat & Shcherbakov (2017).

Maximization equation (1) is equivalent to

$$\max_{G} E\big(u(G)\big)$$

$$\text{s.t}$$

$$E(P(T)G) = x \tag{2}$$

WhereG, denotes all possible $\mathcal{F}(T)$-measurable contingent claims, given by

$$G = X^{\pi}(T)$$

And $P(t)$ is the stochastic discount factor defined by

$$P(t) = \exp\{-\int_0^t R(s)ds - \frac{1}{2}\int_0^t ||\,\phi(s)||\,^2 ds - \int_0^t \phi^{\mathcal{T}}(s)dW(s)\}. \tag{3}$$

Following the same procedures as in Zhang (2007), the optimal wealth was found to be

$$X^{\pi^*}(t) = \frac{xe^{-\left(\frac{\gamma+1}{\gamma}\right)\int_0^t \phi^{\mathcal{T}}(s)dW(s) - \frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)^2 \int_0^t ||\phi(s)||^2 ds}}{\exp\{-\int_0^t R(s)ds - \frac{1}{2}\int_0^t ||\phi(s)||^2 ds - \int_0^t \phi^{\mathcal{T}}(s)dW(s)\}} \tag{4}$$

Suppressing the dependence on t as in Cvitani´c & Karatzas (1992) and Cvitani´c et al. (2006), equation (5) can be written as

$$X^{\pi^*}(t) = \frac{xe^{\left(\frac{\gamma+1}{\gamma}\right)\int_0^t \phi dW(s) - \frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)^2 \int_0^t ||\phi||^2 ds}}{\exp\{-\int_0^t Rds - \frac{1}{2}\int_0^t ||\phi||^2 ds - \int_0^t \phi dW(s)\}}$$

$$= \frac{xe^{\left(\frac{\gamma+1}{\gamma}\right)\phi W(t) - \frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)^2 ||\phi||\,^2 t}}{\exp\{-Rt - \frac{1}{2}||\phi||\,^2 t - \phi W(t)\}}$$

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$$= \left(e^{\left(Rt+\frac{1}{2}||\phi||^2 t+\phi W(t)\right)}\right)\left(xe^{-\left(\frac{\gamma+1}{\gamma}\right)\phi W(t)-\frac{1}{2}\left(\frac{\gamma+1}{\gamma}\right)^2 ||\phi||^{2} t}\right)$$

$$= xe^{\left(Rt-\frac{1}{2\gamma^2}(2\gamma+1)\phi^2 t-\frac{1}{\gamma}\phi W(t)\right)}$$

$$X^{\pi^*}(t) = xe^{\left(Rt-\frac{1}{2\gamma^2}(2\gamma+1)\left(\frac{\mu-R}{\sigma}\right)^2 t-\frac{1}{\gamma}\left(\frac{\mu-R}{\sigma}\right)W(t)\right)} \quad (5)$$

This is the optimal wealth process, and hence the optimal value process of the portfolio (since we have a self-financing portfolio with zero consumption). So, we can conclude our result in the following lemma.

Lemma 2.1.

For power utility given in equation (1), the optimal wealth process (optimal value process) of the portfolio is given by

$$X^{\pi^*}(t) = xe^{\left(Rt-\frac{1}{2\gamma^2}(2\gamma+1)\left(\frac{\mu-R}{\sigma}\right)^2 t-\frac{1}{\gamma}\left(\frac{\mu-R}{\sigma}\right)W(t)\right)} \quad (6)$$

2.2 Forecasting future stock returns

It is possible to create the paths that can be utilized to represent the stock market returns using the wealth process described in Lemma 2.1 above. The distribution of logarithmic returns is preferable for comparison and evaluation of investment performances even though the distributions of simple/arithmetic returns and logarithmic returns are very similar (Panna (2017)). To obtain the model parameters, we used the data set from BofA Merrill Lynch titled BofA Merrill Lynch Asia Emerging Markets Corporate plus Sub-Index Total Return Index Value (with series ID: BAMLEMRACRPIASIATRIV). The sample period is from December 1998 to November 2017. The forecast period starts 14 years after the beginning of the sample, i.e., in January 2013, and ends in November 2017.

Define the logarithmic returns by

$$\xi_t = \ln\left(\frac{S_{t+1}}{S_t}\right)$$
$$= \ln S_{t+1} - \ln S_t$$

Where,

ξ_t Is the today's stock market return, S_{t+1} is the yesterday's stock market closing price and S_t is the today's stock market closing price.

3. RESULTS AND DISCUSSION

The return and risk from the real data were calculated and discovered to be $\mu = 0.0060058717$ and$\sigma = 0.01710579$, respectively. These numbers were then used to forecast the stock market return for the following five years using the constructed model (i.e., equation

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model 6). The predicted return was determined to be $\mu_1 = 0.003615073$ (with an associated risk of $\sigma_1 = 0.006302927$), whereas the actual data showed a return $\mu_2 = 0.003406941$ (with an associated risk of $\sigma_2 = 0.009725064$). The percentage error between the predicted stock return and the actual stock return was found to be 6.11% (a very small percentage error) indicating that our model predicts almost the same value of the stock market return (there is substantial predictability in the stock market returns). From the result, we can conclude that our constructed model is an appropriate one as it produces high returns with less risk (with our constructed model, an investor could have timed the market and gained up to 6.11% over the course of five years).

Table 1: Parameters Used To Forecast Stock Market Returns

Parameter	Definition	Value
γ	Risk-aversion coefficient	0.5
μ	Average stock return	0.006005871
σ	Standard deviation (risk)	0.01710579
μ_1	Forecasted stock return for five years	0.003615073
σ_1	Standard deviation (risk)	0.006302927
μ_2	Actual stock return for five years	0.003406941
σ_2	Standard deviation (risk)	0.009725064
x	Initial capital	1000
R	Interest rate	0.01
PE	Percentage error	6.11%

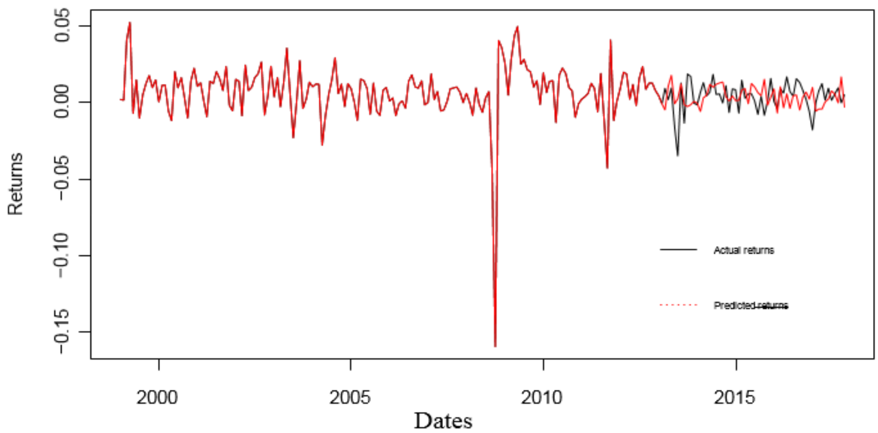

Figure 1: Forecasted returns vs actual returns from the year 2013

4. CONCLUSION

We apply our constructed model to forecast stock market returns. Our method leads to statistical and economic gains for an investor. From the results, we observed that there is substantial predictability in stock market returns, as with our constructed model, an investor could have timed the market and gained up to 6.11% over the course of five years.

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Availability of Data

The data used to support the findings of this study are included withing the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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