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A STUDY ON $\mathbf{M_X / (G_1, G_2) / 1}$ WITH RETRIAL, RENEGING AT THE TIME OF VACATION AND BREAKDOWN PERIODS

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Abstract

In this paper we study a two stage batch arrival queueing model and the server serves his service in a series. We deal with retrial to occur at the time of vacation and breakdown periods. We obtain the steady state equations to find the average queue length and average waiting time in the queue as well as in the system. In addition a few special cases are also considered and numerical illustrations are given to test the feasibility of the model.

Keywords: Batch Arrival; Vacation and Breakdowns; Retrial; Reneging; Steady State Queues.

INTRODUCTION

Queuing theory is the most essential tool in our daily life. Queuing is quite common in many fields, for example in telephone exchange, in a supermarket, at a petrol station, at computer systems etc,. In the beginning of 20th century, first problem of queuing theory was raised by Erlang. After his work, many researchers exited to deal with queuing problems using probabilistic methods. Many of its results have been used in operations research, computer science, telecommunication, traffic engineering and reliability theory etc.

Queuing models with server vacations are more realistic and flexible in studying real world waiting line systems. It has wide applications that include call centers with multi-task employees manufacturing, telecommunication networks etc. It was first introduced by Levy and Yechiali [16]. A few of previous works on vacation queues are analysed by the authors B. T. Doshi [3], William, Patrick and Meckkinley [1] and Choudhury Gautam, Kalita and Selvamuthu [2].

The arrival of customers may be groups or batches. Such a system is called batch arrival queuing system. It occurs in hotels, supermarkets, theaters, bank etc,. Batch arrival queue with multiple vacation policy was first studied by Baba. Y [4]. Later batch arrival with single vacation policy was studied by Choudhury [7, 8]. N. Kempa [13, 14] analysed about this batch arrival queuing systems with both multiple and single vacation policy.

Customers may lose their patience and leave the queue due to the length of queue. These types of queues are called impatient queues or queues with impatient customers. A few of early works on

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these impatient queues are discussed by the authors Barrer. D.Y [5], Feller W [10]. One of the earliest works on balking and reneging was done by Haight [9]. Another early work on reneging was done by Anker and Gafarian[6].

Retrial queues (queues with repeated attempts) have been widely used to model many problems arising in telephone switching systems, telecommunication networks, and computer networks. The detailed overviews of the related references with retrial queues can be found in the recent book of Falin and Templeton [12]. Here we consider on single retrial with batch arrivals in two stages of heterogeneous service. This first batch arrival retrial queuing model was introduced by Falin[11] with the rule " If the server is busy at the arrival epoch, then the whole batch joins the retrial group, whereas the server is free, then one of the arriving units starts its service and the rest join the retrial group". Recently these has been a fast development in the literature on retrial queues. Senthilkumar and Arumuganathan [15] have analysed the batch arrival single server retrial queue in which the server serves two phases of heterogeneous service and receives general vacation time under Bernoulli schedule.

In this paper we deal with a batch arrival queue and the service is provided in two stages, one by one in succession with Poisson arrival and general service distribution. We develop this model by including a new assumptions retrial and vacations. Customers retry for service after a period of time when server breakdowns or when the vacation time of the server. This is a very practical assumption and frequently we approach over such queuing systems in the real circumstances.

The Mathematical Description

Let λ be the Poisson arrival rate for batch arrival customers.

We assume retrial queues that a customer finds the server is busy or the server is on vacation then he leave the system and repeat his need after some time called "retrial time". At the time of trials the blocked customer joins a pool called "orbit". The intervals between successive repeated attempts are exponentially distributed with rate '$m\theta$'when the number of customers in the orbit is 'm'.Hence the total arrival rate to the system is $\lambda + m\theta$.

The first order probability of the batch arrival customers of size i at a small interval of time

$(x, x$ +dt] is $\lambda + m\theta h_i dt$ $(i = 1,2,3, \dots.)$ where $0 \leq h_i \leq 1$ and $\sum_{i=1}^{\infty} h_i = 1$.

The server gives two stages of different services one by one in succession. Batch arrival customers shall get the service at 2 stages one by one in succession, defined as the first stage (FS) and second stage (SS) respectively. The service discipline is supposed to be on a First in First out (FIFO or FCFS) basis.

Let the service time s_j $(j = 1,2)$ of the jth stage service follows general probability distribution. Then the distribution function and the probability density functions are denoted by $T_j(S_j)$ and $t_j(s_j)$ respectively. Similarly $E({S_j}^n)$ denotes the nth moment of the service time s_j $(j = 1,2)$.

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The conditional probability of stage j service at the interval $(x, x$ +dt] be $\mu_j(x)$, given elapsed time is x such that $\mu_j(x) = \frac{t_j(x)}{1-T_j(x)}, j = 1,2$ and

$t_j(s_j) = \mu_j(x) \exp[-\int_0^s \mu_j(x)dx], j = 1,2$

The server is assumed to take vacation with probability 'p' when the second stage service (SS) of a unit is complete or may continue to give service with probability (1-p). If the vacation period of the server is over then he joins the system to continue service of the waiting customers.

Let us consider the vacation period to be a random variable with distribution function G(r), the density function g(r) and the nth moment $E(R^n), (n = 1,2, \dots).$We consider that $\psi(x)$ be the conditional probability of a vacation period in the interval$(x, x$ +dt],given elapsed time is x, so that $\psi(x) = \frac{G(x)}{1-G(x)}$ and

$g(r) = \psi(r) exp[-\int_0^r \psi(x)dx]$

Here we assume reneging (leave the queue after joining) during vacation and breakdown periods and it is assumed to follow exponential distribution with parameter $\eta. i.e, f(t) = \eta e^{-\eta t}dt, \eta > 0$ and ηdt is the probability of a reneging customers at the interval (t,t+dt].

The customer getting service at breakdown returns back to the head of the queue when the system is breakdown at random. We consider that the interval between breakdowns occur according to a Poisson process with mean rate $\gamma > 0$.Then the repair time follows general distribution with distribution function $C(x)$ and density function$c(x)$. The corresponding conditional probability of completion of the repair process is $\omega(x)$, such that $\omega(x) = \frac{C(x)}{1-C(x)}$ and $C(k) = \eta(k) exp\left[-\int_0^k \eta(x)dx\right]$

Definitions and Notations

We consider the steady state occurs and define

$B_{n,j}(x)$ =Probability that there are 'n' ($n \geq 1$) customers in the system including one customer in type 'j' service, $j = 1,2$ and elapsed service time is x.

$\therefore B_{n,j} = \int_0^\infty B_{n,j}(x)dx$ is the corresponding steady state probability of irrespective of elapsed time x.

$K_n(x)$ =Probability that there are 'n' ($n \geq 0$)customers in the queue and the server is on vacation time x.

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$\therefore$ $K_n = \int_0^{\infty} K_n(x)dx$ is the corresponding steady state probability irrespective of elapsed vacation time x.

$Q_n(x)$ =Probability that there are 'n' ($n \geq 0$) customers in the queue and the server is under repairs since the elapsed service time.

$\therefore$ $Q_n = \int_0^{\infty} Q_n(x)dx$ is the corresponding steady probability irrespective of elapsed time x.

I =Steady state probability of the server is idle as the server takes vacation.

$B_j(x,y) = \sum_{n=1}^{\infty} y^n B_{n,j}(x)$ $\qquad$ $B_j(y) = \sum_{n=1}^{\infty} y^n B_{n,j}$ $\quad$ Where $|y| \leq 1, j = 1,2$

$Q(x,y) = \sum_{n=1}^{\infty} y^n Q_n(x)$ $\qquad$ $Q(y) = \sum_{n=1}^{\infty} y^n Q_n$ $\;$;$|y| \leq 1$

$K(x,y) = \sum_{n=1}^{\infty} y^n K_n(x)$ $\qquad$ $K(y) = \sum_{n=1}^{\infty} y^n K_n$ $\;$;$|y| \leq 1$ and $H(y) = \sum_{i=1}^{\infty} y^i h_i$

Equations Governing System

$\frac{d\, B_{n,1}(x)}{dx}+\{(\lambda+m\theta)+\mu_1(x)\}B_{n,1}(x) = (\lambda+m\theta)\sum_{i=1}^{n} h_i B_{n-i,1}(x), n \geq 1$-----1

$\frac{d\, B_{n,2}(x)}{dx}+\{(\lambda+m\theta)+\mu_2(x)\}B_{n,2}(x) = (\lambda+m\theta)\sum_{i=1}^{n} h_i B_{n-i,2}(x), n \geq 1$-----2

$\frac{d\, Q_n(x)}{dx}+\{(\lambda+m\theta)+\omega(x)+\eta\}Q_n(x) = (\lambda+m\theta)\sum_{i=1}^{\infty} h_i Q_{n-i}(x)+\eta Q_{n+1}, n \geq 1$ ----------3

$\frac{d\, R_0(x)}{dx}+\{(\lambda+m\theta)+\omega(x)\}Q_0(x) = \eta Q_0(x)$ ----------4

$\frac{d\, K_n(x)}{dx}+\{(\lambda+m\theta)+\psi(x)+\eta\}K_n(x) = (\lambda+m\theta)\sum_{i=1}^{n} h_i K_{n-i}(x)+\eta K_{n+1}, n \geq 1$ ----------5

$\frac{d\, K_0(x)}{dx}+\{(\lambda+m\theta)+\psi(x)\}K_0(x) = \eta K_0(x)$ ----------6

$(\lambda+m\theta)I = (1-p)\int_0^{\infty} B_{0,2}(x)\mu_2(x)dx + \int_0^{\infty} K_0(x)\psi(x)dx + \int_0^{\infty} Q_0(x)\omega(x)dx$-----------7

The boundary conditions are

$B_{n,1}(o) = (\lambda+m\theta)H_n I + (1-p)\int_0^{\infty} B_{n+1,2}(x)\mu_2(x)dx + \int_0^{\infty} Q_{n+1}(x)\omega(x)dx + \int_0^{\infty} K_n(x)\psi(x)dx, n \geq 1$ -----I

$B_{n,2}(o) = \int_0^{\infty} B_{n,1}(x)\mu_1(x)dx, n \geq 1$ -------------II

$K_n(0) = p\int_0^{\infty} B_{n+1,2}(x)\mu_2(x)dx, n \geq 0$ -----------III

$Q_{n+1}(0) = \gamma\int_0^{\infty} B_{n,1}(x)\, dx + \gamma\int_0^{\infty} B_{n,2}(x)\, dx$

$= \gamma B_{n,1} + \gamma B_{n,2}, n \geq 0$ ---------------IV

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$Q_0(0) = 0$-------------------V

Queue size distribution

Multiply the term r^n with the equations 1, 2, 3 and 4 and take the summation from 1 to ∞ then we get the following equations

$\frac{dB_1(x,y)}{dx} + \{(\lambda + m\theta) - (\lambda + m\theta)H(y) + \mu_1(x) + \gamma\}B_1(x,y) = 0$ ------------8

$\frac{dB_1(x,y)}{dx} + \{(\lambda + m\theta) - (\lambda + m\theta)H(y) + \mu_1(x) + \gamma\}B_1(x,y) = 0$ --------------9

$\frac{dQ(x,y)}{dx} + \left\{(\lambda + m\theta) - (\lambda + m\theta)H(y) + \omega(x) + \eta - \frac{\eta}{y}\right\}Q(x,y) = 0$ ------------10

$\frac{dK(x,y)}{dx} + \left\{(\lambda + m\theta) - (\lambda + m\theta)H(y) + \psi(x) + \eta - \frac{\eta}{y}\right\}Q(x,y) = 0$ --------------11

By integrating the above equations over the limits 0 to x we get,

$B_1(x,y) = B_1(0,y)exp\left[[(\lambda + m\theta) - (\lambda + m\theta)H(y) + \gamma]x - \int_0^\infty \mu_1(t)dt\right]$ ------12

$B_2(x,y) = B_2(0,y)exp\left[[(\lambda + m\theta) - (\lambda + m\theta)H(y) + \gamma]x - \int_0^\infty \mu_2(t)dt\right]$ ------13

$Q(x,y) = Q(0,y)exp\left[\left[(\lambda + m\theta) - (\lambda + m\theta)H(y) + \eta - \frac{\eta}{y}\right]x - \int_0^\infty \omega(t)dt\right]$ ----14

$K(x,y) = K(0,y)exp\left[\left[(\lambda + m\theta) - (\lambda + m\theta)H(y) + \eta - \frac{\eta}{y}\right]x - \int_0^\infty \psi(t)dt\right]$ ----15

By multiplying y^{n+1} with the boundary conditions, take the summation over n and using the probability generating functions (PGF), we get

$yB_1(o,y) = [(\lambda + m\theta)H(y) - (\lambda + m\theta)]I + (1-p)\int_0^\infty B_2(x,y)\,\mu_2(x)dx +$
$\int_0^\infty Q(x,y)\,\omega(x)dx + \int_0^\infty K(x,y)\,\psi(x)dx$ ------16

$B_2(0,y) = \int_0^\infty B_1(x,y)\,\mu_1(x)dx$ -------------------------------17

$yK(0,y) = p\int_0^\infty B_2(x,y)\,\mu_2(x)dx$ ----------------------------18

Multiply y^{n+1} with equation IV, take the summation from 0 to ∞ and using the equation V and PGF's, we get

$Q(0,y) = \gamma y[B_1(y) + B_2(y)]$ --------------------------------------19

Integrating the equations 12, 13, 14, and 15 with respect to x, we get

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$$B_1(y) = B_1(o,y)\left[\frac{1-F_1{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]}{[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]}\right] \text{------------------------}20$$

$$B_2(y) = B_2(o,y)\left[\frac{1-F_2{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]}{[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]}\right] \text{------------------------}21$$

$$K(y) = K(o,y)\left[\frac{1-V^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]}{\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]}\right] \text{-----------------------}22$$

$$Q(y) = Q(o,y)\left[\frac{1-U^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]}{\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]}\right] \text{-----------------------}23$$

Where

$$F_1{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma] = \int_0^\infty e^{-[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]x}\, dF_1(x)$$

$$F_2{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma] = \int_0^\infty e^{-[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]x}\, dF_2(x)$$

$$V^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right] = \int_0^\infty e^{-\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]x} dV(x)$$

$$U^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right] = \int_0^\infty e^{-\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]x} dU(x)$$

The above terms are the Laplace transform of the first stage service time, second stage service time, repair time and vacation time respectively.

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∴ The integrals are

$$\int_0^\infty B_1(x,y)\,\mu_1(x)dx$$

$$\int_0^\infty B_2(x,y)\,\mu_2(x)dx$$

$$\int_0^\infty \mathrm{Q}(x,y)\,\omega(x)dx$$

$$\int_0^\infty \mathrm{K}(x,y)\,\psi(x)dx$$

Integrate the equations 12, 13, 14 and 15 with respect to x by multiplying the corresponding terms $\mu_1(x), \mu_2(x), \omega(x)$ *and* $\psi(x)$ respectively. Then we get the following equations

$\int_0^\infty B_1(x,y)\,\mu_1(x)dx = \mathrm{B}_1\,(0,y)F_1{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]$**--------24**

$\int_0^\infty B_2(x,y)\,\mu_2(x)dx = \mathrm{B}_2\,(0,y)eF_2{}^*[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma]$**---------25**

$\mathrm{Q}(x,y) = \mathrm{Q}(0,y)V^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]$ **---------26**

$\mathrm{K}(x,y) = \mathrm{K}(0,y)U^*\left[(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y}\right]$ **-------------27**

Let

$$(\lambda+m\theta)-(\lambda+m\theta)H(y)+\gamma = b$$

$$(\lambda+m\theta)-(\lambda+m\theta)H(y)+\eta-\frac{\eta}{y} = l$$

By using the above equations in 16, 17 and 18, we get

$yB_1(o,y) = [(\lambda+m\theta)H(y)-(\lambda+m\theta)]\mathrm{I}+(1-\mathrm{p})F_2{}^*(b)B_2(0,y)+\mathrm{Q}(0,y)V^*(l)+ y\mathrm{K}(0,y)U^*(l)$-----------28

$B_2(0,y) = B_1(0,y)F_1{}^*(b)$-------------------------------29

$\mathrm{yK}(0,y) = pB_2(0,y)F_2{}^*(b)$----------------------------30

From the equations 29 and 30, we get

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$\mathrm{y}K(0,y) = pB_1(0,y)F_1^*(b)F_2^*(b)$----------------------31

Using 19, 20 and 21, we get

$Q(0,y) = \frac{\gamma y}{b}\left[B_1(0,y)[1 - F_1^*(b)] + B_2(0,y)[1 - F_2^*(b)]\right]$ ---------------32

To get $B_1(0,y)$ use the equations 29, 31 and 32 in 28, then

$B_1(0,y) = \frac{b[(\lambda+m\theta)H(y)-(\lambda+m\theta)]\mathrm{I}}{A(y)}$ ------------------------------33

Where

$A(y) = b[y - (1-p)F_1^*(b) - pF_1^*(b)F_2^*(b)U^*(l)] - \gamma y V^*(l)\ [1 - F_1^*(b)F_2^*(b)]$-------------34

$B_2(0,y) = \frac{b[(\lambda+m\theta)H(y)-(\lambda+m\theta)]F_1^*(b)\mathrm{I}}{A(y)}$ --------------------------------35

$K(0,y) = \frac{pb[(\lambda+m\theta)H(y)-(\lambda+m\theta)]F_1^*(b)F_2^*(b)\mathrm{I}}{A(y)}$ --------------------------------36

By substituting the above equations in 20, 21, 22 and 23, we get

$B_1(y) = \frac{[(\lambda+m\theta)H(y)-(\lambda+m\theta)][1-F_1^*(b)]\mathrm{I}}{A(y)}$ --------------------------------37

$B_2(y) = \frac{[(\lambda+m\theta)H(y)-(\lambda+m\theta)]F_1^*(b)[1-F_2^*(b)]\mathrm{I}}{A(y)}$ ------------------------38

$Q(y) = \frac{\gamma y[(\lambda+m\theta)H(y)-(\lambda+m\theta)][1-F_1^*(b)F_2^*(b)]\mathrm{I}}{A(y)}\left[\frac{1-V^*(l)}{l}\right]$ --------------------------39

$K(y) = \frac{pb[(\lambda+m\theta)H(y)-(\lambda+m\theta)]F_1^*(b)F_2^*(b)\mathrm{I}}{A(y)}\left[\frac{1-U^*(l)}{l}\right]$ -------------------------40

Let $B_q(y)$ denote the PGF of queue size irrespective of the state of the system

$$\therefore B_q(y) = B_1(y) + B_2(y) + Q(y) + K(y)$$

$B_q(y) = \frac{D(y)}{A(y)}$ --------------41

By using the normalizing condition $B_q(y) + I = 1$, we can find the probability of idle time I.

At $y = 1$, equation 38 is indeterminant form,so we use L'Hospital's Rule, then

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$$B_1(1) = \frac{(\lambda+m\theta)E(L)[1-F_1{}^*(\gamma)]\mathrm{I}}{-[(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)]+[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)-p\gamma[(\lambda+m\theta)E(L)-\eta]E(K)F_1{}^*(\gamma)F_2{}^*(\gamma)}$$ ----------------------42

This is the steady state probability that the server is providing service at stage 1.

Similarly, the steady state probability that the server is providing service at stage 2 is

$$B_2(1) = \frac{(\lambda+m\theta)E(L)F_1{}^*(\gamma)[1-F_2{}^*(\gamma)]\mathrm{I}}{-[(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)]+[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)-p\gamma[(\lambda+m\theta)E(L)-\eta]E(K)]F_1{}^*(\gamma)F_2{}^*(\gamma)}$$

-----------------------43

$$Q(1) = \frac{\gamma(\lambda+m\theta)E(L)E(Q)[1-F_1{}^*(\gamma)F_2{}^*(\gamma)]\mathrm{I}}{-[(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)]+[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)-p\gamma[(\lambda+m\theta)E(L)-\eta]E(K)]F_1{}^*(\gamma)F_2{}^*(\gamma)}$$

-------------------------44

$$K(1) = \frac{p\gamma(\lambda+m\theta)E(L)E(K)F_1{}^*(\gamma)F_2{}^*(\gamma)\mathrm{I}}{-[(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)]+[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta]E(Q)-p\gamma[(\lambda+m\theta)E(L)-\eta]E(K)]F_1{}^*(\gamma)F_2{}^*(\gamma)}$$

---------------------------45

Where H (1) =1,

The mean arriving batch of customers is $H'(1) = E(L)$,

The mean repair time is $-V^{*\prime}(1) = E(L)$ and

The mean vacation time $U^{*\prime}(0) = E(K)$.

Therefore,

$$I = 1 - \left[\frac{(\lambda+m\theta)E(L)\left[\left(1+\gamma E(Q)\right)-\left(1+\gamma E(Q)-p\gamma E(K)F_1{}^*(\gamma)F_2{}^*(\gamma)\right)\right]}{\gamma\eta E(Q)[1-F_1{}^*(\gamma)F_2{}^*(\gamma)]+p\gamma\eta F_1{}^*(\gamma)F_2{}^*(\gamma)}\right]$$ ---------------46

$$\rho = \left[\frac{(\lambda+m\theta)E(L)\left[\left(1+\gamma E(Q)\right)-\left(1+\gamma E(Q)-p\gamma E(K)\right)F_1{}^*(\gamma)F_2{}^*(\gamma)\right]}{\gamma\eta E(Q)[1-F_1{}^*(\gamma)F_2{}^*(\gamma)]+p\gamma\eta F_1{}^*(\gamma)F_2{}^*(\gamma)}\right]$$ ---------------------47

Mean length of Queue and mean waiting time

The mean queue size is indeterminate form at y=1, that is $L_q = \frac{dB_q(y)}{dy}\Big|y = 1$ then we apply L' Hospital's rule twice

$$L_q = \lim_{y=1} \frac{A'(y)D''(y)-D'(y)A''(y)}{2[A'(y)]^2}$$ ------------------------48

$$D'(1) = I\left[(\lambda+m\theta)E(L)\left(1+\gamma E(Q)\right)-(\lambda+m\theta)E(L)\left[\left(1+\gamma E(Q)-p\gamma E(K)\right)\right]F_1{}^*(\gamma)F_2{}^*(\gamma)\right]$$ ---------49

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$D''(1) = I[(\lambda + m\theta)E(L/L-1)[[1 - F_1{}^*(\gamma)F_2{}^*(\gamma)] + \gamma[1 - F_1{}^*(\gamma)F_2{}^*(\gamma)]E(Q) + p\gamma F_1{}^*(\gamma)F_2{}^*(\gamma)E(K)] - (\lambda + m\theta)E(L)[[1 + \gamma E(Q) - p\gamma E(K)]\left[F_1{}^{*'}(\gamma)F_2{}^*(\gamma) + F_2{}^{*'}(\gamma)F_1{}^*(\gamma)\right] + \gamma[(\lambda + m\theta)E(L) - \eta]E(Q^2)[1\text{-}F_1{}^*(\gamma)F_2{}^*(\gamma)] + [(\lambda + m\theta)E(L) - \eta]\, E(K^2)\, F_1{}^*(\gamma)F_2{}^*(\gamma)]]$ ----------------------50

$A'(1) = -[(\lambda + m\theta)E(L) + \gamma[(\lambda + m\theta)E(L) - \eta]E(Q)] + \left[(\lambda + m\theta)E(L) + \gamma[(\lambda + m\theta)E(L) - \eta]E(Q) - p\gamma\left[[(\lambda + m\theta)E(L) - \eta]E(K) + \gamma\right]F_1{}^*(\gamma)F_2{}^*(\gamma)\right]$ ----------------51

$A''(1) = -(\lambda + m\theta)E(L/L-1)\left[[1 + \gamma E(Q)] - F_1{}^*(\gamma)F_2{}^*(\gamma)[1 - \gamma E(Q) + p\gamma E(K)]\right] - 2\gamma\eta[1 - F_1{}^*(\gamma)F_2{}^*(\gamma)] + \gamma[(\lambda + m\theta)E(L) - \eta]^2E(Q^2)[1 - F_1{}^*(\gamma)F_2{}^*(\gamma)] - p\gamma[2\eta E(K) - [(\lambda + m\theta)E(L) - \eta]^2E(K^2)]F_1{}^*(\gamma)F_2{}^*(\gamma) + \left[(\lambda + m\theta)E(L) + \gamma[(\lambda + m\theta)E(L) - \eta]E(Q) - p\gamma\left[[(\lambda + m\theta)E(L) - \eta]E(K)\right]\right]\left[F_1{}^{*'}(\gamma)F_2{}^*(\gamma) + F_2{}^{*'}(\gamma)F_1{}^*(\gamma)\right]$------------------------52

Where

$E(Q^2)$, $E(K^2)$ are the second moments of repair time and vacation time respectively.

$E(L/L-1)$ is the second factorial moment of batch arrival customers.

By using the above equations we get the average queue length L_q and average waiting time $W_q = \frac{L_q}{\lambda + m\theta}$ .We can also find the average queue length of the system $L_s = L_q + \rho$ and the average waiting time in the system $W_s = \frac{L_s}{\lambda + m\theta}$.

<u>Special Cases</u>

<u>Case (i): No Retrial Queues</u>

$$B_1(y) = \frac{[\lambda H(y) - \lambda][1 - F_1{}^*(b)]\mathrm{I}}{A(y)}$$

$$B_2(y) = \frac{[\lambda H(y) - \lambda]F_1{}^*(b)[1 - F_2{}^*(b)]\mathrm{I}}{A(y)}$$

$$Q(y) = \frac{\gamma y[\lambda H(y) - \lambda][1 - F_1{}^*(b)F_2{}^*(b)]\mathrm{I}}{A(y)}\left[\frac{1 - V^*(l)}{l}\right]$$

$$K(y) = \frac{pb[\lambda H(y) - \lambda]F_1{}^*(b)F_2{}^*(b)\mathrm{I}}{A(y)}\left[\frac{1 - U^*(l)}{l}\right]$$

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Let $B_q(y)$ denote the PGF of queue size irrespective of the state of the system

$$\therefore B_q(y) = B_1(y) + B_2(y) + Q(y) + K(y)$$

$$B_q(y) = \frac{D(y)}{A(y)}$$

By using the normalizing condition $B_q(y) + I = 1$, we can find the probability of idle time I.

At $y = 1$, we get indeterminant form, so we use L'Hospital's Rule, then

$$I = 1 - \left[\frac{\lambda E(L)\left[\left(1 + \gamma E(Q)\right) - \left(1 + \gamma E(Q) - p\gamma E(K)\right)F_1^*(\gamma)F_2^*(\gamma)\right]}{\gamma\eta E(Q)[1 - F_1^*(\gamma)F_2^*(\gamma)] + p\gamma\eta F_1^*(\gamma)F_2^*(\gamma)}\right]$$

$$\rho = \left[\frac{\lambda E(L)\left[\left(1 + \gamma E(Q)\right) - \left(1 + \gamma E(Q) - p\gamma E(K)\right)F_1^*(\gamma)F_2^*(\gamma)\right]}{\gamma\eta E(Q)[1 - F_1^*(\gamma)F_2^*(\gamma)] + p\gamma\eta F_1^*(\gamma)F_2^*(\gamma)}\right]$$

The mean queue size is indeterminate form at y=1, that is $L_q = \frac{dB_q(y)}{dy}\Big|y = 1$ then we apply L' Hospital's rule twice

$$L_q = \lim_{y=1} \frac{A'(y)D''(y) - D'(y)A''(y)}{2[A'(y)]^2}$$

$$D'(1) = I\left[\lambda E(L)\left(1 + \gamma E(Q)\right) - \lambda E(L)\left[\left(1 + \gamma E(Q) - p\gamma E(K)\right)\right]F_1^*(\gamma)F_2^*(\gamma)\right]$$

$$D''(1) = I[\lambda E(L/L-1)[[1 - F_1^*(\gamma)F_2^*(\gamma)] + \gamma[1 - F_1^*(\gamma)F_2^*(\gamma)]E(Q) + p\gamma F_1^*(\gamma)F_2^*(\gamma)E(K)] - \lambda E(L)[[1 + \gamma E(Q) - p\gamma E(K)]\left[F_1^{*\prime}(\gamma)F_2^*(\gamma) + F_2^{*\prime}(\gamma)F_1^*(\gamma)\right] + \gamma[\lambda E(L) - \eta]E(Q^2)[1 - F_1^*(\gamma)F_2^*(\gamma)] + [\lambda E(L) - \eta]\, E(K^2)\, F_1^*(\gamma)F_2^*(\gamma)]]$$

$$A'(1) = -[\lambda E(L) + \gamma[\lambda E(L) - \eta]E(Q)] + \left[\lambda E(L) + \gamma[\lambda E(L) - \eta]E(Q) - p\gamma\left[[\lambda E(L) - \eta]E(K) + \gamma\right]F_1^*(\gamma)F_2^*(\gamma)\right]$$

$$\begin{aligned} A''(1) = {} & -\lambda E(L/L-1)\left[[1 + \gamma E(Q)] - F_1^*(\gamma)F_2^*(\gamma)[1 - \gamma E(Q) + p\gamma E(K)]\right] \\ & - 2\gamma\eta[1 - F_1^*(\gamma)F_2^*(\gamma)] + \gamma[\lambda E(L) - \eta]^2 E(Q^2)[1 - F_1^*(\gamma)F_2^*(\gamma)] \\ & - p\gamma[2\eta E(K) - [\lambda E(L) - \eta]^2 E(K^2)]F_1^*(\gamma)F_2^*(\gamma) \\ & + \Big[\lambda E(L) + \gamma[\lambda E(L) - \eta]E(Q) \\ & - p\gamma\left[[(\lambda + m\theta)E(L) - \eta]E(K)\right]\Big]\left[F_1^{*\prime}(\gamma)F_2^*(\gamma) + F_2^{*\prime}(\gamma)F_1^*(\gamma)\right] \end{aligned}$$

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We get the average queue length L_q and average waiting time $W_q = \frac{L_q}{\lambda}$ .We can also find the average queue length of the system $L_s = L_q + \rho$ and the average waiting time in the system $W_s = \frac{L_s}{\lambda}$.

Case (ii): No Server Vacations

In this case we get K(y)=0 and also p=0. Therefore we get the following results

$$B_1(y) = \frac{[(\lambda + m\theta)H(y) - (\lambda + m\theta)][1 - {F_1}^*(b)]\mathrm{I}}{A(y)}$$

$$B_2(y) = \frac{[(\lambda + m\theta)H(y) - (\lambda + m\theta)]{F_1}^*(b)[1 - {F_2}^*(b)]\mathrm{I}}{A(y)}$$

$$Q(y) = \frac{\gamma y[(\lambda + m\theta)H(y) - (\lambda + m\theta)][1 - {F_1}^*(b){F_2}^*(b)]\mathrm{I}}{A(y)}\left[\frac{1 - V^*(l)}{l}\right]$$

And

$$A(y) = b[y - {F_1}^*(b){F_2}^*(b)] - \gamma y V^*(l){F_1}^*(b){F_2}^*(b)$$

$$I = 1 - \left[\frac{(\lambda + m\theta)E(L)\big[\big(1 + \gamma E(Q)\big) - \big(1 + \gamma E(Q)\big){F_1}^*(\gamma){F_2}^*(\gamma)\big]}{\gamma\eta E(Q)[1 - {F_1}^*(\gamma){F_2}^*(\gamma)]}\right]$$

$$\rho = \left[\frac{(\lambda + m\theta)E(L)\big[\big(1 + \gamma E(Q)\big) - \big(1 + \gamma E(Q)\big){F_1}^*(\gamma){F_2}^*(\gamma)\big]}{\gamma\eta E(Q)[1 - {F_1}^*(\gamma){F_2}^*(\gamma)]}\right]$$

We can also find the mean queue size L_q and mean waiting time W_q by using the above results.

Case (iii): No Reneging and no system breakdowns

In this case we have $\gamma = 0 \text{ } and \text{ } \eta = 0$ and also $b = l = (\lambda + m\theta) - (\lambda + m\theta)H(y)$.

Therefore

$$B_1(y) = \frac{[{F_1}^*(b) - 1]\mathrm{I}}{A(y)}$$

$$B_2(y) = \frac{{F_1}^*(b)[{F_2}^*(b) - 1]\mathrm{I}}{A(y)}$$

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$$K(y) = \frac{pF_1{}^*(b)F_2{}^*(b)\mathrm{I}}{A(y)}[U^*(b) - 1]$$

$$A(y) = y - [(1-p) + pU^*(b)]F_1{}^*(b)F_2{}^*(b)$$

The PGF of queue size is

$$B_q(y) = \frac{[(1-p) + \ pU^*(b)]F_1{}^*(b)F_2{}^*(b)}{A(y)}$$

The probability of idle time I is

$I = 1 - (\lambda + m\theta)E(L)[E(S_1) + E(S_2) + pE(K)]$ and

$\rho = (\lambda + m\theta)E(L)[E(S_1) + E(S_2) + pE(K)] < 1$

Hence the system is reduced to a two stage batch arrival vacation queue.

Case (iv): Service and Vacation time are exponential

In this case we consider exponential service time and exponential vacation time. The service rates are $\mu_{1>0}, \mu_{2>0}$.

Similarly the repair and vacation rates are $\omega > 0, \psi > 0$.

Hence

$F_1{}^*(b) = \frac{\mu_1}{\mu_1+b}$ $\qquad$ $F_2{}^*(b) = \frac{\mu_2}{\mu_2+b}$

$V^*(l) = \frac{\omega}{\omega+l}$ $\qquad$ $U^*(l) = \frac{\psi}{\psi+l}$

$E(Q) = \frac{1}{\omega}$ $\qquad$ $E(Q^2) = \frac{2}{\omega^2}$

$E(K) = \frac{1}{\psi}$ $\qquad$ $E(K^2) = \frac{2}{\psi^2}$

Where $b = (\lambda + m\theta) - (\lambda + m\theta)H(y) + \gamma$

$l = (\lambda + m\theta) - (\lambda + m\theta)H(y) + \eta - \frac{\eta}{y}$

Then, we get

$$B_1(y) = \frac{[(\lambda + m\theta)H(y) - (\lambda + m\theta)]\left[1 - \frac{\mu_1}{\mu_1 + b}\right]\mathrm{I}}{A(y)}$$

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$$B_2(y) = \frac{[(\lambda + m\theta)H(y) - (\lambda + m\theta)] \frac{\mu_1}{\mu_1 + b} [1 - \frac{\mu_2}{\mu_2 + b}] \mathrm{I}}{A(y)}$$

$$Q(y) = \frac{\gamma[(\lambda + m\theta)H(y) - (\lambda + m\theta)] \left[1 - \frac{\mu_1}{\mu_1 + b} \frac{\mu_2}{\mu_2 + b}\right] \frac{1}{\omega + l} \mathrm{I}}{A(y)}$$

$$K(y) = \frac{pb[(\lambda + m\theta)H(y) - (\lambda + m\theta)] \frac{\mu_1 \mu_2}{(\mu_1 + b)(\mu_2 + b)(\psi + l)} \mathrm{I}}{A(y)}$$

and

$$A(y) = [(\lambda + m\theta)H(y) - (\lambda + m\theta)] \left[y - [(1 - p) + p \frac{\psi}{\psi + l}] \frac{\mu_1}{\mu_1 + b} \frac{\mu_2}{\mu_2 + b}\right] - \gamma y \left[1 - \frac{\mu_1}{\mu_1 + b} \frac{\mu_2}{\mu_2 + b}\right] \frac{\omega}{\omega + l}$$

$$B_1(1) = \frac{(\lambda+m\theta)E(L)1-\frac{\mu_1}{\mu_1+\gamma}\mathrm{I}}{-\left[(\lambda+m\theta)E(L)+\frac{\gamma[(\lambda+m\theta)E(L)-\eta]}{\omega}\right]+\left[[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta][\frac{1}{\omega}-\frac{p}{\psi}]\right][\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}]}$$

This is the steady state probability that the server is providing service at stage 1.

Similarly, the steady state probability that the server is providing service at stage 2 is

$$B_2(1) = \frac{(\lambda+m\theta)E(L)\frac{\mu_1}{\mu_1+\gamma}\left[1-\frac{\mu_2}{\mu_2+\gamma}\right]\mathrm{I}}{-\left[(\lambda+m\theta)E(L)+\frac{\gamma[(\lambda+m\theta)E(L)-\eta]}{\omega}\right]+\left[[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta][\frac{1}{\omega}-\frac{p}{\psi}]\right][\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}]}$$

$$Q(1) = \frac{\frac{\gamma(\lambda+m\theta)E(L)}{\omega}\left[1-\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right]\mathrm{I}}{-\left[(\lambda+m\theta)E(L)+\frac{\gamma[(\lambda+m\theta)E(L)-\eta]}{\omega}\right]+\left[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta][\frac{1}{\omega}-\frac{p}{\psi}]\right][\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}]}$$

This is the probability that the system is under repair.

$$K(1) = \frac{\frac{p\gamma(\lambda+m\theta)E(L)}{\psi}\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\mathrm{I}}{-\left[(\lambda+m\theta)E(L)+\frac{\gamma[(\lambda+m\theta)E(L)-\eta]}{\omega}\right]+\left[\gamma+(\lambda+m\theta)E(L)+\gamma[(\lambda+m\theta)E(L)-\eta][\frac{1}{\omega}-\frac{p}{\psi}]\right][\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}]}$$

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This is the probability that the server is on vacation.

Idle time probability

$$I = 1 - (\lambda + m\theta)E(L)\frac{\left[1+\frac{\gamma}{\omega}\right] - \left[1+\frac{\gamma}{\omega}-\frac{p\gamma}{\psi}\right]\left[\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right]}{\frac{\gamma\eta}{\omega}\left[1-\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right] + p\gamma\eta\left[\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right]}$$

We can also find the average queue length and average waiting time by using

$L_q = \lim_{y=1}\frac{A'(y)D''(y)-D'(y)A''(y)}{2[A'(y)]^2}$ and $W_q = \frac{L_q}{\lambda+m\theta}$

$$D'(1) = I\left[(\lambda+m\theta)E(L)\left(1+\frac{\gamma}{\omega}\right) - (\lambda+m\theta)E(L)\left[1+\frac{\gamma}{\omega}-\frac{p\gamma}{\psi}\right]\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right]$$

$$\begin{aligned} D''(1) = I\Bigg[&(\lambda+m\theta)E(L/L-1)\left[\left[1+\frac{\gamma}{\omega}\right]\left[1-\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right]+\frac{p\gamma}{\psi}\frac{\mu_1}{\mu_1+\gamma}\frac{\mu_2}{\mu_2+\gamma}\right] \\ &- 2(\lambda+m\theta)E(L)\Bigg[\left[\frac{\gamma(\lambda+m\theta)E(L)-\eta}{\omega^2}\right]\left[1-\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}\right] \\ &+ \frac{p\gamma(\lambda+m\theta)E(L)-\eta}{\psi^2}\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}\Bigg]\Bigg] \end{aligned}$$

$A'(1) = -\left[(\lambda+m\theta)E(L) + \left[\frac{\gamma(\lambda+m\theta)E(L)-\eta}{\omega}\right]\right] + \left[\gamma + (\lambda+m\theta)E(L) + \left[\frac{\gamma(\lambda+m\theta)E(L)-\eta}{\omega}\right] - \frac{p\gamma[[(\lambda+m\theta)E(L)-\eta]+\gamma]}{\psi}\right]\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}$

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$$A''(1) = -(\lambda + m\theta)E(L/L-1)\left[\left[1+\frac{\gamma}{\omega}\right]+\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}\left[1-\frac{\gamma}{\omega}+\frac{p\gamma}{\psi}\right]\right]$$
$$-2\left[\frac{\gamma\eta}{\omega}+\frac{[(\lambda+m\theta)E(L)-\eta]^2}{\omega^2}\right]\left[1-\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}\right]$$
$$+-2p\gamma\left[\frac{\eta}{\psi}-\frac{[(\lambda+m\theta)E(L)-\eta]^2}{\psi^2}\right]\frac{\mu_1\mu_2}{\mu_1+\gamma\mu_2+\gamma}$$
$$-\left[(\lambda+m\theta)E(L)+\frac{\gamma[(\lambda+m\theta)E(L)-\eta]}{\omega}-\frac{p\gamma[(\lambda+m\theta)E(L)-\eta]}{\psi}\right.$$
$$\left.+\gamma\right]\left[\frac{\mu_1\mu_2}{(\mu_1+\gamma)^2(\mu_2+\gamma)}+\frac{\mu_1\mu_2}{(\mu_1+\gamma)(\mu_2+\gamma)^2}\right]$$

NUMERICAL ILLUSTRATION

To identify the effect of distinct parameters especially retrial, reneging and breakdown on the different states of the server, the utilization factor and the proportion of idle time, we measure some numerical results. Here we consider the service time, vacation time and repair time are exponentially distributed for the feasibility of our results and the arrivals come one by one. So we get $E(L) = 1$ and $E(L/L-1) = 0$ with the arrival rate $\lambda + m\theta = 2$, $\mu_1 = 3$ and $\mu_2 = 7$.All the parameters are selected to satisfy the steady state conditions. Further we take $\omega = 9, \psi = 8$ while η takes the values 4, 7 and 8 and γ varies from 1 to 4.

Table 1: Effect of the parameters

λ+mΘ	γ	η	I	ρ	Lq	Ls	Wq=Lq/λ+mΘ	Ws=Ls/λ+mΘ
2	1	4	0.5878	0.4122	3.4882	3.9005	1.744119572	1.950236593
2	2	4	0.6136	0.3864	3.1408	3.5272	1.570392594	1.76361026
2	3	4	0.6324	0.3676	3.1678	3.5354	1.583891981	1.76771551
2	4	4	0.6455	0.3545	3.1792	3.5337	1.589607155	1.766845961
2	1	7	0.3359	0.6641	2.5964	3.2605	1.298207223	1.630274092
2	2	7	0.3506	0.6494	1.9515	2.6009	0.975747242	1.300443051
2	3	7	0.3613	0.6387	1.6756	2.3143	0.837798957	1.157126688
2	4	7	0.3689	0.6311	1.4980	2.1291	0.74898666	1.064551692
2	1	8	0.2939	0.7061	2.4082	3.1143	1.204094256	1.557152767
2	2	8	0.3068	0.6932	1.7662	2.4594	0.883086863	1.229695696
2	3	8	0.3162	0.6838	1.4862	2.1701	0.743124738	1.085036502
2	4	8	0.3228	0.6772	1.3118	1.9890	0.655889407	0.99450881

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Figure 1: Comparison of mean queue lengths Ls and Lq

Effect of $\eta = 4$ and γ varies from 1 to 4 on the mean queue size Ls and Lq

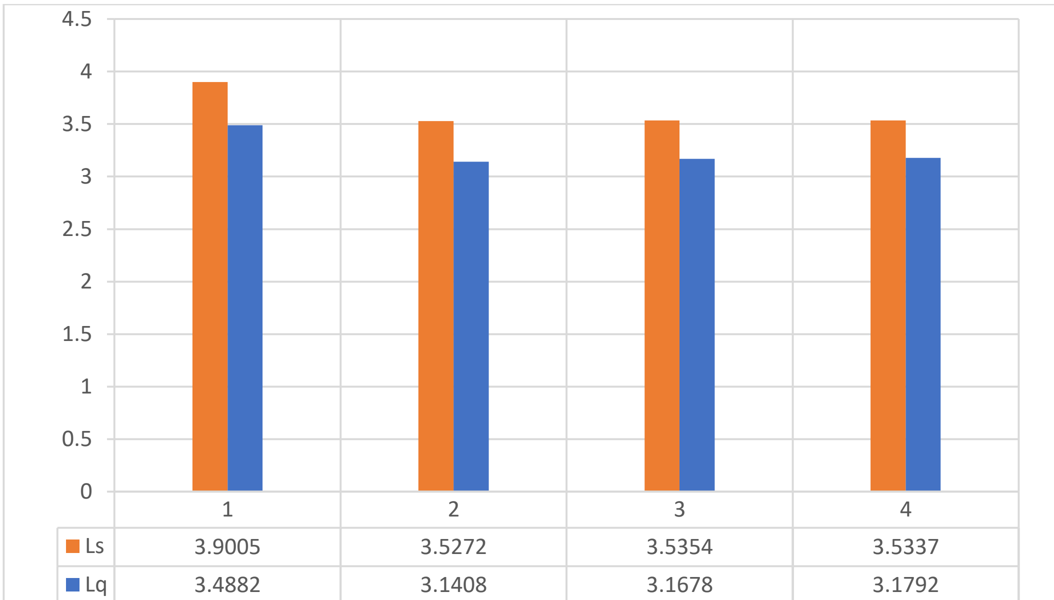

Figure 2: Comparison mean waiting times Ws and Wq

Effect of $\eta = 7$ and γ varies from 1 to 4 on the mean queue size Ws and Wq

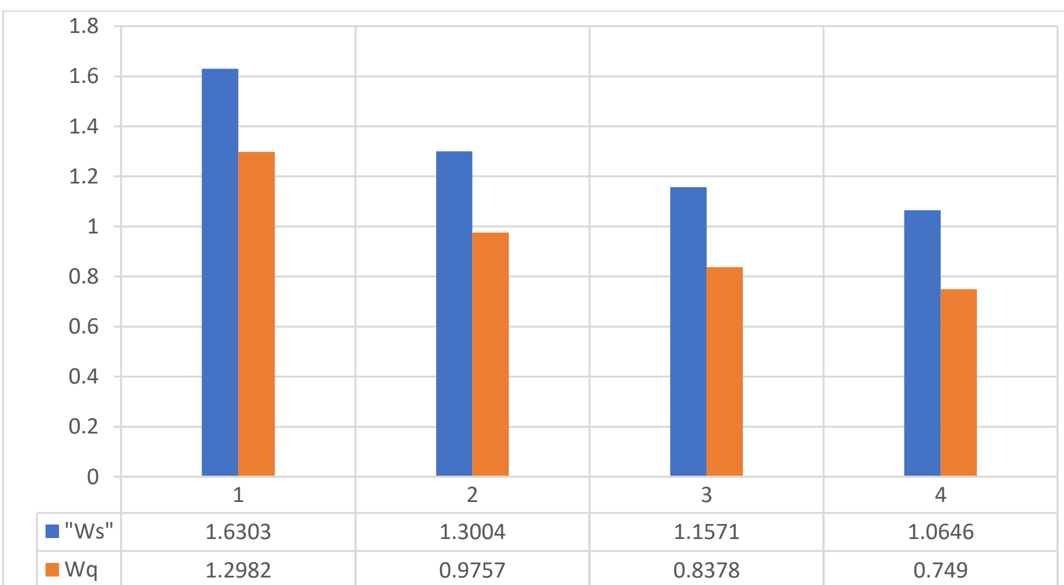

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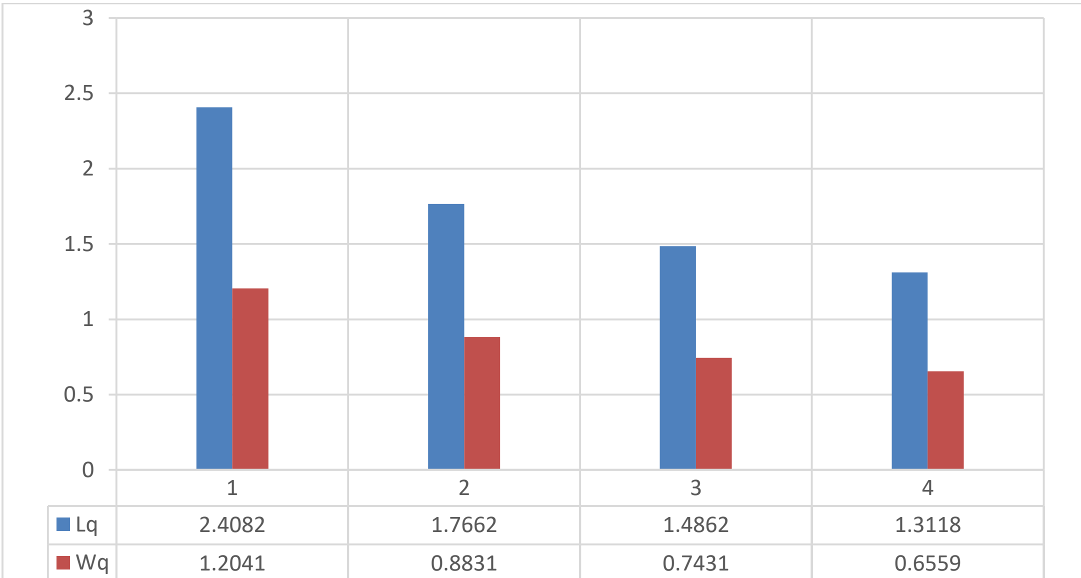
Figure 3: Comparison of mean queue length Lq and mean waiting time Wq

Effect of $\eta = 8$ and γ varies from 1 to 4 on the mean queue size Lq and Wq



From Table: 1, we identify that the utilization factor increases when the increase of the reneging parameter η, with the range values of breakdown parameter γ. At the same time the probability of idle time of the server decreases. Figure 1 and 2 show the comparison of mean queue lengths Ls and Lq and mean waiting times Ws and Wq. Figure 3 is the effect of retrial, reneging and breakdown period. It also indicates that the probability of the idle time of the server decreases and the utilization factor increases due to the retrial, reneging and breakdown of the system.

Table 2: Effectiveness of the arrival rate

$\lambda + m\theta$	L_s	L_q	w_s	w_q
1.5	1.6301	1.0709	1.0867	0.7139
1.6	1.8408	1.3110	1.1505	0.8194
1.7	2.1245	1.6241	1.2497	0.9554
1.8	2.5158	2.0448	1.3977	1.1360
1.9	3.0728	2.6312	1.6173	1.3849
2	3.9005	3.4882	1.9502	1.7441

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Figure 4: Comparison of Lq and Wq

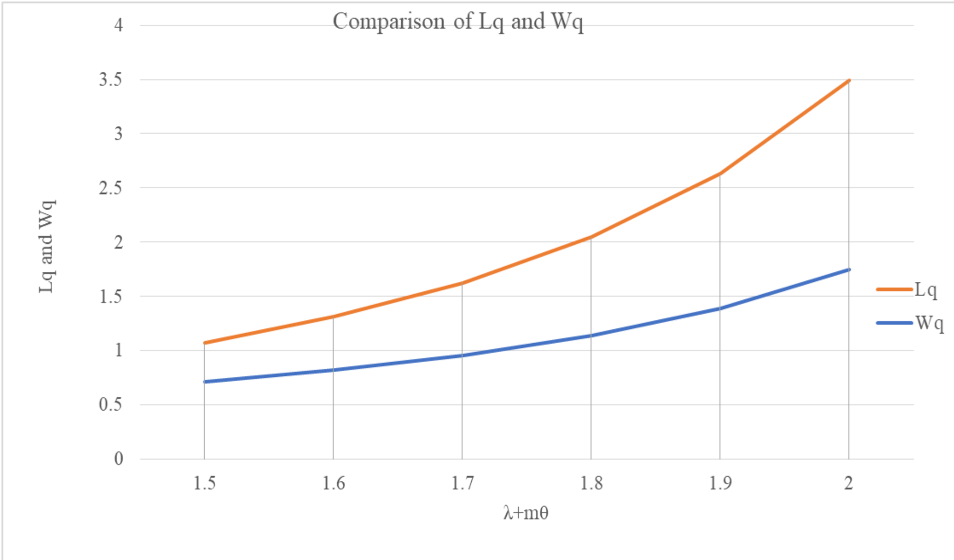

Figure 5: Comparison of Ls and Ws

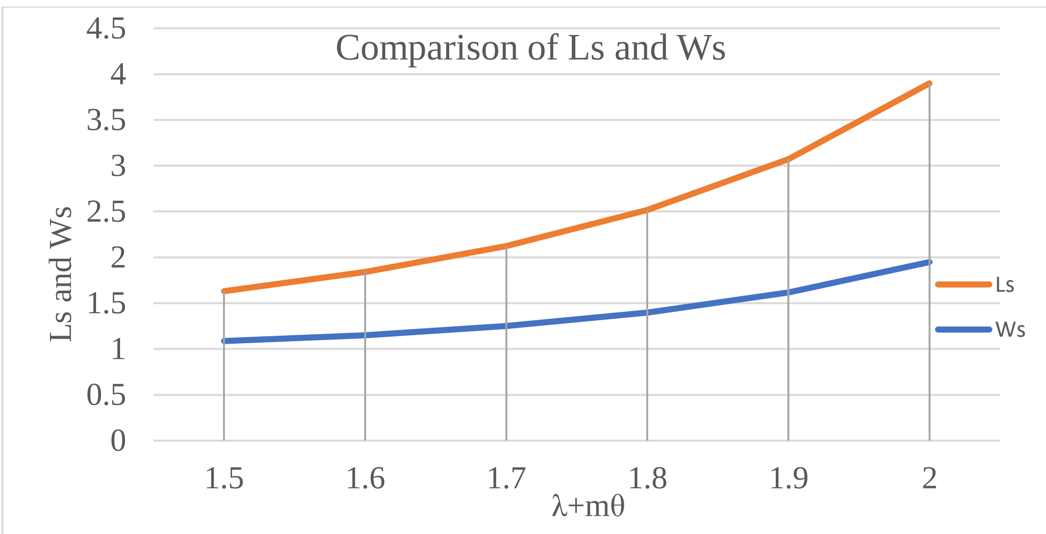

Table 2 shows that the mean queue length increases while the mean waiting time decreases due to the increase of arrival rate. Figure 4 is the comparison of mean queue length Lq and mean waiting time in the queue Wq. Figure 5 is the comparison of the average queue length in the system Ls and the average waiting time in the system Ws.

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CONCLUSION

In this article, we studied two stage batch arrivals with single server queuing model with retrial, reneging during vacation and breakdown periods. The supplementary variable technique was used for the derivation of probability generating function of the states under the steady state condition. We analyzed the effects of retrial, reneging, server vacation and the breakdowns and the mean queue length, mean waiting time are also calculated. The numerical illustrations are presented to test the correctness of the model. It shows, the results are coinciding with previous study.

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