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MODELING AND FORECASTING RISK IN ISLAMIC STOCK MARKETS: A COMPREHENSIVE ANALYSIS OF ONE-DAY-AHEAD VaR AND ES USING GARCH MODELS

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Abstract

This paper evaluates the one-day-ahead Value at Risk (VaR) and Expected Shortfall (ES) of two Islamic stock indexes, namely Dow Jones and FTSE. The analysis takes into consideration the presence of volatility clustering, volatility asymmetry, and volatility persistence in the data. Four GARCH-type models, including two fractionally integrated models, were assessed, assuming three alternative distributions (normal, Student-t, and skewed Student-t distributions). The paper considered four GARCH-type models, and the AR (1) - FIEGARCH model under a skewed Student-t distribution was found to perform the best among them. We have computed one-day ahead VaR and (ES) for both short and long trading positions. Back testing results show very clearly that the skewed Student-t FIEGARCH model provides the best results for both short and long VaR estimations.

Keywords: Long-Range Memory; Value at Risk; Asymmetry; Fat Tail.

JEL Classification: G11; G12; G15

1. INTRODUCTION

The significance of managing risk has grown exponentially for both managers and financial decision-makers. Making informed decisions at the opportune moment necessitates a deep understanding of financial market dynamics. Investors must keep abreast of market evolution, identify risk factors, and employ effective risk measurement strategies to safeguard against potential losses. A robust risk measurement model should inherently account for the stylized facts of financial assets. Value-at-Risk (VaR) has emerged as a widely adopted financial risk measure, quantifying the potential monetary loss within a specific holding period. VaR's straightforward methodology empowers managers and investors to govern their portfolio risk effectively, enabling optimal decision-making and the formulation of appropriate risk management policies. Consequently, VaR has garnered considerable interest from investors, portfolio managers, and financial institution supervisors, as underscored by the Basle Committee on Banking Supervision in 1996. Researchers and practitioners (Dimson and Marsh, 1995, 1997; Cordell and King, 1995; Gjerde and Semmen, 1995; Berger et al., 1995) recognize the value of VaR in determining the amount of resources institutions need to allocate as a guarantee against their risk exposure. The literature features a plethora of VaR methods, emphasizing the importance of evaluating and selecting the most suitable one based on specific portfolio characteristics. An effective risk measure model must consider the stylized facts of financial asset series, including volatility clustering, fat tails, skewness, and long-range memory phenomena. Since the introduction of the ARCH model by Engle in 1982, numerous

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GARCH-type models have been developed to address the volatility clustering phenomenon. These models enable the forecasting of future variance values by incorporating past squared deviations and variance values. Recent empirical studies (Mabrouk 2017, Mabrouk and Aloui, 2010; Aloui, 2008; Angelidis et al., 2007; Bali and Theodossiou, 2007; Degiannakis, 2004; Kang and Yoon, 2007; Marzo and Zagalia, 2007; So and Yu, 2006; Sriananthakumar and Silvapulle, 2003; Tang and Shieh, 2006; Wu and Shieh, 2007; Mabrouk 2016) assert that financial time series data exhibit long-range memory in variance behavior, fat tails, and skewness. In light of these findings, selecting an appropriate VaR model necessitates a correct specification of the chosen GARCH-type model. The studies conducted by Bouoiyour and Selmi (2016); Katsiampa (2017), as well as Charle and Darne-Lemna (2018), involve comparing certain GARCH-type models without simultaneously examining the relevant properties of long memory and asymmetric effects associated with financial asset return series. Conversely, Baur and Dimpfl (2018); Charfeddine and Maouchi (2018); Peng et al. (2018); and Caporale and Zekokh (2019) attempted to select the most appropriate model or a superior set of GARCH volatility models for examining various financial assets but failed to address properties relevant to both long memory and asymmetric effects with different error distributions. In this current work, our aim is to compare four models within the GARCH class, taking into consideration both leverage effects and long memory.

The recognition of financial data characteristics, as highlighted by the literature, underscores the importance of utilizing these models for accurate risk measurement and management. In this study, the focus is on exploring the dynamics of two Islamic stock indexes by employing Value-at-Risk (VaR) and Expected Shortfall (ES) calculations based on GARCH-type models. The researchers delve into assessing four alternative GARCH-type models, including two fractionary integrated models, with the specific aim of determining whether a more accurate estimation of one-day ahead VaR and ES can be achieved when time series exhibit long memory in the variance dynamics. The paper draws on prior empirical studies and takes into consideration three distributions: normal, Student-t, and skewed Student-t distributions. The latter two distributions account for various stylized facts in financial time series data behavior, such as excess kurtosis, heavy tails, and skewness. The study ultimately computes VaR and ES for both short and long trading positions, evaluating their performance across in-sample and out-of-sample periods. By incorporating GARCH-type models and alternative distributions, the researchers aim to provide insights into effective risk management strategies tailored to the characteristics of Islamic stock indexes. Finally, we computed the VaR and ES for both short and long trading positions then we assess their performance for both in-sample and out-of-sample periods.

The subsequent sections of our paper are organized as follows. In Section 2, we outline the four GARCH-type models utilized in our study and elucidate the error's density models, encompassing normal, Student-t, and skewed Student-t distributions. Section 3 is dedicated to introducing the Value-at-Risk (VaR) model, detailing how it can be computed using the specified GARCH-type models, and highlighting the statistical accuracy of VaR estimations derived from these models. Empirical findings are expounded in sections 4 and 5, shedding light on the practical application of the proposed models. Finally, Section 6 provides a

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conclusion, summarizing the key insights gleaned from our study. This structure allows for a comprehensive exploration of the GARCH-type models, their associated error's density models, and the efficacy of VaR estimations, culminating in a cohesive understanding of the dynamics governing the Islamic stock indexes under consideration.

2. THE GARCH-TYPE MODELS

2.1 GARCH Model

The GARCH model, an extension of the ARCH model proposed by Engle (1982) and generalized by Bollerslev (1986), provides a powerful framework for modeling the volatility of financial time series. The Generalized ARCH (GARCH) model is conceptualized as an infinite ARCH, offering a streamlined approach to diminish the number of parameters involved in the ARCH model. The GARCH (p, q) model is mathematically expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_j \, \sigma_{t-j}^2 \qquad (1)$$

The lag operator allows us to specify GARCH model as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (2)$$

Where:

- σ_t^2 represents the conditional variance at time t,
- ω is the constant term,
- α_i and β_j are the model parameters,
- ϵ_{t-i}^2 denotes the squared innovation at time $t - i$, and
- σ_{t-j}^2 Is the conditional variance at time$t - j$.

This formulation allows for a flexible representation of volatility dynamics by considering the impact of past squared innovations and past conditional variances on the current conditional variance. The GARCH model serves as a fundamental building block for more intricate GARCH-type models, contributing significantly to the modeling of financial market volatility. Bollerslev (1986) has shown that the GARCH model is a short memory model since its autocorrelation function decay slowly with a hyperbolic rate.

2.2 The Exponential GARCH Model

The exponential GARCH (EGARCH) is one model that allows for the asymmetric effect of news. Under EGARCH, the logged conditional variance is a function of its own lagged values as well as of the error terms. The EGARCH model has been developed by Nelson (1991) to capture the leverage effects in the volatility. By leverage effect we mean that the falling returns adds to the volatility in the market in comparison to the positive returns. The decreasing returns

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reduces the equity value which further increases the volatility in the equity markets, as studied by Black (1976).

$$logh_t = \alpha_0 + \alpha_1 \left(\frac{|\varepsilon_{t-1}|}{h_{t-1}} - \sqrt{\frac{2}{\pi}} \right) + \delta \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 h_{t-1} + \lambda \varepsilon_{t-1}^2 \quad (3)$$

Where δ is an asymmetry coefficient and the presence of leverage effect will be there when $\delta < 0$ and found to be significant. The α_1 and β_1 are the ARCH terms and the GARCH terms respectively, where the ARCH term exhibits the impact of the news or information on the conditional volatility and the GARCH term exhibits the persistency level in the volatility.

2.3 The Fractional Integrated GARCH Model

Given the prevalent observation that financial time series often exhibit a long memory process in variance dynamics, Baillie, Bollerslev, and Mikkelsen (1996) introduced the Fractional Integrated GARCH (FIGARCH) model to aptly capture this characteristic. This GARCH-type model introduces a fractional parameter d that enables the model to differentiate between short memory and infinite long memory processes. The FIGARCH model proves particularly adept at discerning the varying degrees of memory in conditional variance behavior. Formally, the $\mathrm{FIGARCH}(p, d, q)$ process is specified as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left(\frac{i-1}{i} \right)^d \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4)$$

Where:

- σ_t^2 is the conditional variance at time t,
- ω represents the constant term,
- α_i and β_j are the model parameters,
- ϵ_{t-i}^2 denotes the squared innovation at time $t-i$,
- σ_{t-j}^2 is the conditional variance at time $t-j$, and
- d is the fractional parameter that characterizes the degree of long memory in the process.

This formulation allows the FIGARCH model to effectively capture the nuanced aspects of both short and long memory in the conditional variance behavior, making it a valuable tool for modeling financial time series with distinct memory patterns.

2.4 The Fractional Integrated Exponential GARCH Model

Introduced by Bollerslev and Mikkelsen (1996), the Fractional Integrated Exponential GARCH (FIEGARCH) model is designed to provide a nuanced representation of volatility dynamics, incorporating both fractional integration and exponential smoothing.

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The FIEGARCH (p, d, q) model is expressed as follows:

$$\sigma_t^2 = \omega e^{\left(\sum_{i=1}^{p} \alpha_i \left(\frac{i-1}{i}\right)^d \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2\right)} \quad (5)$$

Where:

- σ_t^2 denotes the conditional variance at time t,
- ω is the constant term,
- α_i and β_j represent the model parameters,
- ϵ_{t-i}^2 signifies the squared innovation at time $t-i$,
- σ_{t-j}^2 is the conditional variance at time $t-j$,
- d Is the fractional parameter that characterizes the degree of long memory in the process.

The FIEGARCH model incorporates the exponential term in the conditional variance equation, providing a smoothing effect that can be valuable in capturing long memory patterns in financial time series data. This model offers a flexible framework for simultaneously addressing both fractional integration and exponential smoothing in the context of volatility modeling.

2.5 The Error's Density Models

In the context of error density models, the paper considers different distributions to account for various characteristics observed in financial time series data. Assuming the random variable z follows a standard normal distribution$N(0,1)$, the log-likelihood of the normal distribution Norm L is expressed as:

$$L_{\text{Norm}} = -\frac{1}{2}\sum_{t=1}^{T}[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2] \quad (6)$$

Where T is the number of observations. However, recognizing the inadequacy of assuming normality for economic time series, the paper introduces alternative distributions. To accommodate fat-tailed residuals, the Student-t distribution is incorporated. If the random variable z follows a Student-t distribution$\text{ST}(0,1,\nu)$, the log-likelihood function Stud L is defined as:

$$L_{\text{Stud}} = T\left[\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln[\pi(\nu-2)]\right] - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+\nu)\ln\left(1+\frac{z_t^2}{\sigma_t^2(\nu-2)}\right)\right] \quad (7)$$

Where$2 < \nu \leq \infty$, and$\Gamma(.)$is the gamma function. The Student − t distribution Introduces an additional parameter ν representing the degrees of freedom, capturing the fat-tailed nature of the density.

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To jointly account for excess skewness and kurtosis, the paper includes the skewed Student-t distribution proposed by Lambert and Laurent (2001). If z follows a skewed Student-t distributionSKST$(0,1,k,\nu)$, the log-likelihood SkStL is defined as:

$$L_{\mathrm{SkSt}} = T\left[\ln\Gamma\left(\frac{\nu+1}{2}\right) - \ln\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\ln[\pi(\nu-2)] + \ln\left(\frac{2}{k+\frac{1}{k}}\right) + \ln(s)\right] - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+\nu)\ln\left[1 + \frac{\left(1+(sz_t+m)\right)^2}{(\nu-2)k^{-2I_t}}\right]\right] \quad (8)$$

Where $I_t = 1$ if $z_t \geq \frac{m}{s}$ or $I_t = -1$ if$z_t < \frac{m}{s}$. Here, k is an asymmetry parameter, and $m = m(k,\nu)$ and $s = \sqrt{s^2(k,\nu)}$ are the mean and standard deviation of the skewed Student-t distribution.

These error density models provide a comprehensive framework to capture the distributional characteristics of financial time series data, considering normal, Student-t, and skewed Student-t distributions. The choice among these models depends on the empirical characteristics observed in the data.

2.6 The Value-at-risk and the Expected Shortfall

In this sub-section, the paper proceeds to present the values of Expected Shortfall (ES) and Value-at-Risk (VaR) utilizing a Fractional Integrated Exponential GARCH (FIEGARCH) model with skewed Student-t distribution for innovations. The FIEGARCH model, incorporating both fractional integration and exponential smoothing, is employed to capture the intricate dynamics of volatility. Furthermore, the use of skewed Student-t distribution for innovations allows for the modeling of asymmetry, fat tails, and excess skewness in the financial time series data.

The Expected Shortfall (ES) and Value-at-Risk (VaR) are key metrics in risk management, providing insights into the potential downside risks associated with a given portfolio or financial instrument. These metrics play a crucial role in decision-making processes by quantifying the level of risk exposure.

The utilization of the FIEGARCH model with skewed Student-t distribution underscores a comprehensive approach to risk assessment, considering both the long memory in variance dynamics and the stylized facts associated with financial time series behavior. The ensuing analysis is poised to offer valuable insights into the risk characteristics of the financial data under consideration.

The formulas for calculating Expected Shortfall (ES) and Value-at-Risk (VaR) using a Fractional Integrated Exponential GARCH (FIEGARCH) model with skewed Student-t distribution for innovations.

Expected Shortfall (ES) Formula:

The Expected Shortfall at a certain confidence level α for a distribution can be calculated by taking the conditional expectation of losses exceeding the Value-at-Risk at the same confidence level. Mathematically, ES is expressed as:

$$ES_\alpha = E[L \mid L \geq VaR_\alpha] \quad (9)$$

Where L represents the loss distribution an ↓ $^{\tau}R_\alpha$ is the Value-at-Risk at confidence levelα.

Value-at-Risk (VaR) Formula:

The Value-at-Risk at a certain confidence level α for a distribution represents the maximum potential loss within that confidence level. For a skewed Student-t distribution, VaR can be computed as:

$$VaR_\alpha = -s \cdot \sqrt{\frac{\nu - 2}{\nu}} \cdot t_{\nu,\alpha} \quad (10)$$

Where s is the standard deviation, ν is the degrees of freedom parameter in the skewed Student-t distribution, and $t_{\nu,\alpha}$ is the α-quantile of the Student-t distribution with ν degrees of freedom.

2.7 Test of Accuracy of VaR Model

Back testing the accuracy for the estimated VaR is crucial. The VaR quality estimation depends on the methodology of computation of VaR. Therefore, to investigate the VaR performance we have computed the empirical failure rates for both short and long trading positions. The prescribed probability is ranging from 0.25% to 5%. In reality, the failure rate is the number of times in which returns exceed (in absolute value) the forecasted VaR. If the model is said to be correctly specified, when the failure rate is equal to the specified VaR's level. In our study, the back testing VaR is based on Kupiec (1995) test and the Dynamic Quantile (DQ) test of Engle and Manganelli (2002). In order to test the accuracy and to evaluate the performance of the model-based VaR estimates, Kupiec (1995) provided a likelihood ratio test (LR_{UC}) for testing whether the failure rate of the model is statistically equal to the expected one (unconditional coverage). Consider that $N = \sum_{t=1}^{T} I_t$ is the number of exceptions in the sample size T. Then,

$$I_{t+1} = \begin{cases} 1 & \text{if} \quad r_{t+1} < VaR_{t+1|t(\alpha)} \\ 0 & \text{if} \quad r_{t+1} \geq VaR_{t+1|t(\alpha)} \end{cases} \quad (11)$$

Follows a binomial distribution, $N \sim B(T, \alpha)$. If $p = E(\frac{N}{T})$ is the expected exception frequency (i.e. the expected ratio of violations), then the hypothesis for testing whether the failure rate of the model is equal to the expected one is expressed as follows: $H_0: \alpha = \alpha_0$. α_0 Is the prescribed VaR level. Thus, the appropriate likelihood ratio statistic in the presence of the null hypothesis is given by:

$$LR_{uc} = -2\log\{\alpha_0^N (1 - \alpha_0)^{T-N}\} + 2\log\left\{(\frac{N}{T})^N (1 - (\frac{N}{T}))^{T-N}\right\} \quad (12)$$

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Under the null hypothesis, LR_{uc} has a $\chi^2(1)$ as an asymptotical distribution. Thus, a preferred model for VaR prediction should provide the property that the unconditional coverage measured by $p = E(\frac{N}{T})$ equals the desired coverage level p_0.

Engle and Manganelli developed the Dynamic Quantile (DQ) test building upon a linear regression model based on the process of centered hit function:

$$\delta_t^{\alpha} = \mathrm{Hit}_t(\alpha) \equiv I(y_t < -\mathrm{VaR}_t(\alpha) \mid \Omega_{t-1}) - \alpha \quad (13)$$

Conditional on pre-sample values, the dynamic of the hit function is modeled as:

$$\delta_t^{\alpha} = \theta_0 + \sum_{i=1}^{p} \theta_i \delta_{t=i}^{(\alpha)} + \sum_{\tau=1}^{m} \vartheta_i \delta_{t=i}^{(\tau)} + \mu_t \quad (14)$$

Where μ_t is an IID process. The DQ test is defined under the hypothesis that the regressors in Eq. (27) have no explanatory power:

$$H_0 = \Psi = \left(\theta_0, \theta_1, \dots, \theta_p, \vartheta_0, \vartheta_1, \dots, \vartheta_m\right)^T = 0 \quad (15)$$

For back testing, the DQ test statistic, in association with Wald statics, is as follows:

$$DQ = \frac{\hat{\Psi}^T X^T \hat{\Psi}}{\alpha(1-\alpha)} \xrightarrow{\ell} \chi^2_{1+p+m} \quad (16)$$

Where X denotes he regressors matrix in Eq. (27)

3. DATA AND PRELIMINARY ANALYSIS

The data consist of daily closing prices for two stocks indexes which are Islamic Dow Jones and the Islamic FTSE. The sample period of our study and the number of observation are provided in the table below

Table 1: Data

stock index	***sample period***	***Observations***
Islamic DOW JONES	01/22/2007 – 04/01/2016	2329
Islamic FTSE	01/22/2007 – 04/01/2016	2206

For each series, the log-returns is expressed (in %) as,

$$r_t = 100 * \ln\left(\frac{S_t}{S_{t-1}}\right) \quad (17)$$

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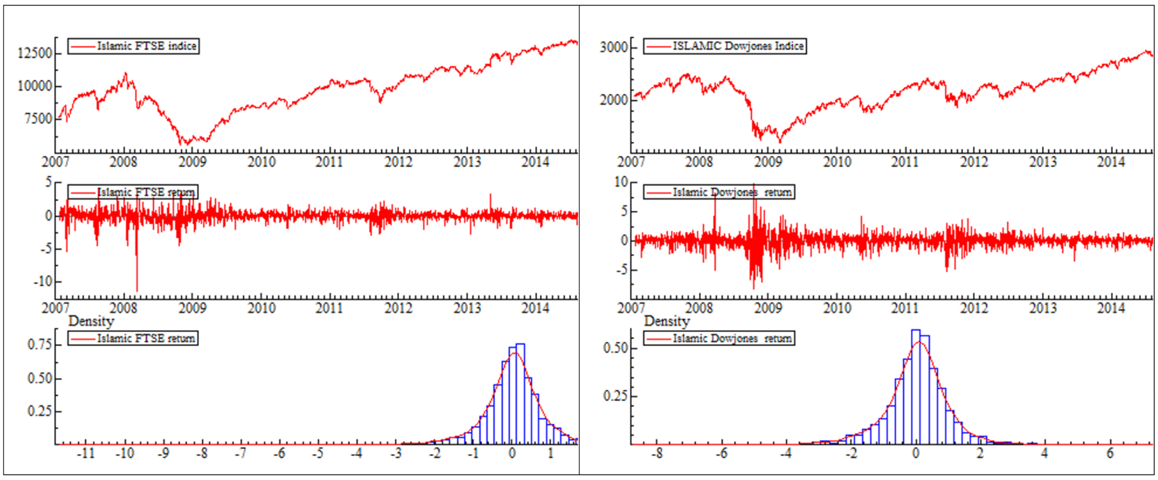
Table 2: Descriptive Statistics

stock index	*Mean*	*Mediane*	*Maximum*	*Minimum*	*SD*	*Skewness*	*Kurtosis*	*Jarque – Béra*
I – DOW JONES	0.0121	0.0649	9.77	−8.18	1.138	−0.311	9.84	9443
I – FTSE	0.0223	0.0541	4.07	−11.32	0.842	−1.508	18.38	31887

Notes: S.D. is the standard deviation. For all the time series, the descriptive statistics for cash daily returns are expressed in percentage.

As it's shown on the table above, these statistics provide insights into the average returns, variability, skewness, and kurtosis of the daily returns for the Islamic Dow Jones and FTSE stock indexes during the specified sample period. The negative skewness and higher kurtosis suggest non-normality and the potential presence of extreme values in the return distributions. The same conclusion is confirmed by the Jarque – Bera statistic which indicates the non-normality of our time series[1].

Graphical Analysis



The graphical analysis presented in Figure 1 reveals a characteristic known as volatility clustering in all sample return series. The presence of volatility clustering is evident from the graphical representation. This phenomenon is characterized by periods of low volatility followed by periods of high volatility. During periods of low volatility, the returns appear to be relatively stable and close to each other. Conversely, during periods of high volatility, the returns exhibit larger fluctuations, indicating increased market uncertainty and variability. The observed volatility clustering confirms the presence of the Autoregressive Conditional Heteroskedasticity (ARCH) effect in the return series. ARCH effect implies that the volatility (or variance) of the returns is not constant over time but varies with past squared returns. Volatility clustering is a key stylized fact in financial time series data, and its observation aligns with empirical findings in financial markets. Investors and analysts often use this information to make informed decisions during periods of heightened market uncertainty.

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3.1 Unit Root and Stationarity Tests

In the analysis of the Islamic Dow Jones (I-DOW JONES) and Islamic FTSE (I-FTSE) stock indexes, unit root tests (Augmented Dickey-Fuller and Phillips-Perron) and stationarity tests (Kwiatkowski-Phillips-Schmidt-Shin) were conducted. The results are presented in Table 3:

Table 3: ADF, PP and KPSS Tests

	ADF	PP	$KPSS$
$I-DOW\ JONES$	-48.81	-68.53	0.3997
$I-FTSE$	-50.55	-73.64	0.2617

Notes: MacKinnon's 1% critical value is –3.435 for the ADF and PP tests. The KPSS critical value is 0.739 at the 1% significance level.

As it is given by the table above, the ADF and PP tests both reject the null hypothesis of the presence of a unit root. The critical values for both tests are well below the test statistics, indicating strong evidence against the existence of a unit root. This implies that the stock index returns time series are stationary after differencing. The KPSS test supports the stationarity of the stock index returns time series. The test statistic is less than the critical value, leading to the rejection of the null hypothesis of non-stationarity. The KPSS test complements the ADF and PP tests by providing evidence in favor of stationarity. The combined results from the ADF, PP, and KPSS tests indicate that the stock index returns time series for both I-DOW JONES and I-FTSE are stationary. This suggests that after differencing, the time series exhibit stable statistical properties, which is essential for reliable time series analysis and modeling.

3.2 Long Memory Tests

The assessment of long-range memory is crucial in understanding the persistence and memory characteristics of financial time series data. The study employed two long-memory tests, namely the log-periodogram regression (GPH) of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric estimate (GSP) of Robinson and Henry (1998). The results are presented in Table 4:

Table 4: Long Range Memory Tests

	$\lvert r_t \rvert$		r_t^2	
	$I-DOW\ JONES$	$I-FTSE$	$I-DOW\ JONES$	$I-FTSE$
$GPH\ Test$				
$m=T^{0.5}$	0.38	0.36	0.39	0.29
$m=T^{0.6}$	0.42	0.33	0.37	0.30
$m=T^{0.7}$	0.41	0.34	0.35	0.228
$GSP\ Test$				
$m=\frac{T}{2}$	0.291	0.266	0.229	0.202
$m=\frac{T}{4}$	0.351	0.312	0.279	0.291
$m=\frac{T}{8}$	0.472	0.398	0.364	0.346

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The table above displays results of long memory tests including two tests which are GPH test for three BANDWITH $m = T^{0.5}$; $m = T^{0.6}$ et $m = T^{0.7}$ and GSP for three BANDWITH $\mathrm{m} = \frac{\mathrm{T}}{2}$; $\mathrm{m} = \frac{\mathrm{T}}{4}$ et $\mathrm{m} = \frac{\mathrm{T}}{8}$. As it is shown, the GPH test results for different bandwidths (m) consistently reject the null hypothesis of short memory.The rejection of the null hypothesis indicates the presence of long-range memory in the time series of absolute returns and daily squared volatility returns for I-DOW JONES and I-FTSE. Similar to the GPH test, the GSP test rejects the null hypothesis of short memory for different bandwidths (m).The rejection of the null hypothesis supports the conclusion that the time series are governed by a long memory process. The results of both GPH and GSP tests consistently suggest the presence of long-range memory in the time series of absolute returns and daily squared volatility returns for I-DOW JONES and I-FTSE. These findings motivate the exploration of fractionally integrated models to better capture the long memory dynamics observed in the data.

4. EMPIRICAL RESULTS

4.1. Estimates GARCH-type Models

Results of GARCH, EGARCH, FIGARCH and FIEGARCH models under normal, Student-t and skewed Student-t distributions are provided in Tables 5–8

Table 5: AR (1)-GARCH (1-1) Model Estimation

	Islamic FTSE			Islamic Dow Jones		
	N	*t*	*skt*	*N*	*t*	*skt*
Cst(M)	0.0481***	0.047***	0.031**	0.048***	0.068***	0.043**
	(0.015)	(0.012)	(0.013)	(0.018)	(0.016)	(0.017)
AR(1)	0.148***	0.121***	0.116***	0.129***	0.113***	0.099***
	(0.024)	(0.021)	(0.021)	(0.022)	(0.019)	(0.02)
Cst(V)	0.013***	0.012***	0.011***	0.016 ***	0.016***	0.014***
	(0.004)	(0.003)	(0.003)	(0.005)	(0.004)	(0.004)
ARCH(Alpha1)	0.154***	0.134***	0.127***	0.109***	0.113***	0.107***
	(0.034)	(0.023)	(0.022)	(0.021)	(0.018)	(0.017)
GARCH(Beta1)	0.838***	0.855***	0.862	0.877***	0.878***	0.883***
	(0.027)	(0.022)	(0.021)	(0.02)	(0.017)	(0.016)
Student(DF)	—	5.100***	5.188***	—	5.514***	5.942***
		(0.565)	(0.583)		(0.647)	(0.76)
Asymmetry	—	—	-0.091***	—	—	-0.114***
			(0.028)			(0.025)
$\ln(\ell)$	-2313.632	-2217.69	-2212.72	-3036.852	-2980.759	-2972.165
AIC	2.103067	2.016951	2.013356	2.613275	2.565944	2.559420

Notes: $\ln(\ell)$is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

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The table 5 presents estimation results for GARCH models under three alternative distributions (normal, Student-t, and skewed Student-t) for Islamic FTSE and Islamic Dow Jones stock indexes. We note that ARCH and GARCH coefficients are consistently positive for all-time series, indicating the presence of volatility clustering and persistence. The condition for the existence of conditional variance, $\alpha_1 + \beta_1 < 1$, holds for both stock returns. This condition is crucial for the GARCH model's stability. The exponential decay of autocorrelations with a decay factor of $\alpha_1 + \beta_1$ supports the short memory nature of the GARCH (1, 1) model, as outlined by Bollerslev (1986). Financial returns are not normally distributed, necessitating the consideration of stylized facts like fat tails and skewness. Estimating GARCH models under different distributions (normal, Student-t, and skewed Student-t) allows for a more accurate representation of the underlying dynamics. Evaluation metrics, such as log-likelihood and the Akaike Information Criterion (AIC), suggest that GARCH models under a skewed Student-t distribution outperform other distributions (normal and Student-t). GARCH models effectively capture the dynamics of the time series data for Islamic FTSE and Islamic Dow Jones stock indexes. The choice of distribution significantly affects the model's ability to address stylized facts, with the skewed Student-t distribution proving most suitable for these financial time series.

Table 6: AR (1)-EGARCH (1-1) Model Estimation

	Islamic FTSE			Islamic Dow Jones		
	N	*t*	*skt*	*N*	*t*	*skt*
Cst(M)	0.023***	0.033	0.015	0.007	0.037***	0.008
	(0.005)	(0.025)	(0.013)	(0.01)	(0.012)	(__)
AR(1)	0.143***	0.120**	0.120***	0.111***	0.104***	0.097
	(0.022)	(0.060)	(0.022)	(0.007)	(0.013)	(__)
Cst(V)	-0.239	-0.824***	-2.038***	0.057	-0.483**	-1.633
	(0.316)	(0.220)	(0.508)	(0.189)	(0.221)	(__)
ARCH(Alpha1)	-0.120	-0.021	-0.038	0.013	0.06	0.08
	(0.207)	(0.224)	(0.213)	(0.286)	(0.268)	(__)
GARCH(Beta1)	0.978***	0.977***	0.979***	0.982***	0.982***	0.981
	(0.008)	(0.007)	(0.007)	(0.006)	(0.006)	(__)
EGARCH(Theta1)	-0.102***	-0.086***	-0.086***	-0.125***	-0.139***	-0.138
	(0.027)	(0.025)	(0.021)	(0.037)	(0.037)	(__)
EGARCH(Theta2)	0.244***	0.213***	0.210***	0.128***	0.13***	0.122
	(0.050)	(0.046)	(0.044)	(0.033)	(0.026)	(__)
Student(DF)	__	5.369***	5.423***	__	6.029***	6.377
		(0.664)	(0.632)		(0.752)	(__)
Asymmetry	__	__	-0.099***	—	–	-0.151
			(0.028)			(__)
ln(ℓ)	-2288.65	-2205.81	-2199.94	-2979.784	-2931.180	-2916.734
AIC	2.082225	2.008366	2.003573	2.565966	2.526902	2.513517

Notes: ln(ℓ)is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

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Table 6 reports the EGARCH model estimates results for our times series under the same three distributions (normal, Student-t and skewed Student-t). In reality, this model considers for leverage effect in volatility, clustering volatility and asymmetry. Those stylized facts are very important since all our time series dynamics support those facts. The results given above showed very clearly that the EGARCH model performs very well compared to the GARCH model. Furthermore, under a skewed Student-t distribution, the EGARCH provides the best adequate model for all our time series.

In reality, return volatility changes quite slowly over time. Indeed, the autocorrelation function decays hyperbolically as shown in Ding, Granger and Engle (1993) among others. Therefore, the effects of a shock can take a considerable time to decay. So, when we consider a stationary process, the propagation of shocks decays very quickly (at an exponential rate). But when the process is a unit root the shocks effect is infinite. Thus, a factionary integrated model can be a good solution to take into account the long memory (long-run dependence.) in the return volatility. Estimates results of long memory GARCH-type models are given in table 7 and 8

Table 7.0: AR (1)-FIGARCH (1-d-1) model estimation

	Islamic FTSE			Islamic Dow Jones		
	N	*t*	*skt*	*N*	*t*	*skt*
Cst(M)	0.050***	0.049***	0.033***	0.044**	0.065***	0.040**
	(0.015)	(0.012)	(0.013)	(0.017)	(0.016)	(0.017)
AR(1)	0.149***	0.124***	0.12***	0.129 ***	0.114***	0.099***
	(0.024)	(0.021)	(0.021)	(0.021)	(0.018)	(0.019)
Cst(V)	0.031**	0.033**	0.03**	0.03**	0.034***	0.028**
	(0.014)	(0.015)	(0.014)	(0.012)	(0.012)	(0.011)
d-Figarch	0.538***	0.408***	0.404***	0.544***	0.601***	0.579***
	(0.119)	(0.069)	(0.07)	(0.148)	(0.128)	(0.113)
ARCH(Phi 1)	0.012	-0.087	-0.055	-0.017	-0.019	-0.013
	(0.128)	(0.177)	(0.179)	(0.088)	(0.064)	(0.061)
GARCH(Beta1)	0.409**	0.193	0.222	0.488**	0.558***	0.547***
	(0.187)	(0.22)	(0.227)	(0.198)	(0.139)	(0.127)
Student(DF)	—	5.209***	5.311***	—	5.529***	5.993***
		(0.548)	(0.572)		(0.651)	(0.784)
Asymmetry	—	—	-0.088***	—	—	-0.124***
			(0.028)			(0.026)
$\ln(\ell)$	-2308.04	-2209.85	-2205.17	-3022.346	-2968.895	-2958.962
AIC	2.098900	2.010751	2.007414	2.601672	2.556610	2.548937

Notes: $\ln(\ell)$is the value of the maximized log-likelihood. AIC Is the Akaike (1974) Information criterion. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

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Table 7 presents the estimation results of the AR (1)-FIGARCH (1-d-1) model under three alternative innovation distributions (normal, Student-t, and skewed Student-t) for Islamic FTSE and Islamic Dow Jones. The results show significant coefficients for these parameters, indicating the importance of considering long memory and volatility clustering in the model. The skewed Student-t distribution shows superiority in capturing the stylized facts of financial time series, including heavy tails and skewness. The estimated fractionally integrated parameter (d) ranges from 0.40 to 0.60, indicating that the time series is governed by a long memory process. This suggests that return volatility changes slowly over time, supporting the idea that shocks can take a considerable time to decay. The log-likelihood and the Akaike Information Criterion (AIC) are used to evaluate model performance. Lower AIC values indicate better model fit. The results suggest that the FIGARCH model, especially under a skewed Student-t distribution, outperforms GARCH and EGARCH models in capturing the dynamics of the time series.

Table 8: AR (1)-FIEGARCH (1-d-1) Model Estimation

	Islamic FTSE			Islamic Dow Jones		
	N	*t*	*skt*	*N*	*t*	*skt*
Cst(M)	0.021	0.023***	0.025*	0.001	0.012	-0.029*
	(0.016)	(0.007)	(0.015)	( )	(0.017)	(0.015)
AR(1)	0.138***	0.125***	0.125***	0.103	0.107***	0.096***
	(0.027)	(0.022)	(0.021)	( )	(0.018)	(0.02)
Cst(V)	0.616	-0.172	-0.111	0.20	-0.375	0.026
	(0.501)	(0.38)	(0.441)	( )	(0.43)	(0.254)
d-Figarch	0.611***	0.648***	0.647***	0.557	0.573***	0.551***
	(0.064)	(0.054)	(0.054)	( )	(0.04)	(0.051)
ARCH(Phil)	0.564	1.067**	1.07**	0.16	-0.002	0.031
	(0.509)	(0.463)	(0.468)	( )	(0.346)	(0.313)
GARCH(Beta1)	0.13	-0.131	-0.129	0.69	0.586***	0.583***
	(0.342)	(0.247)	(0.249)	( )	(0.169)	(0.156)
EGARCH(Theta1)	-0.13***	-0.108***	-0.108***	-0.164	-0.184***	-0.203***
	(0.032)	(0.025)	(0.026)	( )	(0.041)	(0.035)
EGARCH(Theta2)	0.206***	0.192***	0.191***	0.084	0.08***	0.021
	(0.047)	(0.044)	(0.044)	( )	(0.025)	(0.023)
Student(DF)	—	5.492***	5.504***	—	6.812***	7.051***
		(0.703)	(0.706)		(0.994)	(1.132)
Asymmetry	—	—	0.003	—	—	-0.182***
			(0.014)			(0.033)
ln(ℓ)	-2267.49	-2192.69	-2192.66	-2950.503	-2908.441	-2892.280
AIC	2.0639	2.0038	1.9978	2.5416	2.5196	2.5196

Notes: ln(ℓ)is the value of the maximized log-likelihood. AIC Is the Akaike (1974) Information criterion. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

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Table 8 provides the estimation results of the AR (1)-FIEGARCH (1-d-1) model under three alternative innovation distributions (normal, Student-t, and skewed Student-t) for Islamic FTSE and Islamic Dow Jones.

The results show significant coefficients for the FIEGARCH parameters, indicating the importance of considering both leverage effects and long memory in the model.

The model is estimated under three different innovation distributions.

The skewed Student-t distribution continues to demonstrate superiority in capturing the stylized facts of financial time series, including heavy tails and skewness.

The estimated fractionally integrated parameter (d) ranges from 0.4 to 0.6, confirming the presence of long memory in the financial time series.

The long memory phenomenon is significant as the value of d is between zero and one, indicating that return volatility changes quite slowly over time.

The log-likelihood and the Akaike Information Criterion (AIC) are used to evaluate model performance. Lower AIC values indicate better model fit.

The FIEGARCH model outperforms other models, suggesting that it is the most suitable model for capturing the dynamics of the financial time series.

The model considers both leverage effects and long-range memory, providing a more accurate representation of the financial time series compared to simpler models.

4.2 The Value at Risk Analysis

4.2.1 The In-sample VaR Estimation Results

In this sub-section, we estimate the one-day-ahead VaR and ES for the AR (1)-FIEGARCH model under the three alternative innovation's distributions (normal, Student-t and skewed Student-t) for two Islamic stock index returns.

Indeed, we have computed the Kupiec's (1995) LR tests and the Dynamic Quantile (DQ) test of Engle and Manganelli (2002).

The VaR levels range (α) from 0.05 to 0.01 for short and it ranges from 0.95 to 0.99 for the long trading positions. In addition, the Expected Shortfall (ES) is computed for both short and long trading positions for the mentioned levels.

As we knew, the failure rate for the short trading position denotes the percentage of positive returns larger than the VaR prediction. However for the long trading positions, the failure rate is the percentage of negative returns smaller than the VaR prediction.

Those results are reported in the following Table.

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Table 9: In sample VaR Back testing results based on AR (1)-FIEGARCH (1.d.1)

	Panel a.	Islamic Dow Jones indice												
	Short trading position							Long trading position						
	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value	ESF	Quantile	Failure rate	Kupiec LRT	P-value	DQT	Pvalue	ESF
	0.950	0.966	14.191	0.000	18.760	0.008	2.255	0.050	0.063	8.346	0.003	17.502	0.014	-2.208
	0.975	0.983	7.336	0.006	9.507	0.218	2.599	0.025	0.043	27.713	1.40e-007	49.562	1.76e-008	-2.378
	0.990	0.991	0.848	0.357	2.477	0.928	3.151	0.010	0.022	26.499	2.63e-007	42.818	3.61e-007	-2.699
	0.995	0.995	0.036	0.849	0.394	0.999	3.029	0.005	0.012	18.356	1.83e-005	31.388	5.27e-005	-3.025
	0.997	0.997	0.005	0.940	0.097	1.000	3.703	0.0025	0.008	18.674	1.55e-005	32.839	2.83e-005	-3.062
St	0.950	0.965	13.395	0.000	16.904	0.018	2.264	0.050	0.070	17.559	2.78e-005	26.735	0.0003	-2.094
	0.975	0.988	21.352	3.82e-006	17.966	0.012	2.705	0.025	0.037	12.719	0.000	22.943	0.001	-2.363
	0.990	0.996	15.851	6.85e-005	11.536	0.116	3.487	0.010	0.015	6.016	0.014	13.516	0.060	-2.728
	0.995	0.998	9.177	0.002	6.455	0.487	4.662	0.005	0.005	0.011	0.916	0.342	0.999	-3.709
	0.997	0.998	1.667	0.196	1.389	0.985	4.662	0.0025	0.002	0.121	0.727	1.432	0.849	-3.371
Skt	0.950	0.947	0.387	0.533	5.990	0.540	1.959	0.050	0.052	0.387	0.01	6.986	0.430	-2.266
	0.975	0.974	0.011	0.915	4.355	0.738	2.397	0.025	0.024	0.025	0.03	3.813	0.800	-2.661
	0.990	0.991	0.490	0.483	2.548	0.923	3.106	0.010	0.008	0.848	0.787	1.465	0.983	-3.295
	0.995	0.996	0.653	0.419	0.903	0.996	3.109	0.005	0.003	2.169	0.419	2.274	0.943	-3.506
	0.997	0.997	0.121	0.727	0.308	0.999	3.564	0.0025	0.000	3.373	0.940	2.693	0.911	-4.260

	Panel b.	Islamic FTSE indice												
	Short trading position							Long trading position						
	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value	ESF	Quantile	Failure rate	Kupiec LRT	P-value	DQT	Pvalue	ESF
	0.950	0.958	3.361	0.066	14.190	0.047	1.572	0.050	0.051	0.071	0.788	8.742	0.271	-1.757
	0.975	0.979	1.640	0.200	6.470	0.485	1.723	0.025	0.031	3.798	0.051	7.597	0.369	-2.020
N	0.990	0.990	0.198	0.655	1.087	0.993	2.151	0.010	0.016	8.505	0.0033	4.458	1.41e-005	-2.468
	0.995	0.995	0.098	0.753	0.426	0.999	2.389	0.005	0.013	20.291	6.65e-006	56.703	6.84e-010	-2.591
	0.997	0.996	0.986	0.320	1.602	0.978	2.466	0.0025	0.009	22.669	1.92e-006	74.420	1.88e-013	-2.690
	0.950	0.957	2.648	0.103	15.630	0.028	1.553	0.050	0.057	2.268	0.132	15.014	0.035	-1.738
St	0.975	0.980	2.471	0.115	11.608	0.114	1.824	0.025	0.031	3.798	0.051	13.934	0.052	-2.110
	0.990	0.992	1.853	0.173	7.025	0.426	2.397	0.010	0.011	0.675	0.411	17.138	0.016	-2.699
	0.995	0.997	2.760	0.096	2.347	0.938	2.789	0.005	0.004	5.70e-005	0.993	19.071	0.007	-3.280
	0.997	0.999	5.620	0.017	3.756	0.807	3.247	0.0025	0.002	0.042	0.837	0.180	0.999	-4.100
	0.950	0.955	1.249	0.263	13.926	0.052	1.534	0.050	0.052	0.212	0.644	6.926	0.436	-1.752
Skt	0.975	0.981	4.068	0.043	7.630	0.366	1.789	0.025	0.028	0.845	0.357	7.122	0.416	-2.066
	0.990	0.995	10.048	0.001	7.940	0.337	2.396	0.010	0.012	1.494	0.221	25.372	0.0006	-2.682
	0.995	0.997	4.159	0.041	3.433	0.842	2.610	0.005	0.005	0.084	0.771	16.604	0.020	-3.368
	0.997	0.999	5.620	0.017	3.747	0.808	3.247	0.0025	0.003	0.986	0.320	1.332	0.987	-3.675

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Table 9 reports in-sample VaR back testing results for the AR (1)-FIEGARCH (1.d.1) model under three alternative innovation distributions (normal, Student-t, and skewed Student-t) for Islamic Dow Jones and Islamic FTSE. Key elements of the table include quantile values, failure rates, Kupiec's LR tests, p-values, Dynamic Quantile (DQ) tests, p-values, and Expected Shortfall (ES) values for both short and long trading positions at various VaR levels.

Panel a. for Islamic Dow Jones, for both short and long trading position, we note: High failure rates, significant Kupiec LRT, and DQ test results indicate poor performance of the normal distribution.

The Student-t Distribution improved performance compared to the normal distribution but still not satisfactory.

The Skewed Student-t Distribution outperforms other distributions, indicating that it effectively models fat-tailed and skewed returns. Panel b. for Islamic FTSE, we note similar patterns as observed in Islamic Dow Jones, with the skewed Student-t distribution outperforming others for both short and long trading position.

Overall, the normal distribution-based VaR models exhibit poor performance due to the neglect of fat tails and skewness in the return distribution.

The Student-t distribution improves results, but the skewed Student-t distribution consistently outperforms, providing the most satisfactory results for both short and long trading positions.

The ability of the skewed Student-t distribution to capture fat tails and skewness contributes to its superiority in modeling extreme events and tail risk.

These findings emphasize the importance of considering alternative distributions, especially those that account for fat tails and skewness, in improving the accuracy of VaR models.

The skewed Student-t distribution, in particular, proves to be a robust choice for capturing the complexities of financial return distributions.

4.2.2 The out-of-sample VaR Estimation Results

As we know Value-at-Risk target is to quantify the potential losses in a definite horizon. Indeed, VaR model is based on forecasting risk which has to be made for a holding period forecast h.

In our study we have tested the short and long VaR out-of-sample for one day horizon. Therefore, the skewed-Student-t FIEGARCH model under three alternative innovations' distribution was assessed to predict the one-day-ahead VaR.

Indeed, we considered 1000 observations of the out-of-sample. Our forecast updated the FIEGARCH model parameters every 50 observations in the out-of-sample period.

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Table 10: Out-of-sample VaR Back Testing Results based on AR (1)-FIEGARCH (1.d.1)

Panel a	Islamic Dow Jones indice													
	Short trading position							Long trading position						
	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value	ESF	Quantile	Failure rate	Kupiec LRT	P-value	DQT	Pvalue	ESF
N	0.950	0.980	12.392	0.000	10.135	0.181	1.353	0.050	0.063	1.786	0.181	9.379	0.226	-1.516
	0.975	0.994	10.776	0.001	7.763	0.353	1.811	0.025	0.037	2.891	0.089	8.744	0.271	-1.684
	0.990	0.998	4.877	0.027	3.401	0.845	1.325	0.010	0.017	2.548	0.110	13.09	0.069	-1.924
	0.995	1.000	__	0.000	2.53 2	0.924	__	0.005	0.007	0.740	0.389	1.140	0.992	-1.925
	0.997	1.000	__	0.000	1.263	0.989	__	0.0025	0.005	0.731	0.188	5.615	0.585	-1.588
St	0.950	0.986	19.150	1.20e-005	14.000	0.051	1.470	0.050	0.057	0.576	0.447	7.775	0.352	-1.542
	0.975	0.996	14.065	0.000	9.154	0.241	1.725	0.025	0.025	0.012	0.909	4.350	0.738	-1.750
	0.990	1.000	__	0.000	5.090	0.648	__	0.010	0.003	2.401	0.121	1.943	0.962	-1.811
	0.995	1.000	__	0.000	2.532	0.924	__	0.005	0.000	__	0.000	2.532	0.924	__
	0.997	1.000	__	0.000	1.263	0.989	__	0.0025	0.000	__	0.000	1.263	0.989	__
Skt	0.950	0.952	0.061	0.804	6.247	0.511	1.266	0.050	0.045	0.208	0.648	7.634	0.365	-1.642
	0.975	0.982	1.169	0.279	1.725	0.973	1.366	0.025	0.017	0.169	0.279	6.928	0.436	-2.022
	0.990	0.996	2.401	0.121	1.962	0.961	1.725	0.010	0.003	2.401	0.121	1.928	0.963	-1.811
	0.995	1.000	__	0.000	2.532	0.924	__	0.005	0.000	__	0.000	2.532	0.924	__
	0.997	1.000	__	0.000	1.263	0.989	__	0.0025	0.000	__	0.000	1.263	0.989	__

Panel b.	Islamic FTSE indice													
	Short trading position							Long trading position						
	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value	ESF	Quantile	Failure rate	Kupiec LRT	P-value	DQT	Pvalue	ESF
N	0.950	0.966	3.156	0.075	9.632	0.210	1.459	0.050	0.043	0.446	0.504	4.511	0.719	-1.431
	0.975	0.978	0.217	0.640	4.370	0.736	1.609	0.025	0.023	0.029	0.863	5.759	0.568	-1.811
	0.990	0.990	0.0003	0.985	19.668	0.006	1.796	0.010	0.011	0.174	0.676	1.109	0.992	-2.077
	0.995	0.994	0.086	0.768	0.235	0.999	1.994	0.005	0.011	3.474	0.062	6.676	0.463	-2.077
	0.997	0.994	1.731	0.188	2.690	0.912	1.994	0.0025	0.007	3.776	0.051	6.664	0.464	-2.043
St	0.950	0.962	1.748	0.186	11.325	0.125	1.414	0.050	0.051	0.026	0.870	21.692	0.002	-1.428
	0.975	0.984	1.974	0.159	6.206	0.515	1.651	0.025	0.021	0.217	0.640	6.428	0.490	-1.859
	0.990	0.994	0.975	0.323	0.920	0.996	1.994	0.010	0.009	0.0003	0.985	0.221	0.999	-1.971
	0.995	0.998	1.196	0.274	0.975	0.995	2.375	0.005	0.003	0.116	0.733	0.204	0.999	-2.681
	0.997	1.000	__	0.000	1.263	0.989	__	0.001	0.001	0.057	0.809	0.340	0.999	-3.369
Skt	0.950	0.962	1.748	0.186	8.815	0.266	1.469	0.050	0.047	0.061	0.804	8.010	0.331	-1.448
	0.975	0.986	3.034	0.081	8.279	0.308	1.764	0.025	0.021	0.217	0.640	5.189	0.636	-1.817
	0.990	0.994	0.975	0.323	0.940	0.995	1.994	0.010	0.009	0.0003	0.985	0.221	0.999	-1.971
	0.995	0.998	1.196	0.274	0.976	0.995	2.375	0.001	0.003	0.116	0.733	0.224	0.999	-2.681
	10.997	1.000	__	0.000	1.263	0.989	__	0.0025	0.003	0.369	0.543	0.650	0.998	-2.681

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Table 10 provides out-of-sample VaR back testing results based on the AR (1)-FIEGARCH (1.d.1) model for the Islamic Dow Jones and FTSE indices. The table includes various statistics such as quantiles, failure rates, Kupiec's LR tests, p-values, Dynamic Quantile (DQ) tests, p-values, and Expected Shortfall (ES) values for both short and long trading positions at different VaR levels. In panel a, we note that the skewed Student-t distribution consistently outperforms other distributions, providing low failure rates and satisfactory Kupiec LRT and DQ test results for short and long VaR.

The normal distribution shows reasonable performance, indicating that it can be a viable option in out-of-sample scenarios. In panel b we have similar patterns observed as in Panel a, with the skewed Student-t distribution consistently providing better results. The skewed Student-t distribution continues to be the best-performing distribution, showing its effectiveness in capturing fat-tailed and skewed returns. Overall we conclude that, Out-of-sample VaR estimates are similar to in-sample results, with the skewed Student-t FIEGARCH model consistently exhibiting superior performance.

The skewed Student-t distribution demonstrates its ability to improve VaR estimation quality, with the null hypothesis of the correct model not being rejected. Unlike in-sample VaR results, the out-of-sample VaR under a normal distribution performs relatively well, suggesting that, in practical scenarios, the normal distribution may provide acceptable results compared to the Student-t distribution. The conservative nature of the Student-t distribution observed in in-sample results is less pronounced in out-of-sample scenarios.

These findings suggest that the skewed Student-t FIEGARCH model remains effective for out-of-sample VaR estimation, providing accurate and reliable forecasts of potential losses in financial markets. The inclusion of both fat tails and skewness in the model contributes to its robustness and superior performance compared to alternative distributions.

5. CONCLUSION

In conclusion, this paper has undertaken a comprehensive analysis of Value-at-Risk (VaR) and Expected Shortfall (ES) for two Islamic stock index return series. The primary focus was on addressing volatility clustering, a prevalent characteristic in the sample return series. To capture the persistence of volatility, four GARCH-type models, including two fractionally integrated models, were evaluated.

The assessment involved considering three alternative distributions—normal, Student-t, and skewed Student-t distributions. The findings of this study highlight the superiority of the skewed Student-t FIEGARCH model over other models. This superiority is attributed to its capacity to simultaneously account for asymmetry, long memory, and the leverage effect. The computation of VaR for both short and long trading positions, with a one-day horizon, further reinforced the efficacy of the skewed Student-t FIEGARCH model. Back testing results demonstrated the model's robust performance, making it a valuable tool for risk measurement and hedging in financial markets.

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The implications of these findings are significant for risk managers, providing them with insights into the choice of models for measuring and mitigating financial risk. The incorporation of asymmetry, long memory, and leverage effect in the skewed Student-t FIEGARCH model contributes to its accuracy in forecasting potential losses. This research contributes to the broader understanding of risk modeling and management, particularly in the context of Islamic stock indices. Overall, the skewed Student-t FIEGARCH model emerges as a powerful and effective tool for financial risk assessment, offering valuable applications in real-world risk management scenarios.

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