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ON THE GEOMETRY OF MATROIDS AND FLOWS ON THEM

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Abstract

Discrete version of manifolds lead to some combinatorial structures of the manifold and it is this aspect of the problem that lead to rich investigations. Matroids came to fore purely as combinatorial theme with a firm geometry behind it. In our investigations we have considered discrete versions of low dimensional topological manifolds and developed few ideas relating to certain flows on them to locate invariant sets. Mainly the Anasov diffeomorphisms. It will be presented in two parts the first paper will be the description of matriods and their role in Linear algebraic setting and then graph theoretic setting. Not much is dealt when it comes to the geometry but we assume that the underlying manifold is smooth .We have specifically given one interesting case for the stability of the invariant sets. Also we have a family of algebraic varieties to this we want to attach a family of combinatorial objects and these are our convex polytopes

Keywords: Matroids, Topological Manifolds, Convex Polytopes

AMS Classification: 52B40, 52B20, 52B05

1. INTRODUCTION

In this paper we have discussed the combinatorial aspects of Geometry and Topology coming from algebraic geometric background. All the manifolds that we have in our mind are smooth manifolds of dimension utmost four

The problem of classification of topological spaces for 2- dimensional manifolds met with a great success and it was continued for higher dimensional analogus. For case m=3 the Poincare conjucture was an open problem till recently and now that this conjucture was settled by Peralman, around 2000 people started looking at Geometry and topology related problem differently.

So in our paper we have given an exposition to highlight the convexity related sets of Euclidean space R^n. In this context we have discussed complex polytopes. Differential Geometry is something that characterizes topological manifolds into differential manifolds. In this sense any arbitrary smooth space is locally like R^n for some suitable n, the dimension of manifold, under algebraic geometric setting we have an analogus characterization as smooth algebraic varieties. We would like to highlight the convexity of the subsets of R^n in terms of algebraic geometry notions. The examples for such varieties are the unit circle and the unit sphere.

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Loosely speaking the differential geometric view point are 1 & 2 dimensional smooth compact manifolds and sphere being 2-dimensional one which is closed oriented in terms of manifolds which are closed oriented and 2 dimensional they are all classified upto homeomorphism as sphere with k-handles. So a sphere with 1-handle is our Torus which is S_1XS_1 and then one would see them upto homeomorphism by attaching as many handles as one would wish

2. A COMBINATORIAL PROBLEM

We have developed a combinatorial problem for a geometric observation the idea comes from the background of algebraic geometry notions that is of a family of 'Algebraic varieties' It is developed on the following border context

Proposition 2.1: Suppose we have a family of algebraic varieties to this we want to attach a family of combinatorial objects and these are our "Convex polytopes" since they encircle the geometric information about the algebraic varieties.

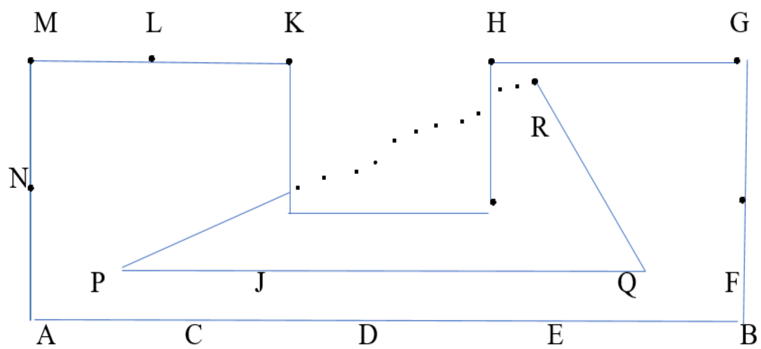
Proposition 2.2: In the algebraic geometry area these problems fall under the category of Modili problems. We will describe this problem in its simplest terms and then focus our attention on convex polytopes.

Remark 2.3: Proposition 2.1 and 2.2 is all about the modili space in lines in R^2 and in general in R^n while R^n is characterized as an n-dimensional Euclidean space and as such any subset in R^n say $E \subset R^n$ under Euclidean setting will enjoy the same status as R^n, over R otherwise, we are interested in an arbitrary set of R^n

Example 2.4: A circle (unit circle) in R^2 an unit sphere S^2 in R^3 these are all Framzr geometric objects and at the same time are the subsets of R^2, and their description in terms of convexity and polytopes, need clarification.

Example 2.5: A set E in R^n is said to be convex if for any two points x, y in E, the line segment joining them given by $\alpha x+(1-\alpha)y$ for α such that $0<\alpha<1$ lies entirely in E.

Example 2.6: If R^n is R^2 ie for n=2 if it is a 2-dimensional plane and imagine a set of R^2 whose vertices are as shown below



$S \subset R^2$

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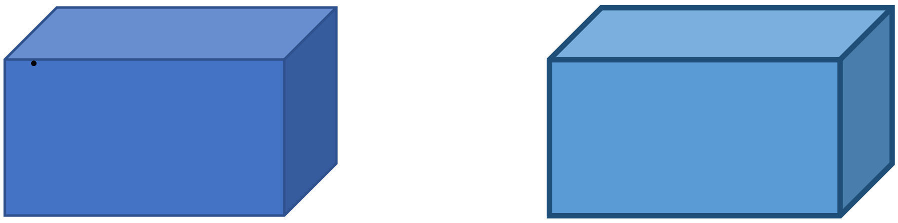
Let P ,Q and R are the interior points of Sin R^2 then the line joining PQ,PR and QR except PR both PQ & QR as line segments completely lie in S But not PR because the dotted lines do not form the part of S joining P & R

Example 2.7: In a 3-dimensional case imagine a set S^1 in R^3 as an unit circle, with a sub part removed as shown in following figure



i) S uint convex cube in R3

ii) Cube S not convex removed unit portion

Remark 2.8: A convex hull is always convex. If A$\subset$ R^n, convA = the 'convex hull' of A is convex

3. ALGEBRAIC VARIETY

Polynomials F(x) over R is an expression of the form $F(x) = a_0+a_1x+\ldots\ldots.a_nx^n$

$a_0, a_1,\ldots\ldots.a_n\in$ R x is an indeterminate

If $\alpha \in$R is such that $f(\alpha)$ =0 then we say that α is a zero if F(x) If $f(x_1,x_2,\ldots\ldots.x_n)$ =0 is a polynomial in n variables over R then $f(\alpha_1,\ldots\ldots\ldots\alpha_n)$ =0 for $(\alpha_1,\ldots\ldots.\alpha_n)\in R^n$

then $\alpha :(\alpha_1,\ldots..\alpha_n)\in R^n$ will form a zero of $f(x_1,\ldots\ldots..x_n)$: f(x) $(x:x_1,\ldots\ldots x_n)$ x_i is a variable if $f(x_1,x_2)$ =0 then $f(\alpha)$=0 implies $\alpha(\alpha1, \alpha2) \in R^2$ & g(X_1,X_2,X_3) =0 then $g(\beta)$ =0 implies $\beta(\beta_1,\beta_2 , \beta_3) \in R^3$. Find all such $\alpha, \beta's$ from the zero set associated with f and g in R^2&R^3. If we dig a bit in detail the underlying set is a polynomial ring R $[X_1,X_2]$ & $R[X_1,\ldots\ldots Xn]$ over R

V(R) : $\{(a_1,\ldots\ldots\alpha_n)$: fi $(\alpha_1,\ldots\ldots\alpha_n)\}$ =0 for all i, , i= 1,2,…….n, $\alpha_i \in R$ } is called the zero set

Recall S_1 the unit circle

As a subset of R^2 $S^1 =\{(x,y)\in R^2, x^2+y^2=1$ or $x^2+y^2-1=0\}$

$S^2 \subset R^3$, $S^2 =\{(x,y,z)\in R^3$ / $x^2+y^2+z^2-1=0\}$

The zero set of S^1 in R^2 (over R) is

$S^1 = \{\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$: t∈R such that (x(t),y(t)) t∈R, $x^2(t) +y^2(t) -1=0\}$These are well known parametrized of the subset of R^2 & R^3 as zero set and the same time a smooth 'algebraic variety'

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Example 3.1: If $y^2 = x^3$ a polynomial in two variables f(x,y) =0 ie $y^2 - x^3 = 0$ over R then V(R) $= \{(t^2, t^3): t \in R\}$ is the zero set . But it is not smooth at (0,0) this has a ……..(see the following fig). In general $y^2 = x^3 + ax + b$ is an algebraic curve (Elliptic curves) an algebraic variety is a rapidly studied geometric objects for variety of applicants ringing from number theory to fluid mechanics. Analytical system to theoretical problems

Remark 3.2: With these pre requisities we want to now understand the combinatorial nature of algebraic genetic objects

Lemma 3.3: We started with objects in algebraic geometry setting as the zero sets, and examples for them were the unit circle, its higher analogue a 2- sphere , Curve given by f(x,y) $= y^2 - x^3 = 0$

Lemma 3.4: Convexity in algebraic geometry is very much noted in the geometric observation which we will highlite. As said earlier, we are not claming any result of our own but the exposition would help to at the Modili problem associated with the Modile space.

Proposition 3.5: Let U be finite set in Z^n Since $Z^n = \{ (m_1, \ldots m_n) : m_i \in Z \;\; i = 1,2 \ldots n\}$ then U may be convex on U may not be convex but its convex hull is convU U is always convex Let $x \in U$ and $\{ \in R^n$ then the dot product x } and its maximum and minimum is attained on the boundary of conv U then we have the following proposition

Proposition 3.6: With U & R^n as above the K- hold sums $\{1/k(x_1 + x_2 + \ldots x_k) : x_i \in U\}$ converge to the convex hull of U as $k \to \infty$

4. CONCLUSION

The discussion is all about the convex polytope their characterization in terms of algebraic varieties as in terms of algebraic geometry as zero sets of polynomial and later we have also presented few characterization involving the subsets of n-dimensional Euclidean space R^n. We have the proposition which clearly sums up as an important problem in algebraic geometry known as Modili problems. Thus we have considered algebraic varieties arising from compact Riemann surface of higher genera.

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