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AN APPROACH FOR SOLVING BILEVEL LINEAR FRACTIONAL PROGRAMMING PROBLEMS USING PENTAGONAL INTUITIONISTIC FUZZY NUMBER

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Abstract

This study presents a novel methodology for addressing intuitionistic fuzzy bilevel linear fractional programming problems (IFBLFPP). The proposed approach utilizes pentagonal intuitionistic fuzzy numbers to represent the cost coefficients of the objective function, resource constraints, and technological coefficients. To solve the IFBLFPP, the problem is first transformed into an intuitionistic fuzzy bilevel linear programming problem (IFBLPP), which is subsequently converted into a crisp bilevel linear fractional programming problem (CBLFPP) through a rigorously defined accuracy function. Several theorems are established to demonstrate that an efficient solution of the CBLFPP also serves as an efficient solution for the IFBLFPP. By applying Zimmermann's technique along with suitable non-linear membership functions, the CBLFPP is further simplified into a single-objective linear programming problem. The practicality and effectiveness of the proposed methodology are illustrated through a numerical example.

Keywords: Pentagonal Intuitionistic Fuzzy Number, Efficient Solution, Linear Membership Function, Bilevel Linear Programming.

1. INTRODUCTION

The Bilevel Linear Fractional Programming Problem (BLFPP) holds significant relevance in various real-world applications, as it facilitates the simultaneous optimization of multiple ratios pertaining to physical and economic quantities. This problem is applicable across diverse fields, including financial and corporate planning (e.g., debt-to-equity ratios, profit-to-investment ratios), production planning (e.g., investment-to-sales ratios), healthcare and hospital management (e.g., cost-to-patient ratios, nurse-to-patient ratios), and university planning (e.g., student-to-teacher ratios). In such problems, it is typically assumed that the coefficients of the objective functions, constraints, and available resources are known with absolute certainty.

However, the coefficients in these scenarios may not always be precise due to various factors such as measurement errors, market fluctuations, or uncontrollable variables like climate, traffic, or customer behavior. This uncertainty often leads decision-makers (DMs) to hesitate when determining the desired levels of the objective function and problem parameters. Consequently, DMs must contend with uncertainty and indecision. Fortunately, intuitionistic fuzzy linear fractional programming provides an effective approach for modeling such scenarios. This method enables the incorporation of imprecise or uncertain information, allowing DMs to make informed decisions under conditions of uncertainty.

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In the realm of linear fractional programming (LFP), a variety of methods have been put forth to tackle the associated problems. Initially, esteemed researchers such as Charne and Cooper [1], Craven [2], and Schaible [3] focused on solving single objective LFP problems. Charne and Cooper introduced a method of variable transformation to address linear fractional problems, while Schaible and Craven offered detailed suggestions for LF optimization. Additionally, Antczak [4] proposed a modified objective method for solving nonlinear multi-objective fractional programming problems. These contributions have significantly advanced the field of LFP problem-solving techniques.

In the field of optimization, the concept of decision-making in a fuzzy environment was first introduced by Bellman and Zadeh [5]. Following this, numerous researchers have explored fuzzy linear fractional programming (FLFP) problems. Notable contributions have been made by Luhandjula [6], Dutta et al. [7,8], Chakraborty and Gupta [9], Pal et al. [10], Bhati and Singh [11], Guzel and Sivri [12], Guzel [13], Li et al. [14], Mehlawat et al. [15], Nachammai et al. [16], Pop and Stancu-Minasian [17], Duran Toksari [18], Jain et al. [19], Stanojević and Stancu-Minasian [20], Das et al. [21,22], Das and Edalatpanah [23], Das and Mondal [24], Mehra et al. [25], and Veeramani and Sumathi [26]. These authors have developed various methodologies for solving FLFP problems. Campos and Muñoz [27] and Zimmermann [28] introduced fuzzy programming techniques to solve crisp multi-objective linear programming problems. Additionally, Campos and Muñoz [27], along with Fortems and Roubens [29], have applied ranking functions to reduce fuzzy multi-objective linear programming problems to crisp equivalents.

In the field of optimization, the pioneering work on decision-making in a fuzzy environment was introduced by Bellman and Zadeh [5]. Since then, numerous scholars have explored fuzzy linear fractional programming (FLFP) problems. Notable contributors to this area include Luhandjula [6], Dutta et al. [7,8], Chakraborty and Gupta [9], Pal et al. [10], Bhati and Singh [11], Guzel and Sivri [12], Guzel [13], Li et al. [14], Mehlawat et al. [15], Nachammai et al. [16], Pop and Stancu-Minasian [17], Duran Toksari [18], Jain et al. [19], Stanojević and Stancu-Minasian [20], Das et al. [21,22], Das and Edalatpanah [23], Das and Mondal [24], Mehra et al. [25], and Veeramani and Sumathi [26]. These researchers have proposed various methodologies for solving FLFP problems. Additionally, Campos and Muñoz, along with Zimmermann, introduced fuzzy programming techniques to address crisp multi-objective linear programming problems. Campos and Muñoz, as well as Fortems and Roubens, successfully converted fuzzy multi-objective linear programming problems into crisp equivalents using ranking functions.

These studies highlight a range of innovative approaches and techniques designed to address the challenges associated with fuzzy multi-objective linear fractional programming (FMOLFP) and related problems. Veeramani and Sumathi [35] introduced a fuzzy approach for solving single-objective fully fuzzy linear fractional optimization problems. Arya and Singh [36, 37], along with Arya et al. [38], proposed a fuzzy method to handle deterministic multi-objective linear fractional (MOLF) optimization problems. Singh and Yadav [39] developed a methodology for solving intuitionistic fuzzy linear fractional programming

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(IFLFP) problems by transforming them into equivalent crisp multi-objective linear fractional optimization problems (MOLFOP). To address these issues using triangular intuitionistic fuzzy parameters, Singh and Yadav [39] employed a fractional programming method originally developed by Charnes and Cooper in 1962 [1], utilizing component-wise optimization techniques to achieve a compromise solution.

These studies contribute valuable insights to the field of fuzzy linear fractional programming and offer practical solutions to the challenges inherent in this specialized area. Veeramani and Sumathi's innovative approach introduces a fuzzy framework that effectively addresses single-objective fully fuzzy linear fractional optimization problems. Arya et al. propose a fuzzy method that successfully tackles deterministic linear fractional optimization problems. Singh and Yadav, in contrast, focus on solving intuitionistic fuzzy linear fractional programming problems by transforming them into crisp multi-objective linear fractional optimization problems. By leveraging the fractional programming method developed by Charnes and Cooper, they effectively manage problems involving triangular intuitionistic fuzzy parameters. Their use of component-wise optimization ultimately results in a balanced compromise solution.

2. DEFINITIONS AND PRELIMINARIES

2.1. Intuitionistic Fuzzy number

Definition 1 ([40]). Let X be a universe of discourse. An intuitionistic fuzzy set (IFS) $\tilde{A}^I$ in X is defined by a set of ordered triple $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), V_{\tilde{A}^I}(x)\right); x \in X\right\}$, where $\mu_{\tilde{A}^I}$, $V_{\tilde{A}^I}$: X → [0 , 1] are functions such that $0 \leq \mu_{\tilde{A}^I}(x) + V_{\tilde{A}^I} \leq 1$, $\forall\ x \in X$. The value of $\mu_{\tilde{A}^I}(x)$ represents the degree of membership of the element x belongs to X being in $\tilde{A}^I$ and $V_{\tilde{A}^I}$ is the degree of non – membership of the element x belongs to X being in $\tilde{A}^I$. π (x) = 1 - $\mu_{\tilde{A}^I}(x) - V_{\tilde{A}^I}(x)$, for all $x \in$ X is called degree of hesitation for $x \in$ X being in $\tilde{A}^I$.

Definition 2 ([39, 41]). An intuitionistic fuzzy set $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), V_{\tilde{A}^I}(x)\right); x \in X\right\}$ is called an intuitionistic fuzzy number (IFN) if the following hold:

(i). There exists m ∈ R such that $\mu_{\tilde{A}^I}(m) = 1$ and $V_{\tilde{A}^I}(m) = 0$ (m is called the mean value of $\tilde{A}^I$) i.e., it is normal.

(ii). $\mu_{\tilde{A}^I}$ and $V_{\tilde{A}^I}$ are piece-wise continuous functions from R to the closed interval [0, 1] and $0 \leq \mu_{\tilde{A}^I}(x) + V_{\tilde{A}^I} \leq 1$, for all $x \in R$, with

$$\mu_{\tilde{A}^I}(x) = \begin{cases} g_1(x), & m - a \leq x < m \\ 1, & x = m \\ h_1(x), & m < x \leq m + b \\ 0, & otherwise \end{cases}$$

Where $g_1(x)$ and $h_1(x)$ are piece-wise continuous, strictly increasing and strictly decreasing function in [m – a, m) and (m, m + b] respectively and

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$$V_{\tilde{A}^I}(x) = \begin{cases} g_2(x), & m - a^1 \le x < m; 0 \le g_1(x) + g_2(x) \le 1 \\ 0, & x = m \\ h_2(x), & m < x \le m + b^1; 0 \le h_1(x) + h_2(x) \le 1 \\ 1, & otherwise \end{cases}$$

Where $g_2(x)$ and $h_2(x)$ are piecewise continuous, strictly decreasing and strictly increasing function in [m − a^1 , m] and [m, m + b^1] respectively.

The IFN $\tilde{A}^I$ is represented by $\tilde{A}^I$ = (m; a, b;a^1, b^1)

Remark 1.

1. IFN is a convex set for the membership function $\mu_{\tilde{A}^I}(x)$ i.e. $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda) x_2) \ge$ min $(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$ for all x_1 , x_2 ∈ R and λ ∈ [0, 1].

2. It is a concave set for the non-membership function $V_{\tilde{A}^I}(x)$ i.e. $V_{\tilde{A}^I}\, x_1 + (1-\lambda)\, x_2) \ge$ max $(V_{\tilde{A}^I}(x_1), V_{\tilde{A}^I}(x_2))$, for all x_1 , x_2 ∈ R and λ ∈ [0, 1].

2.2. Pentagonal Fuzzy number (PFN) and Pentagonal Intuitionistic Fuzzy Number (PIFN)

Inaccuracies in measurement techniques and instrument reliability can lead to difficulties in accurate determining data related to real-world problems. For example, when measuring weather temperature and humidity simultaneously, variations in atmospheric temperature around 34°C can impact humidity levels. This variability gives rise to a unique type of fuzzy number known as the pentagonal fuzzy number. A pentagonal fuzzy number is a 5-tuple subset of the real number set R, characterized by five parameters. Denoted as $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$, where a_3 represents the midpoint, and (a_1, a_2) and (a_4, a_5) denote the left and right endpoints of a_3, respectively. In situations where complexities arise due to disturbances in the environment for various reasons, the phenomenon can be effectively modeled using pentagonal intuitionistic fuzzy numbers.

A pentagonal intuitionistic fuzzy number of a intuitionistic fuzzy set $\tilde{A}^I$ is defined as $\tilde{A}^{PI} = (a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$, where a_i, a_i^I ∈ R,i = 1, 2, 3, 4, 5

Definition 3 (Pentagonal Fuzzy Number (PFN) [41]). A pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ with membership function $\mu_{\tilde{A}}$ (x) defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} w_1\left(\frac{x - a_1}{a_2 - a_1}\right), a_1 \le x \le a_2 \\ 1 - (1 - w_1)\frac{x - a_3}{a_3 - a_2}, a_2 \le x \le a_3 \\ 1, \quad x = a_3 \\ 1 - (1 - w_2)\frac{x - a_3}{a_4 - a_3}, a_3 \le x \le a_4 \\ w_2 \frac{x - a_5}{a_4 - a_5}, a_4 \le x \le a_5 \\ 0, \quad otherwise \end{cases}$$

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Withw_1, w_2 ∈ [0, 1] and should satisfy the following condition:

i. $\mu_{\tilde{A}}$ (x) is a piece wise continuous function having the range of interval [0,1]

ii. $\mu_{\tilde{A}}$ (x) is strictly increasing and continuous function on $[a_1, a_2]$ and $[a_2, a_3]$

iii. $\mu_{\tilde{A}}$ (x) is strictly decreasing and continuous function on $[a_3, a_4]$ and $[a_4, a_5]$ Now, we can redefine pentagonal intuitionistic fuzzy number as follows:

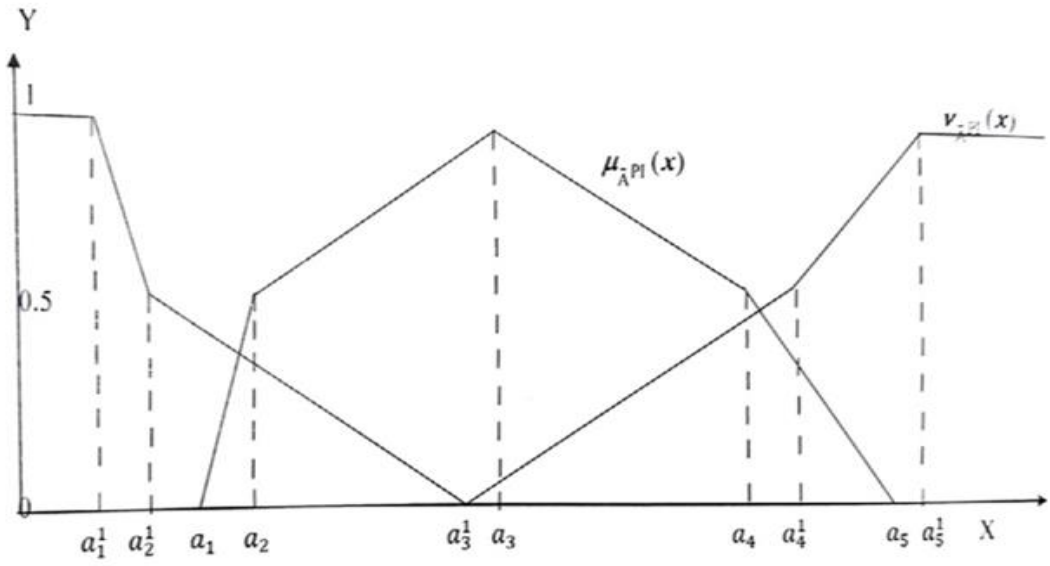

Fig 1: Pentagonal Intuitionistic Fuzzy Number

Definition 4 (Pentagonal Intuitionistic Fuzzy Number (PIFN)). A pentagonal intuitionistic fuzzy number of a intuitionistic fuzzy set $\tilde{A}^I$ is defined as $\tilde{A}^{PI}$ = $(a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$, where $a_i, a_i^I \in$ R,i = 1, 2, 3, 4, 5 and whose membership and non-membership function are defined as

$$M\tilde{A}^{PI} = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_4-x}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ \frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ 0, & x \ge a_5 \end{cases}$$

And

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$$V\tilde{A}^{PI}(x) = \begin{cases} 1, & x < a_1^1 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{a_2^1 - x}{a_2^1 - a_1^1}\right), & a_1^1 \le x \le a_2^1 \\ \frac{1}{2}\left(\frac{a_3^1 - x}{a_3^1 - a_2^1}\right), & a_2^1 \le x \le a_3^1 \\ 0, & x = a_3^1 \\ \frac{1}{2}\left(\frac{x - a_3^1}{a_4^1 - a_3^1}\right), & a_3^1 \le x \le a_4^1 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_4^1}{a_5^1 - a_4^1}\right), & a_4^1 \le x \le a_5^1 \\ 1, & x > a_5^1 \end{cases}$$

The membership function of a PIFN can be shown in Fig. 1.

Definition 5 (Arithmetic Operations on PIFNs). Let $\tilde{A}^{PI} = (a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$ and $\tilde{B}^{PI} = (b_1, b_2, b_3, b_4, b_5; b_1^1, b_2^1, b_3^1, b_4^1, b_5^1)$, be two pentagonal intuitionistic fuzzy number, then the arithmetic operation are as follows:

Addition: $\tilde{A}^{PI} \oplus \tilde{B}^{PI} =$

$(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5; a_1^1 + b_1^1, a_2^1 + b_2^1, a_3^1 + b_3^1, a_4^1 + b_4^1, a_5^1 + b_5^1)$

Subtraction: $\tilde{A}^{PI} \ominus \tilde{B}^{PI} =$

$(a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1; a_1^1 - b_5^1, a_2^1 - b_4^1, a_3^1 - b_3^1, a_4^1 - b_2^1, a_5^1 - b_1^1)$

Multiplication: $\tilde{A}^{PI} \otimes \tilde{B}^{PI} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5; a_1^1 b_1^1, a_2^1 b_2^1, a_3^1 b_3^1, a_4^1 b_4^1, a_5^1 b_5^1)$

Division: $\tilde{A}^{PI} \oslash \tilde{B}^{PI} = \left(\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1}; \frac{a_1^1}{b_5^1}, \frac{a_2^1}{b_4^1}, \frac{a_3^1}{b_3^1}, \frac{a_4^1}{b_2^1}, \frac{a_5^1}{b_1^1}\right)$

Scalar multiplication:

i. $k\tilde{A}^{PI} = (ka_1, ka_2, ka_3, ka_4, ka_5; ka_1^1, ka_2^1, ka_3^1, ka_4^1, ka_5^1)$, k > 0

ii. $k\tilde{A}^{PI} = (ka_5, ka_4, ka_3, ka_2, ka_1; ka_5^1, ka_4^1, ka_3^1, ka_2^1, ka_1^1)$, K < 0.

Definition 6 (Ordering of PIFNs). Let $\tilde{A}^{PI} = (a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$ and $\tilde{B}^{PI} = (b_1, b_2, b_3, b_4, b_5; b_1^1, b_2^1, b_3^1, b_4^1, b_5^1)$ be two PIFNs and we define the ordering based on the components of PIFNs as follows:

1. $\tilde{A}^{PI} \ge \tilde{B}^{PI} \Rightarrow (a_1 \ge b_1, a_2 \ge b_2, a_3 \ge b_3, a_4 \ge b_4, a_5 \ge b_5;$
$a_1^1 \ge b_1^1, a_2^1 \ge b_2^1, a_3^1 \ge b_3^1, a_4^1 \ge b_4^1, a_5^1 \ge b_5^1)$

2. $\tilde{A}^{PI} \le \tilde{B}^{PI} \Rightarrow (a_1 \le b_1, a_2 \le b_2, a_3 \le b_3, a_4 \le b_4, a_5 \le b_5;$
$a_1^1 \le b_1^1, a_2^1 \le b_2^1, a_3^1 \le b_3^1, a_4^1 \le b_4^1, a_5^1 \le b_5^1)$

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3. $\tilde{A}^{PI} = \tilde{B}^{PI} \Rightarrow (\ a_1 = b_1 \ , a_2 = b_2 \ , a_3 = b_3 \ , a_4 = b_4 \ , a_5 = b_5;$

$a_1^1 = b_1^1 \ , a_2^1 = b_2^1 \ , a_3^1 = b_3^1 \ , a_4^1 = b_4^1 \ , a_5^1 = b_5^1)$

4. min $(\tilde{A}^{PI} \ , \tilde{B}^{PI}) = \tilde{A}^{PI}$, if $\tilde{A}^{PI} \leq \tilde{B}^{PI}$ or $\tilde{B}^{PI} \geq \ \tilde{A}^{PI}$

5. max $(\tilde{A}^{PI} \ , \tilde{B}^{PI}) = \tilde{A}^{PI}$, if $\tilde{A}^{PI} \geq \tilde{B}^{PI}$ or $\tilde{B}^{PI} \leq \ \tilde{A}^{PI}$

2.3. Accuracy function and ordering of PIFN

Definition 7. Let $\tilde{A}^{PI} = (a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$ be a PIFN. The score function for the membership function $M\tilde{A}^{PI}$ is denoted by S $(M\tilde{A}^{PI})$ and is defined by S $(M\tilde{A}^{PI}) = (a_1, 2a_2, 3a_3, 2a_4, a_5)/9$. The score function for the non-membership function $V\tilde{A}^{PI}$ is denoted by S $(V\tilde{A}^{PI})$ and is defined by S $(V\tilde{A}^{PI}) = (a_1^1 + 2a_2^1 + 3a_3^1 + 2a_4^1 + a_5^1)/9$. The accuracy function of $\tilde{A}^{PI}$ denoted by f $(\tilde{A}^{PI})$ and

Defined by f $(\tilde{A}^{PI}) = \left(\frac{S\,(M\tilde{A}^{PI}) + S\,(V\tilde{A}^{PI})}{2}\right) = \left(\frac{(a_1 + 2a_2 + 3a_3 + 2a_4 + a_5) + a_1^1 + 2a_2^1 + 3a_3^1 + 2a_4^1 + a_5^1}{18}\right)$

The advantage of the accuracy function lies in its ability to establish an expected interval by utilizing two score functions to evaluate a single quantity. By taking the average of these scores, a more precise approximation can be obtained for comparison purposes. The subsequent theorem demonstrates that the accuracy function operates as a linear function.

Definition 8 (Ordering of Accuracy Function). Let $\tilde{A}^{PI} = (a_1, a_2, a_3, a_4, a_5; a_1^1, a_2^1, a_3^1, a_4^1, a_5^1)$ and $\tilde{B}^{PI} = (b_1, b_2, b_3, b_4, b_5; b_1^1, b_2^1, b_3^1, b_4^1, b_5^1)$ Then

1. f $(\tilde{A}^{PI}) \geq f\ (\tilde{B}^{PI}) \Rightarrow \tilde{A}^{PI} \geq \tilde{B}^{PI}$

2. f $(\tilde{A}^{PI}) \leq f\ (\tilde{B}^{PI}) \Rightarrow \tilde{A}^{PI} \leq f\ \tilde{B}^{PI}$

3. f $(\tilde{A}^{PI}) = f\ (\tilde{B}^{PI}) \Rightarrow \tilde{A}^{PI} = f\ \tilde{B}^{PI}$

4. min $(\tilde{A}^{PI} \ , \tilde{B}^{PI}) \ = \ \tilde{A}^{PI}$, if $\tilde{A}^{PI} \leq \tilde{B}^{PI}$ or $\tilde{B}^{PI} \geq \ \tilde{A}^{PI}$

5. max $(\tilde{A}^{PI} \ , \tilde{B}^{PI}) \ = \tilde{A}^{PI}$, if $\tilde{A}^{PI} \geq \tilde{B}^{PI}$ or $\tilde{B}^{PI} \leq \ \tilde{A}^{PI}$

3. LINEAR FRACTIONAL PROGRAMMING PROBLEM

The general form of LFP problem can be written as

$$\text{Max F (x)} = \frac{\sum_{i=1}^{m} c_i\, x_i + \alpha}{\sum_{i=1}^{m} d_i\, x_i + \beta} = \frac{P\ (x)}{Q\ (x)} \qquad (1)$$

Subject to

$$g_i\ (\text{x}) = \sum_{i=1}^{m} a_{ij}\, x_i \leq \ b_j, j = 1\ , 2\ , 3\ , \ldots\ldots, n_1;$$

$$g_i\ (\text{x}) = \sum_{i=1}^{m} a_{ij}\, x_i \geq \ b_j, j = n_1 + 1, n_1 + 2\ , \ldots \ldots, n_2;$$

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g_i (x) = $\sum_{i=1}^{m} a_{ij}\, x_i = \; b_j, j = n_2 + 1, n_2 + 2\,, \dots \dots, n;$

x = $(x_1\;, x_2\;, \dots\dots, x_m) \geq 0;$

where c_i , d_i , α , β , a_{ij} , b_j ∈ R for i = 1 , 2 , 3 , ……., m , j = 1 , 2 , 2 , 3 , …….. , n ,

x = $(x_1\;, x_2\;, \dots\dots, x_m)$ ∈ R^m .

Let S represent the set of all feasible solutions for equation (1). For a given value of x in S, the expression x ∈ S, Q (x) = $\sum_{i=1}^{m} d_i\, x_i + \beta$ may potentially equal zero. To prevent this scenario, it is necessary to ensure that either Q(x) > 0 for all x in S, or Q(x) < 0 for all x in S.

For the sake of convenience, we will assume that the LFP meets the following condition:

{Q (x) > 0, x ∈ S} (2)

Definition 9: A standard concave-convex fractional programming problem is defined as follows: P(x) is concave on set S with P(τ) ≥ 0 for some τ ∈ S, and Q(x) is convex with Q(x) > 0 for all x ∈ S.

Consider a standard concave-convex fractional program where P(x) is concave and positive for all x in set S, and Q(x) is convex with Q(x) greater than 0.

If in problem (1), Q(x) is concave and positive in set S, and P(x) is concave and negative for each x in S, then -P(x) is convex and positive.

max $_{x \in S} \frac{P\,(x)}{Q\,(x)}$ ⇔ mix $_{x \in S} \frac{-\,P\,(x)}{Q\,(x)}$ ⇔ max $_{x \in S} \frac{P\,(x)}{-\,Q\,(x)}$

The objective is to maximize the function P(x) while minimizing Q(x). This requires maximizing the function G(x) = P(x) - Q(x) under the same constraint as stated in the original problem. Therefore, we are faced with an equivalent problem presented below:

Max G (x) = P (x) − (Q) (3)

Subject to

g_i (x) = $\sum_{i=1}^{m} a_{ij}\, x_i \leq \; b_j, j = 1\,, 2\,, 3\,, \dots\dots, n_1;$

g_i (x) = $\sum_{i=1}^{m} a_{ij}\, x_i \geq \; b_j, j = n_1 + 1, n_1 + 2\,, \dots \dots, n_2;$

g_i (x) = $\sum_{i=1}^{m} a_{ij}\, x_i = \; b_j, j = n_2 + 1, n_2 + 2\,, \dots \dots, n;$

x = $(x_1\;, x_2\;, \dots\dots, x_m) \geq 0;$

where c_i , d_i , α , β , a_{ij} , b_j ∈ R for I = 1 , 2 , 3 , ……., m , j = 1 , 2 , 2 , 3 , …….. , n ,

x = $(x_1\;, x_2\;, \dots\dots, x_m)$ ∈ R^m .

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Definition 10. A solution $\bar{x} \in S$ is an optimal solution of problem (3) iff $G(\bar{x}) \geq G(x)$ for all $x \in S$.

Definition 11. The Bilevel Programming Problem (BPP) is a decision problem in which vector variables x and y are controlled by two decision-makers: the leader and the follower. The variables x (upper level) and y (lower level) are decision variables in this hierarchical optimization structure, commonly found in various applications where the lower level's strategic decisions depend on the upper level's strategic decisions. A typical Bilevel Programming Problem (BPP) can be represented as follows:

$$\min_{x} F(x, y) \tag{4}$$

$$\text{Subject to} \begin{cases} G(x) \leq 0 \\ y \text{ solve} \begin{cases} \min f(x, y) & (BPP) \\ s.t \\ g(x, y) \leq 0 \end{cases} \end{cases}$$

Mathematically, solving a Bi-level Programming Problem (BPP) involves identifying a solution to the problem at the higher level known as the leader's (or ouster's) problem. In this context, for each value of x, y represents the solution to the problem at the lower level, referred to as the follower's (or inner's) problem. Here, $x \in Rn_1$, $y \in Rn_2$; F, f: $Rn_1+n_2 \rightarrow Rm_1$ serve as the objective functions at the upper (and lower) level respectively, while G, g: $Rn_1+n_2 \rightarrow Rm_2$ represent the constraint functions at the upper (and lower) level.

4. INTUITIONISTIC FUZZY BILEVEL LINEAR FRACTIONAL PROGRAMMING PROBLEM

The general form of the Intuitionistic Fuzzy Bilevel Linear Fractional Programming Problem (IFBLFPP) can be expressed as follows:

$$\text{Maximize } \tilde{F}(x) = \left(\frac{\tilde{P}_1(x)}{\tilde{Q}_1(x)}\right) \tag{5}$$

Subject to

$$\text{Maximize } \tilde{F}(x) = \left(\frac{\tilde{P}_2(x)}{\tilde{Q}_2(x)}\right)$$

$$\tilde{g}_i(x) = \sum_{i=1}^{m} \tilde{a}_{ij}\, x_i \leq \tilde{b}_j, j = 1, 2, 3, \ldots\ldots, n_1;$$

$$\tilde{g}_i(x) = \sum_{i=1}^{m} \tilde{a}_{ij}\, x_i \geq \tilde{b}_j, j = n_1 + 1, n_1 + 2, \ldots\ldots, n_2;$$

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$\tilde{g}_i$ (x) = $\sum_{i=1}^{m} \tilde{a}_{ij}\, x_i = \tilde{b}_j, j = n_2+1, n_2+2, \dots \dots, n;$

x = $(x_1, x_2, \dots\dots, x_m) \geq 0;$

where $\tilde{P}_i$ (x) = $\sum_{i=1}^{m} \tilde{C}_{ir}\, x_i \oplus \tilde{\alpha}_r$, $\tilde{Q}_r$ (x) = $\sum_{i=1}^{m} \tilde{d}_{ir}\, x_i \oplus \tilde{\beta}_r$, r = 1, 2 , ……. , K and $\tilde{C}_{ir}$, $\tilde{d}_{ir}$, $\tilde{\alpha}_r$, $\tilde{\beta}_r$, $\tilde{a}_{ij}$, $\tilde{b}_j$, i = 1 , 2 , …….. , m and j = 1 , 2 , ……n are PIFNs.

x = $(x_1, x_2, \dots\dots, x_m) \in R^m$.

4.1. Linearization of (IFBLFPP) to (IFBOLAFPP)

To optimize our results, it is necessary to maximize the function $\tilde{G}_r$ (x) = $\tilde{P}_r$ (x) − $\tilde{Q}_r$ (x) for each value of r ranging from 1 to K, while adhering to the constraint outlined in problem (5) that requires $\tilde{Q}_r$ (x) to be greater than 0.

This leads us to an equivalent problem presented below:

Maximize $\tilde{G}$ (x) = ($\tilde{G}_1$ (x)) (6)

Subject to

Maximize $\tilde{G}$ (x) = ($\tilde{G}_2$ (x))

$\tilde{g}_i$ (x) = $\sum_{i=1}^{m} \tilde{a}_{ij}\, x_i \leq \tilde{b}_j, j = 1, 2, 3, \dots\dots, n_1;$

$\tilde{g}_i$ (x) = $\sum_{i=1}^{m} \tilde{a}_{ij}\, x_i \geq \tilde{b}_j, j = n_1+1, n_1+2, \dots \dots, n_2;$

$\tilde{g}_i$ (x) = $\sum_{i=1}^{m} \tilde{a}_{ij}\, x_i = \tilde{b}_j, j = n_2+1, n_2+2, \dots \dots, n;$

x = $(x_1, x_2, \dots\dots, x_m) \geq 0;$

where $\tilde{G}_r$ (x) = $\tilde{P}_r$ (x) - $\tilde{Q}_r$ (x) , $\tilde{P}_r$ (x) = $\sum_{i=1}^{m} \tilde{C}_{ir}\, x_i \oplus \tilde{\alpha}_r$, $\tilde{Q}_r$ (x) = $\sum_{i=1}^{m} \tilde{d}_{ir}\, x_i \oplus \tilde{\beta}_r$, r = 1, 2 , ……. , K and $\tilde{C}_{ir}$, $\tilde{d}_{ir}$, $\tilde{\alpha}_r$, $\tilde{\beta}_r$, $\tilde{a}_{ij}$, $\tilde{b}_j$, i = 1 , 2 , …….. , m and j = 1 , 2 , ……n are PIFNs.

x = $(x_1, x_2, \dots\dots, x_m) \in R^m$.

Definition 12. Let S represent the set of feasible solutions for equation (6) (equivalent to equation (5)). An efficient solution $\bar{x} \in$ S is defined as a solution of equation (6) where there does not exist another solution x* ∈ S such that $\tilde{G}_r$ (x*) ≥ $\tilde{G}_r$ ($\bar{x}$), r = 1, 2,…., K and $\tilde{G}_r$ (x*) > $\tilde{G}_r$ ($\bar{x}$), for at least one r. This criterion establishes the efficiency of a solution within the set S.

4.2 Steps for Solve Linear Bilevel Programming Problem (LBPP)

Step1: Constraint region of the BLPP:

S = {(x, y) : x ∈ X, y ∈ Y, A1 x + B1 y ≤ b1, A2 x + B2 y ≤ b2}

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Step2: Follower's feasible set for each fixed x ∈ X:

S(x) = {y ∈ Y : B2 y ≤ b2 − A2x}

Step3: Follower's rational reaction set:

P(x) = {y ∈ Y: y ∈ argmin [f (x, ý): ý ∈ S(x)]}

Step4: Inducible Region:

IR = {(x, y) ∈ S, y ∈ P(x)}

Step5: When S and P(x) are non-empty, the BLPP can be written as:

min {F (x, y): (x, y) ∈ IR}

4.3. Conversion of (IFBOLPP) to (crisp BOLFPP)

Now using the accuracy function the model in (6) is transformed to the following crisp BOLFPP.

Maximize G(x) = $((G_1$ (x)) (8)

Subject to

Maximize G(x) = $(G_2$ (x))

$$g_i\,(x) = \sum_{i=1}^{m} a'_{ij}\, x_i \le b'_j, j = 1, 2, 3, \ldots\ldots, n_1;$$

$$g_i\,(x) = \sum_{i=1}^{m} a'_{ij}\, x_i \ge b'_j, j = n_1 + 1, n_1 + 2, \ldots\ldots, n_2;$$

$$g_i\,(x) = \sum_{i=1}^{m} a'_{ij}\, x_i = a'_j, j = n_2 + 1, n_2 + 2, \ldots\ldots, n;$$

$$x = (x_1, x_2, \ldots\ldots, x_m) \ge 0;$$

where G_r (x) = P_r (x) - Q_r (x) , P_r (x) = $\sum_{i=1}^{m} C'_{ir}\, x_i + \alpha'_r$, Q_r (x) = $\sum_{i=1}^{m} d'_{ir}\, x_i + \beta'_r$ and the accuracy functions values as f $(\tilde{C}_{ir}) = C'_{ir}$f $(\tilde{d}_{ir}) = d'_{ir}$, f $(\tilde{\alpha}_r) = \alpha'_r$, f $(\tilde{\beta}_r) = \beta'_r$, f $(\tilde{a}_{ij}) = a'_{ij}$, f $(\tilde{b}_j) = b'_j$ for r = 1 , 2 , …….. K.

To efficiently solve the crisp Bilevel linear programming problem (8), we will follow the steps outlined below:

Step 1: Begin by solving the crisp Bilevel linear programming problem one objective function at a time, while considering all constraints and disregarding any other objective functions. Repeat this process k times for k different objective functions. Let the solutions obtained be denoted as X_1, X_2, ….. , X_K respectively. Define X as the set {X_1 , X_2 , ….. , X_K}. By following this systematic approach, we can effectively tackle the crisp Bilevel linear programming problem and arrive at optimal solutions for each objective function.

Step 2. Find the value of the objective function$G_r\,(x)$, = $P_r\,(x) - Q_r\,(x)$, r = 1, 2,……,K at

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each point in X. Form a payoff matrix.

Step 3. Find the minimum and maximum value of each objective function. Let L_r be the minimum and U_r be the maximum value of $P_r\ (x)$ - $Q_r\ (x)$ i.e. L_r = min {$P_r\ (x) - Q_r\ (x)$: x ∈ X} and U_r = max {$P_r\ (x) - Q_r\ (x)$: x ∈ X}.

Step 4. In order to maximize the objective function $G_r\ (x) = P_r\ (x) - Q_r\ (x)$, r = 1, 2,……, K, where r = 1, 2,……, K, it is essential to strive towards reaching the upper bound for each objective. As the solution approaches the upper bound, the satisfaction level of the Decision Maker (DM) will increase. Ultimately, the DM will be fully satisfied when the objectives reach their upper bounds. Let μ_(U_r) represent the degree of attainability of the upper bound U_r for the objective function $G_r\ (x) = P_r\ (x) - Q_r\ (x)$.

Then $\mu_{U_r}(G_r)$ = 1, 2 ,……, K is defined as

$$\mu_{U_r}(G_r(x)) = \begin{cases} 0, & G_r(x) < L_r \\ \frac{(G_r(x))^t - L_r^t}{U_r^t - L_r^t}, & L_r \le G_r\ (x) \le U_r \\ 1, & G_r\ (x) > U_r \end{cases}$$

where t > 0 is prescribed by the DM.

Our current challenge is to enhance the satisfaction level of the decision maker while adhering to the specified constraint. To achieve this, we will implement Zimmermann's technique.

4.4. Zimmermann's technique

Let λ = min {$\mu_{U_r}(G_r(x))$, r = 1, 2}.

The Balanced Objective Linear Programming Problem (BOLPP) seeks to develop an optimal plan that maximizes the decision maker's (DM) satisfaction by achieving a balanced consideration of both objectives and constraints. The primary goal is to attain a high degree of equilibrium between the objectives and the associated constraints. This concept can be represented by the following model:

max λ

Subject to

$\mu_{U_r}(G_r(x)) \ge \lambda$, r = 1, 2 ;

$\sum_{i=1}^{m} a_{ir}\, x_i \le b_j, j = 1\,, 2\,, \ldots\ldots, n_1;$

$\sum_{i=1}^{m} a_{ir}\, x_i \ge b_j, j = n_1 + 1, n_1 + 2\,, \ldots \ldots, n_2;$

$\sum_{i=1}^{m} a_{ir}\, x_i = b_j, j = n_2 + 1, n_2 + 2\,, \ldots \ldots, n;$

Thus, the problem is reduced to the following single objective NLPP or LPP which can be solved easily by suitable crisp NLP or crisp LPP method.

max λ

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Subject to

$(G_r(x))^t$ - $L_r^t \geq \lambda$ (U_r^t - L_r^t), r = 1,2;

$\sum_{i=1}^{m} a_{ir}\, x_i \leq b_j, j = 1\,, 2\,, \ldots\ldots, n_1;$

$\sum_{i=1}^{m} a_{ir}\, x_i \geq b_j, j = n_1 + 1, n_1 + 2\,, \ldots\ldots, n_2;$

$\sum_{i=1}^{m} a_{ir}\, x_i = b_j, j = n_2 + 1, n_2 + 2\,, \ldots\ldots, n;$

$x_i \geq 0$, i = 1, 2 , ……… , m.

5. APPROACH for (IFBOFLPP)

The solution technique discussed in Section 6 can be summarized as an algorithm as given below:

Step 1: Begin by modeling the intuitionistic fuzzy Bilevel linear fractional programming problem (IFBOFLPP) involving PIFNs, as outlined in problem (5).

Step 2: Proceed to convert the IFBOFLPP from problem (5) to the IFBOLPP as described in (6).

Step 3: Utilize the accuracy function on problem (6) to determine the corresponding CBLFPP (8).

Step 4: Solve the CBLFPP (8) by focusing on one objective function at a time and disregarding all others. Repeat this process k times for k different objective functions. Let the solutions obtained be denoted as X1, X2 respectively, and let X = {X_r : r = 1, 2}.

In this manner, we can systematically address and solve the complex challenges presented by the intuitionistic fuzzy Bilevel linear fractional programming problem, ensuring a thorough and effective approach to finding optimal solutions.

Step 5. Find the value of objective function $G_r\,(x)$ = $P_r\,(x) - Q_r\,(x)$, r = 1, 2 at each point obtained in Step 4.

Step 6. Find the minimum maximum value of each objective function.

Let L_K = min {$P_r\,(x) - Q_r\,(x)$: x ∈ X} and U_r = max {$P_r\,(x) - Q_r\,(x)$: x ∈ X}.

Step 7. Using Zimmermann's technique discussion in Step 6 transfer the CBLFPP into single objective NLPP or LPP.

Step 8. Using any method or software solves the problem.

Table 1: Payoff Matrix

	G_1	G_2
x^1	8.66	4.60
x^2	8.32	7.34
x^3	0.40	0.27

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6. COMPUTATIONAL WORK

A numerical example with two objective functions, three constraints and three variables are considered to illustrate the solution procedure Sahoo et el., [42].

$$\text{Max } \tilde{F}_1(x) = \frac{\tilde{6}x_1+\tilde{7}x_2+\tilde{5}x_3+\tilde{4}}{\tilde{7}x_1+\tilde{5}x_2+\tilde{4}x_3} \quad (9)$$

Subject to

$$\text{Max } \tilde{F}_2(x) = \frac{\tilde{7}x_1+\tilde{8}x_2+\tilde{9}x_3}{\tilde{5}x_1+\tilde{4}x_2+\tilde{6}x_3+\tilde{3}}$$

$\tilde{4}x_1 + \tilde{5}x_2 + \tilde{7}x_3 \geq \tilde{4}$

$\tilde{5}x_1 + \tilde{8}x_2 + \tilde{6}x_3 \leq \widetilde{20}$,

$\tilde{6}x_1 + \tilde{7}x_2 + \tilde{4}x_3 = \widetilde{15}$,

Where $\tilde{3}$ = (1, 2, 3, 4, 5; 1.3, 2.6, 3, 4.1, 5.1), $\tilde{4}$ = (2, 3, 4, 5, 6; 2.2, 3.2, 4, 6, 7)

$\tilde{5}$ = (4, 4.5, 5, 6.2, 7; 3.5, 4, 5.5, 6.5, 7), $\tilde{6}$ = (4, 5, 6, 7, 8; 3, 4, 6, 8, 9),

$\tilde{7}$ = (5, 6, 7, 10, 12; 3, 5, 7, 9, 13), $\tilde{8}$ = (6, 7, 8, 9, 10; 5, 6, 8, 10, 11),

$\tilde{9}$ = (7, 8, 9, 10, 12; 6.1, 7, 9.5, 11, 13)

$\widetilde{20}$ = (17.4, 18, 20, 21, 22; 18.4, 19, 20, 22, 24),

$\widetilde{15}$ = (13, 14, 15, 16, 18; 12.6, 13, 15, 17, 20)

Using method (6) the above problem converted to

$$\text{Max } \tilde{G}_1(x) = (\tilde{6}x_1 + \tilde{7}x_2 + \tilde{5}x_3 + \tilde{4}) - (\tilde{7}x_1 + \tilde{5}x_2 + \tilde{4}x_3) \quad (10)$$

Subject to

$$\text{Max } \tilde{G}_2(x) = (\tilde{7}x_1 + \tilde{8}x_2 + \tilde{9}x_3) - (\tilde{5}x_1 + \tilde{4}x_2 + \tilde{6}x_3 + \tilde{3})$$

$\tilde{4}x_1 + \tilde{5}x_2 + \tilde{7}x_3 \geq \tilde{4}$,

$\tilde{5}x_1 + \tilde{8}x_2 + \tilde{6}x_3 \leq \widetilde{20}$

$\tilde{6}x_1 + \tilde{7}x_2 + \tilde{4}x_3 = \widetilde{15}$

By using accuracy function and simplifying, Problem (10) reduces to

$$\text{Max } G_1(x) = -1.5x_1 + 2.2x_2 + 1.1x_3 + 4.2, \quad (11)$$

Subject to

$\text{Max } G_2(x) = 2.2x_1 + 3.8x_2 + 3.2x_3 - 3.1$,

$4.2x_1 + 5.3x_2 + 7.5x_3 \geq 4.2$,

$5.3x_1 + 8x_2 + 6x_3 \leq 20.1$

$6x_1 + 7.5x_2 + 4.2x_3 = 15.2$

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Solving each objective function w.r.t. all constraints in problem (11) at a time, we get the following:

(i) For the first objective function, the ideal solution is obtained as:

x_1= 0.000000, x_2 = 2.026667, x_3 = 0.000000 and G_1 = 8.658667

(ii) For the second objective function, the ideal solution is obtained as:

x_1 = 0.000000, x_2 = 0.5947368, x_3 = 2.557018 and G_2 = 7.342456

A pay-off matrix is formulated as (see Table 1).

From the pay-off matrix (Table 1) lower bound and upper bound are estimated as

$L_1 = 0.4,\ U_1 = 8.65,\ L_2 = 0.27,\ U_2 = 7.34,$

The membership functions of the objectives G_1 and G_2 are defined as:

$$\mu_{G_1}(x) = \begin{cases} 0, & G_1(\mathrm{x}) < 0.4 \\ \frac{(G_1(\mathrm{x}))^t - (0.4)^t}{(8.65)^t - (0.4)^t}, & 0.4 \le G_1(\mathrm{x}) \le 8.65 \\ 1, & G_1(\mathrm{x}) > 8.65 \end{cases}$$

$$\mu_{G_2}(x) = \begin{cases} 0, & G_2(\mathrm{x}) < 0.27 \\ \frac{(G_2(\mathrm{x}))^t - (0.27)^t}{(7{,}34)^t - (0.27)^t}, & 0.27 \le G_2(\mathrm{x}) \le 7.34 \\ 1, & G_2(\mathrm{x}) > 7.34 \end{cases}$$

By Zimmermann's approach, Problem (11) reduces to

Subject to

$G_1(\mathrm{x}) \ge \lambda,\ G_2(\mathrm{x}) \ge \lambda,$

$4.2x_1 + 5.3x_2 + 7.5x_3 \ge 4.2,$

$5.3x_1 + 8x_2 + 6x_3 \le 20.1,$

$6x_1 + 7.5x_2 + 4.2x_3 = 15.2,$

$x_1, x_2, x_3 \ge 0.$

Thus, problem is reduced to the following single objective NLPP:

$$\max \quad \lambda \qquad (12)$$

Subject to $(-1.5x_1 + 2.2x_2 + 1.1x_3 + 4.2)^{t} - (0.4)^{t} \ge \lambda\,(8.65^{t} - 0.4^{t}),$

$(2.2x_1 + 3.8x_2 + 3.2x_3 - 3.1)^{t} - (0.27)^{t} \ge \lambda\,(7.34^{t} - 0.27^{t}),$

$4.2x_1 + 5.3x_2 + 7.5x_3 \ge 4.2,$

$5.3x_1 + 8x_2 + 6x_3 \le 20.1,$

$6x_1 + 7.5x_2 + 4.2x_3 = 15.2$

$x_1, x_2, x_3 \ge 0.$

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Case 1: Taking t = 0.25 and using LINGO software, the optimal solution of problem (12), is obtained as x_1 = 1.081, x_2 = 0, x_3 = 2.069 with satisfaction level λ = 0.7453.

Case 2: Taking t = 0.5 and using LINGO software, the optimal solution of problem (12), is obtained as x_1 = 0.910, x_2 = 0, x_3 = 2.234 with satisfaction level λ = 0.7145.

Case 3: Taking t = 1 and using LINGO software, the optimal solution of problem (12), is obtained as x_1 = 0.831, x_2 = 0, x_3 = 2.431 with satisfaction level λ = 0.6882.

Case 4: Taking t = 1.5 and using LINGO software, the optimal solution of problem (12), is obtained as x_1 = 0.721, x_2 = 0, x_3 = 2.598 with satisfaction level λ = 0.5358.

Case 5: Taking t = 2 and using LINGO software, the optimal solution of problem (12), is obtained as x_1 = 0.674, x_2 = 0, x_3 = 2.651 with satisfaction level λ = 0.4775.

For different values of t, the comparative study of the obtained fuzzy optimal solutions is given in Tables 2 to 6. For t = 2 we obtained a better solution for the leader objective, whereas for t = 0.25, we find better solution for follower objective.

Table 2: Fuzzy optimal solution for t = 0.25.

X = (1.081, 0, 2.069)	$\tilde{F}_1$(x) = (0.586, 0.739, 1.416, 2.055, 3.162; 0.448, 0.717, 1.382, 2.376, 4.03)	F_1(x) = 1.584
X = (1.081, 0, 2.069)	$\tilde{F}_2$(x) = (0.684, 0.915, 1.258, 1.830, 2.780; 0.508, 0.719, 1.276, 2.140, 3.631)	F_2(x) =1.268

Table 3: Fuzzy optimal solution for t = 0.5.

X = (0.910, 0, 2.234)	$\tilde{F}_1$(x) = (0.591, 0.857, 1.334, 2.045, 3.149; 0.457, 0.723, 1.404, 2.356, 4.009)	F_1(x) = 1.577
X = (0.910, 0, 2.234)	$\tilde{F}_2$(x) = (0.690, 0.923, 1.265, 1.827, 2.783; 0.516, 0.724, 1.288, 2.159, 3.653)	F_2(x) = 1.476

Table 4: Fuzzy optimal solution for t = 1.

X = (0.831, 0, 2.431)	$\tilde{F}_1$(x) = (0.613, 0.884, 1.360, 2.108, 3.290; 0.475, 0.736, 1.438, 2.384, 4.017)	F_1(x) = 1.612
X = (0.831, 0, 2.431)	$\tilde{F}_2$(x) = (0.700, 0.934, 1.274, 1.823, 2.786; 0.528, 0.731, 1.305, 2.187, 3.687)	F_2(x) = 1.488

Table 5: Fuzzy optimal solution for t = 1.5.

X = (0.721,0, 2.598)	$\tilde{F}_1$(x) = (0.660, 0.306, 1.351, 2.151, 3.412; 0.483, 0.745, 1.465, 2.405, 4.022)	F_1(x) = 1.632
X = (0.721, 0, 2.598)	$\tilde{F}_2$(x) = (0.706, 0.941, 1.280, 1.820, 2.789; 0.534, 0.737, 1.317, 2.207, 3.712)	F_2(x) = 1.498

Table 6: Fuzzy optimal solution for t = 2.

X = (0.674, 0, 2.651)	$\tilde{F}_1$(x) = (0.657, 0.924, 1.329, 2.878, 3.439; 0.004, 0.811, 1.475, 2.413, 4.024)	F_1(x) = 1.771
X = (0.674, 0, 2.651)	$\tilde{F}_2$(x) = (0.716, 0.943, 1.281, 1.811, 2.798; 0.546, 0.731, 1.342, 2.295, 3.822)	F_2(x) = 1.591

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7. CONCLUSION

This paper presents a novel approach to addressing the intuitionistic fuzzy bilevel linear fractional programming problem (IFBLFPP). The proposed method involves transforming the IFBLFPP into an intuitionistic fuzzy bilevel linear programming problem (IFBLPP), which is subsequently converted into a crisp bilevel linear programming problem (CBLPP) using a rigorously defined accuracy function. To further simplify the problem, Zimmermann's technique is employed in conjunction with appropriate nonlinear membership functions, reducing the CBLPP to a single-objective linear programming problem (CBLFPP), which can be efficiently solved using a suitable linear programming algorithm.

To demonstrate the effectiveness of the proposed methodology, a numerical example featuring two objective functions involving pentagonal intuitionistic fuzzy numbers (PIFNs) is provided. Additionally, a comparative analysis is conducted, examining various nonlinear membership functions and different values of time parameter ttt. This analysis offers valuable insights into the approach's performance.

In the future, this methodology could potentially be extended to address more complex problems, such as the intuitionistic fuzzy bilevel nonlinear fractional programming problem.

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